Forecasting Cause-Specific Mortality Rates Using the Insights from the Cointegration Analysis

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ABSTRACT

Much like the all-cause mortality, cause-specific mortality rates in countries with similar socio-economic characteristics are likely to follow comparable development patterns. They are also not expected to substantially diverge in the future. We propose to assess the coherence of the past country-specific experiences by the means of the cointegration analysis applied to the mortality time trends extracted by country and cause of death. Indeed, should the time trends of two countries be cointegrated, this would indicate there existed a long-run stationary relation between them, and so, the mortality patterns of these countries were linked to each other in their long-term development. We analyze the data from five developed Western European countries (France, Italy, Netherlands, Spain, and England and Wales), two sexes, and split the mortality rates into five main groups of causes of death (Infectious&Parasitic, Cancer, Circulatory diseases, Respiratory diseases, and External causes). We observe that while in many cases the cause-specific time trends are indeed cointegrated, this is not always the case in spite of the closeness of the studied countries. Further, once we include the countries having the cointegrated time trends in a multipopulational context, such as the Li-Lee mortality model, the forecast results are improved in comparison with the basic Lee-Carter approach.

KEYWORDS

Causes of death, coherence, cointegration, Lee-Carter model, Li-Lee model

1 Introduction

The difficulty to project the mortality rates due to the ongoing increases of life expectancy and the uncertainty related to these increases is a well-known topic in actuarial science. Due to its importance for pension providers and social security systems, this question has attracted a lot of attention from researchers and practitioners alike. A review of the existing models can be found, for example, in Booth and Tickle (2008) including their references. Still, new approaches continue to be developed with an objective to improve our understanding of the mortality rates' evolution as well as the quality of their forecasts.

Among the stochastic mortality models, the Lee-Carter model (Lee and Carter, 1992) together with its various extensions is possibly one of the most widely known and used. Initially applied to the all-cause mortality in a single population context, it was enhanced by different authors to take into account the cohort effects as well as to incorporate multiple populations: an impressive genealogy of the models is provided in Cairns (2013) and an overview of the multipopulational extensions in Villegas et al. (2017).

Originally, modelling efforts concerned the all-cause mortality for the simple reason that these were the only data at hand. As the amount of available statistics grew with time, modelling the cause-specific mortality rates became possible. In this regard, there are two aspects to consider. On the one hand, it seems a natural step to disaggregate the total mortality by causes of death when one knows that the cause-specific mortality rates had vastly different development patterns in the past. Also, as mentioned in Tabeau et al. (1999), while it is practically impossible to make empirically valid assumptions for the total mortality due to the extremely large number of the mortality determinants, trends in the cause-specific mortality can be linked with the risk factors of diseases. Further, as the prevalence of diseases heavily depends on age, aggregate mortality rates forecasts for a specific age group may not be complete. Overall, as Gutterman and Vanderhoof (1998) put it, "we must be able to decompose past trends and recognize their causes to help us feel our way to the future" which is not possible if one works with the aggregate mortality rates. On the other hand, the disaggregated approach has its practical and theoretical drawbacks: the effects of misclassification of deaths by cause, limited length of the available time series, inferior data quality and the dependence structure between the causes are cited among reasons that did not permit to obtain forecasts superior to those obtained for the aggregated mortality rates (Wilmoth, 1995; Booth and Tickle, 2008).

However, the forecasts of the cause-specific mortality rates are needed not only as a path leading to the aggregate mortality, but also in their own value for many purposes such as estimation of the health care and disability costs in the ageing populations (Tabeau et al., 1999). While remaining conscious of the difficulties the cause-specific approach entails, we align with the view that analyzing and modelling the cause-specific mortality rates can improve our understanding of the past and improve our forecasts for the future.

To overcome the problem of the limited and sometimes volatile number of observations by sex, age, and cause, we propose to apply the multipopulational approach in a context of the cause-specific mortality rates. Lyu et al. (2020) note that the cause-specific mortality rates have not yet been modelled in a multipopulational setup to the same extent as this has been the case for the all-cause mortality rates. We believe that the multipopulational modelling for the cause-specific mortality rates could be justified

for two reasons. First, one could expect that similar to the all-cause mortality, there will be resemblance in the development patterns of the cause-specific mortality rates between countries having comparable socioeconomic conditions, medical advances quickly spreading across the developed countries. Second, the multipopulation approach ensures that the forecasts built for the countries included into the group are converging. As there is no reason to expect that the future mortality rates from cancer will be substantially different between France and Spain, for example, the country- and cause-specific mortality rates should not be diverging in the long term, and the multipopulational modelling is a way to ensure the coherence of the forecasts.

The question is then how to choose the countries to be included into the group or, in other words, what can serve as a measure of the sufficient coherence between the experiences of two countries? If certain countries have similar experience in the past, then they should be modelled together in a multipopulational context. Li and Lee (2005) define an explanation ratio for the augmented common factor model. They suggest including countries in the group if the corresponding ratios are "large enough" while stressing that this criteria is left intentionally vague and should be tempered by judgment. In Lyu et al. (2020) the authors are confronted with a similar task of comparing cause-specific experiences of three countries that they propose to solve by using a beta-convergence test from the growth literature. This test verifies whether the cause-specific mortality rates in different countries tend to the same level and improve at the same speed. Both approaches arrive at simple "yes/no" answers that summarize several decades of age-specific observations, and as such, are necessarily simplistic. We propose a complementary approach that consists in using the cointegration analysis to assess if the cause-specific mortality rates in different countries exhibited coherent development in the past. Indeed, should the cancer mortality rates from France and Spain, for example, be cointegrated, this would mean that they were linked in their development in the past and are not expected to wander from each other or diverge in the long run. Should a common development pattern be revealed between these two countries, this would justify using the cause-specific mortality rates in a multipopulational model that explicitly imposes a common trend on the country-specific rates, such as the one proposed by Li and Lee (2005). Specifically, we will analyze the cause-specific mortality experience of several countries by applying the cointegration analysis to the country-specific mortality time trends by cause of death. We believe that this new angle will provide additional evidence whether countries should be modelled together on the cause-specific level or not. For the future, finding a more nuanced answer to the difficult question of comparing country-specific mortality experiences expressed in vast matrices of observations seems to be an interesting and promising research topic.

At the second stage, we verify if the Li-Lee model built for the countries having the cointegrated cause-specific mortality rates allows improving the forecasts in comparison with a benchmark approach, that is the Lee-Carter model. Indeed, we observe that for the male as well as for the female cause-specific mortality rates the Li-Lee model helps to improve the forecasts for most of the countries included in our study. The cointegration analysis can hence deliver a helpful answer to the question regarding the countries to be included into a multipopulational model for the cause-specific mortality rates.

The paper is organized as follows: in Section 2 we briefly present the data used in the study regarding causes and countries chosen for the study. A brief theoretical review of

the mortality models that will be used as well as of the cointegration analysis is exposed in Section 3. The results of the cointegration tests along with the comparison of the forecasts are presented in Section 4. Section 5 concludes.

2 Data

Below we briefly present the main steps of the data preparation process that follows a path similar to the one described in Arnold and Sherris (2016):

- The data comes from the WHO Mortality Database (World Health Organization, 2020) that collects the mid-year population and the death numbers by country, year, sex, age group, and cause of death since its creation in 1950.

- As the WHO database splits the death numbers according to the *primary* cause of death, we will ignore the potential presence of the secondary cause, third cause etc. Also, we would have to signigicanlty change our approach in order to incorporate the information on the secondary cause of death, for example. For this reason, our results would not hold in presense of several causes leading to death.

- In order to limit the extent to which countries' experiences differ due to the social-economic factors, we chose the five most populated Western European countries participating in the database from its onset: France, Italy, Netherlands, Spain, and England and Wales, subsequently shortened to FR (1), IT (2), NL (3), SP (4), and EW (5) respectively. In contrast to Arnold and Glushko (2021a, 2021b) where the authors wanted to have a variety of experiences, in the present work we want the countries' conditions be as close as possible.

- We are going to build a model involving the data from different countries and for this reason, we are obliged to cut the observations for all countries at the shortest available observation window. This corresponded to the time period 1952-2014 at the moment when the data were retrieved (October 2020).

- WHO Mortality database provides the data for the age groups: "deaths at 0 years", "at 1", "at 2", "at 3", "at 4", "5-9 years", ..., "90-94 years", and finally "deaths at 95 years and above". To deal with the age groups, we, first, created two new age groups by grouping together the ages from 1 to 4 and 85 and above. Second, we distributed the number of deaths at unspecified age proportionally among the all known age groups.

- Causes of death are clustered into five main groups: infectious and parasitic diseases, cancer, diseases of the circulatory system, diseases of the respiratory system, and external causes. We define these groups of causes of death under the different versions of the International Classification of Diseases (ICD) as shown in Table 1. Naturally, there is more than one way to perform such grouping, and for the sake of comparability with earlier studies (Arnold and Sherris, 2016; Arnold and Glushko, 2021a), we keep these five groups of causes of death. Cause-specific mortality rates for selected years are shown on Figure A1 in the Appendix.

Table 1: Main groups of causes of death according to the versions of the *International Classification of Diseases*.

Causes of death	ICD 7	ICD 8	ICD 9	ICD 10
IP	001-138	001-136	001-139	A00-B99
Cancer	140-239	140-239	140-239	C00-D48
Circulatory	400-468	390-458	390-437	I00-I99
Respiratory	470-527	460-519	460-519	J00-J98
External	E810-E999	E810-E999	E800-E999	V00-Y89

- We calculate the cause-specific central death (mortality) rates as the number of deaths by age, sex, and cause divided by the mid-year population by age and sex:

$$m_{x,t}^{d,s,c} = d_{x,t}^{d,s,c} / l_{x,t}^{s,c},$$

with

$d_{x,t}^{d,s,c}$ = number of deaths at age x, in year t, for cause of death d, gender s and country c;

$l_{x,t}^{s,c}$ = mid-year population at age x, in year t, gender s and country c;

$m_{x,t}^{d,s,c}$ = central death rate at age x, in year t, for cause of death d, gender s and country c.

- We apply the comparability ratios to ensure the comparability between the observations under the different versions of the ICD. In this way, the discontinuities between the observation periods are removed. Indeed, a comparability ratio makes the average mortality rates of the last two years of a classification equal to the average mortality rates of the first two years of the following classification. Once the mortality rates in every classification are divided by the comparability ratio(s) linking this classification to the previous one(s), observations become comparable across the different versions of the ICD. Further details on the data preparation process involving comparability ratios can be found in Arnold and Sherris (2015).

- For our analysis, we will use the data for the age groups 20 years and older as the cause-specific data for the younger age groups are known to be sparse. Similar approach was taken in Lyu et al. (2020).

- All equations were estimated for the natural logarithms of the cause-specific mortality rates:

$$ln(m_{x,t}^{d,s,c}),$$

where
$x \in \{20-24, 25-29, ..., 80-84, 85+\}, t \in \{1952-2014\}, d \in \{IP, Canc, Circ, Resp, Ext\}$, $s \in \{Males, Females\}, c \in \{FR, IT, NL, SP, EW\}$.

3 Theoretical framework

3.1 Lee-Carter model for the cause-specific mortality rates

As already mentioned, the Lee-Carter model or simply LC (Lee and Carter, 1992) is possibly the most widely used model for the mortality rates and for this reason, it often serves as a comparison point in studies aiming to improve the quality of a forecast. With this objective in mind, we apply the Lee-Carter model to the mortality rates separately for each cause, sex, and country :

$$ln(m_{x,t}^{d,s,c}) = \alpha_x^{d,s,c} + \beta_x^{d,s,c} k_t^{d,s,c} + \epsilon_x^{d,s,c}(t) \tag{1}$$

To ease the notation in what follows, we will sometimes omit the indexes d, s, c. Following the approach proposed by Brouhns et al. (2002), we estimate the parameters α_x, β_x and k_t by maximizing the log-likelihood based on the Poisson model for the number of deaths:

$$L(\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{k}) = \sum_{x,t} (d_{x,t}(\alpha_x + \beta_x k_t) - l_{x,t} exp(\alpha_x + \beta_x k_t)) + \text{constant}, \tag{2}$$

and applying the constrains $\sum_x \hat{\beta}_x = 1$, $\sum_t \hat{k}_t = 0$.

The Box-Jenkins methodology is used to generate the appropriate ARIMA time series model and project k_t.

3.2 Li-Lee model for the cause-specific mortality rates

As an extension of the Lee-Carter model, Li and Lee (2005) proposed the augmented common factor model for the multi-population context:

$$ln(m_{x,t}^{d,s,c}) = \alpha_x^{d,s,c} + B_x^{d,s} K_t^{d,s} + \beta_x^{d,s,c} k_t^{d,s,c} + \epsilon_x^{d,s,c}(t), \tag{3}$$

where $B_x^{d,s} K_t^{d,s}$ is the common factor and $\beta_x^{d,s,c} k_t^{d,s,c}$ is the population-specific factor.

Like the Lee-Carter model, the $\alpha_x^{d,s,c}$ are obtained as the average mortality rates, in this case, by cause:

$$\alpha_x^{d,s,c} = \frac{\sum_t ln(m_{x,t}^{d,s,c})}{T} \tag{4}$$

The remaining model parameters are defined in two steps. First, the $B_x^{d,s}$ and $K_t^{d,s}$ are obtained from applying the ordinary LC model to the aggregate group mortality rates. In this way, the common trend of mortality change is identified. Second, the population-specific factor is obtained from the residual matrix $ln(m_{x,t}^{d,s,c}) - \alpha_x^{d,s,c} - B_x^{d,s} K_t^{d,s}$ to which the strategy of the ordinary LC model is applied. Li and Lee (2005) suggest to assess the performance of this model for a particular population by constructing the explanation ratio as follows:

$$R(c) = 1 - \frac{\sum_{x,t} (ln(m_{x,t}^{d,s,c}) - \alpha_x^{d,s,c} - B_x^{d,s} K_t^{d,s} - \beta_x^{d,s,c} k_t^{d,s,c})^2}{\sum_{x,t} (ln(m_{x,t}^{d,s,c}) - \alpha_x^{d,s,c})^2}. \tag{5}$$

The authors propose to include the population c to the studied group if the explanation ratio $R(c)$ is "large enough", leaving the criteria intentionally vague as other considerations may play a role. For example, a country may not be a part of the group in the past, but its mortality can be expected to follow a similar path in the future. In the current study, we propose to use the cointegration analysis which we briefly present below to assess if two countries should be modelled together using the Li-Lee approach.

Like the Lee-Carter model, we use the Box-Jenkins methodology to generate the appropriate ARIMA time series model and build the forecasts for the K_t and k_t^c.

3.3 Cointegration analysis as a measure of coherence

According to Engle and Granger (1987), the time series $\mathbf{y}_t$ that consist of the n non-stationary variables $(y_{1t}, y_{2t}, ...y_{nt})'$ with $t = 1, ..., T$ are said to be *cointegrated of order 1* or I(1) when there exists a linear combination of its elements $\beta'\mathbf{y}_t$ that is stationary or I(0):

$$\beta_1 y_{1t} + \beta_2 y_{2t} + ... + \beta_2 y_{nt} = z_t, \tag{6}$$

where z_t is a stationary variable of stochastic deviations. Then $\beta' = (\beta_1, \beta_2, ..., \beta_n)$ is said to be a cointegrating vector and $\beta'\mathbf{y}_t$ is a cointegration relation.

Should such a linear combination exist, this means that non-stationary variables remain linked to each other in their long-term development. It is also possible that there is more than one cointegrating vector, so that β becomes a matrix. Each cointegration relation is then linearly independent from the others.

Arnold and Sherris (2015, 2016) studied the age-standardized cause-specific mortality rates within different countries:

$$\mathbf{y}_{t,s,c} = \begin{pmatrix} ln(m_{t,s,c}^{IP}) \\ ln(m_{t,s,c}^{Canc}) \\ ln(m_{t,s,c}^{Circ}) \\ ln(m_{t,s,c}^{Resp}) \\ ln(m_{t,s,c}^{Ext}) \end{pmatrix}$$

and showed that they were non-stationary. They also demonstrated that at least one cointegration relation existed between the variables. This means that the long-term equilibrium relation(s) existed between mortality rates corresponding to different causes inside of a particular country. For this reason, it was possible to build a Vector Error Correction Model (VECM) describing the joint development of the cause-specific mortality rates within every country included into the study. Supposing that there are r cointegration relations, i.e. that there exists a matrix β of rank r such that $\beta'\mathbf{y}_t$ is I(0), the corresponding VECM has then the following form:

$$\Delta\mathbf{y}_t = \mathbf{c} + \mathbf{d}t + \alpha\beta'\mathbf{y}_{t-1} + \sum_{i=1}^{l} \xi_i \Delta\mathbf{y}_{t-i} + \epsilon_t, \ t = 1...T \tag{7}$$

where

- $\mathbf{c}$ and $\mathbf{d}$ are $(n \times 1)$ vectors of constants;
- ξ_i is a $(n \times n)$ matrix of autoregressive coefficients for $i = 1, 2, ..., l$;

- l is number of lags;
- β is a $(n \times r)$ matrix containing r vectors each representing a cointegration relation;
- α is a $(n \times r)$ loading matrix that indicates how a particular variable is impacted by the cointegration relation;
- ϵ_t is a $(n \times 1)$ vector of white noise errors.

More details on the VECM can be found in such extensive references on the subject as Hamilton (1994) and Lütkepohl (2005).

For our part, we would like to apply the cointegration analysis from a different perspective by studying the possible cointegration relations between the cause-specific mortality rates corresponding to the same causes, but coming from different countries. For this, we will study all possible pairwise combination of countries. At the same time, age-specific mortality time series present a challenge from a modelling perspective, because to the best of our knowledge, the cointegration tests have been developed for the time series with dimension n less than 12 (Osterwald-Lenum 1992). For this reason, the cointegration tests cannot be applied to the age-specific mortality time series directly. In Arnold and Sherris (2015, 2016) the authors overcame this difficulty by using the age-standardized mortality rates. In the present study we want to apply an alternative approach and study the cointegration between the time trends extracted from the cause-specific mortality rates, $k_t^{d,s,c}$ as defined in (1). So, we will test if the following time series are cointegrated:

$$\mathbf{y}_{t,s,d} = \begin{pmatrix} k_t^{d,s,c_1} \\ k_t^{d,s,c_2} \end{pmatrix},$$

where $c_1 \neq c_2$ and $c_1, c_2 \in \{FR, IT, NL, SP, EW\}$.

To achieve this, we use the trace and the maximum eigenvalue tests developed by Johansen (1995) and test for the existence of the cointegration relation for $\mathbf{y}_{t,s,d}$, i.e. that $\mathbf{\Pi y}_{t,s,d} = \alpha\beta'\mathbf{y}_{t,s,d}$ is stationary. Should this be the case, we proceed with testing for the form of the deterministic terms in (7), also developed by Johansen, and at the later stage, with assessing the quality of the fit for every identified VECM using the usual residuals tests. In this way, we verify, first, if the cointegration relation exists, and, second, that the resulting VECM has a good fit.

As suggested by Johansen (1995), we will consider the following cases where $\mathbf{d} = \alpha\rho + \alpha_\perp\gamma$ and $\alpha\alpha_\perp = 0$ to distinguish between the possible forms of the deterministic elements in the VECM:

- NT: no trend in the VECM, but a linear trend in the levels of the variables: $\mathbf{c} \neq 0, \rho = 0, \gamma = 0,$ hence $\mathbf{d} = 0$,
- TC: linear trend in the cointegration relation combined with a linear trend in the levels of the variables (i.e., no linear trend in the differenced variables): $\mathbf{c} \neq 0, \rho \neq 0, \gamma = 0,$ hence $\mathbf{d} = \alpha\rho$,
- QT: linear trend in the differenced variables, thus a quadratic trend in the levels of the variables : $\mathbf{c} \neq 0, \rho \neq 0, \gamma \neq 0,$ hence $\mathbf{d} = \alpha\rho + \alpha_\perp\gamma$.

In what follows, we will use the abbreviations NT, TC and QT to describe the VECM that was chosen, if any, for every tested pair of countries.

Should it be possible to identify a cointegration relation (here at most 1) as well as a VECM having normally distributed and non-correlated residuals, then this would mean that the particular cause-specific mortality rates from two countries experienced a similar development in the past. This observation may then justify the creation of the corresponding group of the countries to be included into the Li-Lee model. By comparing with the historical mortality rates (backtesting) we will be able to see if the existence of the cointegration relation between the time trends extracted from the cause-specific mortality rates of two countries can improve the forecasts of the corresponding cause-specific mortality rates.

4 Application

4.1 Cointegration relations in cause-specific mortality experiences

To decide if cause-specific experiences of two countries are close enough, as a first step, we extract the country-, cause- and sex-specific time trends $k_t^{d,s,c}$ from the model (1). The time trends corresponding to the Infectious&Parasitic diseases for males in five countries are shown on Figure 1. The charts for the rest of the causes can be found on Figures A2-A10 in the Appendix. As we can see, on the one hand, there is a general pattern to which all countries tend to. On the other hand, certain differences can be observed between the countries. Hence, we need a formal procedure that could allow us to judge whether the experiences of two countries are close enough to justify the application of the Li-Lee model, i.e. a measure of coherence between the country-specific experiences.

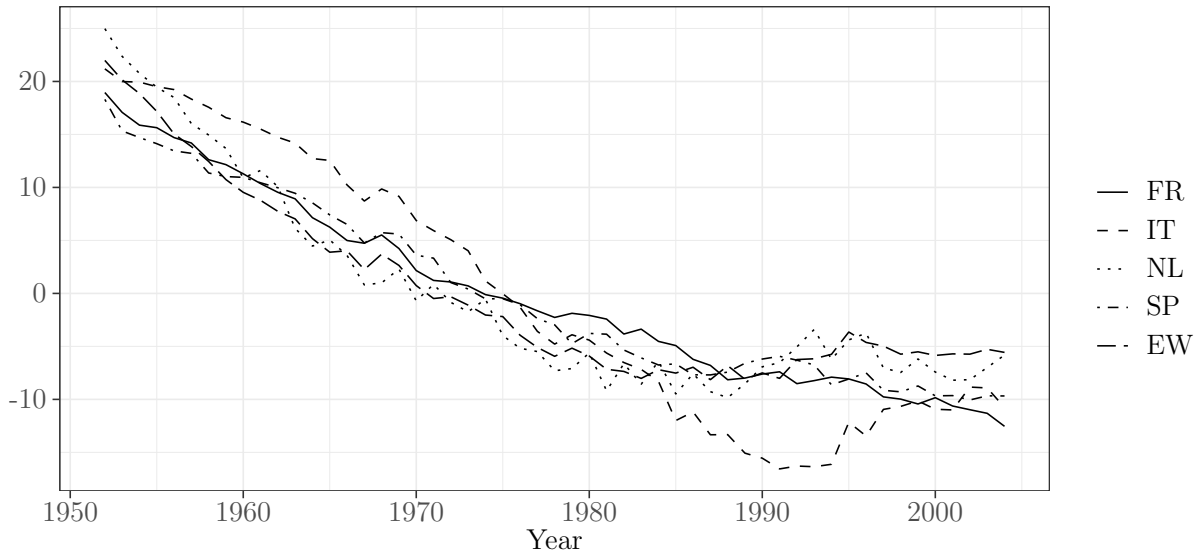

Figure 1: Time trends by country for the IP diseases, males.

For this, we check if the time trends corresponding to a particular cause are coin-

tegrated between a pair of countries. If yes, this is an indication that the information contained in the cause-specific mortality rates of one country can enrich the model and improve the forecast of the second country from the pair and vice versa.

To achieve this, we apply the Johansen test and show the number of cointegration relations and the resulting VECM, if any, in the Table 2 below for males. We see that for the Infectious&Parasitic diseases the country-specific time trends are cointegrated in 7 pairs (out of 10). Similar observations hold for the Cancer and the Respiratory diseases (6 out of 10 pairs). The External causes happen slightly less often to be cointegrated (in 5 out 10 pairs), but the cointegration is observed more frequently for the Circulatory diseases (in 9 out of 10 pairs).

The corresponding results for the female country-specific time trends are shown in Table A1 in the Appendix. Similar to the male time series, the female time trends for the Infectious&Parasitic diseases are cointegrated in 8 pairs out of 10, in 5 out of 10 pairs for the Cancer, the Circulatory and the Respiratory diseases, and only in 3 pairs out of 10 for the External causes. The detailed results of the maximum eigenvalue and trace tests as well as of the tests for the form of the deterministic elements are available from the authors upon request. Then, the quality of the model fit was assessed using the autocorrelation and the normality tests and the results are shown in Tables A2 and A3 of the Appendix.

Table 2: Number of cointegration relation and the form of the VECM, if any, describing the relation between the country-specific time trends, males.

Countries	IP	Canc	Circ	Resp	Ext
1 & 2	1 CR, QT l=1	1 CR, NT l=1	0 CR	1 CR, TC l=0	1 CR, NT l=0
1 & 3	1 CR, QT l=0	0 CR	1 CR, TC l=0	1 CR, TC l=1	0 CR
1 & 4	0 CR	1 CR, NT l=1	1 CR, NT l=0	0 CR	0 CR
1 & 5	1 CR, NT l=1	1 CR, QT l=1	1 CR, NT l=1	0 CR	1 CR, QT l=0
2 & 3	1 CR, QT l=1	0 CR	1 CR, QT l=0	1 CR, TC l=0	1 CR, TC l=1
2 & 4	0 CR	0 CR	1 CR, NT l=0	1 CR, TC l=1	0 CR
2 & 5	0 CR	1 CR, NT l=1	1 CR, QT l=1	0 CR	1 CR, NT l=0
3 & 4	1 CR, NT l=0	0 CR	1 CR, QT l=0	1 CR, TC l=0	0 CR
3 & 5	1 CR, QT l=1	1 CR, QT l=1	1 CR, NT l=1	1 CR, NT l=0	0 CR
4 & 5	1 CR, NT l=0	1 CR, NT l=1	1 CR, QT l=0	0 CR	1 CR TC, l=0

Note: CR = cointegration relation; QT = quadratic trend in the levels of the variables; TC = linear trend in the cointegration relation; NT = no trend; l = number of lags.

As mentioned in the introduction, our approach that consists in measuring the simi-

larity between country-specific experiences using cointegration analysis is complementary to those proposed by Lyu et al. (2020) and Li and Lee (2005). In the former study the authors analyze the cause-specific experiences in France, Netherlands and Belgium. The first two countries are included in our study as well (countries 1 and 3 in the Table 2). Lyu et al. (2020) arrive at the conclusion that there was no diverging pattern in the cause-specific mortality among all countries and for all causes analysed in their study. Our results cannot be directly compared with those in Lyu et al. (2020) due to a diverging definition of causes of death and a different observation period. Still, we see that the experiences of France and Netherlands can be called similar in terms of the cointegration analysis for three causes of death (IP, Circulatory and Respiratory) for male as well for female datasets (Tables 2 and A1). Our study thus reveals that there are causes for which the experiences of these two countries have not been as close as one could think.

To follow the approach proposed by Li and Lee (2005), we calculated the explanation ratios for mortality rates by cause and country as per the Li-Lee model (3) that we applied to every pair of countries mentioned in the Table 2. The explanation ratios for males are shown in the Table 3 and for the females in the Table A4 in the Appendix. We can see that apart from the External cause in some cases (e.g., the country 4 in the pairs 1&4, 2&4 and 3&4 and 4&5 for males), the application of the Li-Lee model to the rest of the cause- and country-specific mortality rates results in a "large enough" explanation ratios. This observation suggests that all pairs of countries should be modelled together in a multipopulational setting according to the Li and Lee (2005) approach whereas the cointegration analysis delivers a more nuanced answer.

Cointegration in the set of three countries

It should be also noted that the cointegration analysis can be applied to the systems having three or more variables. In the case of the cause-specific mortality rates, once the countries have been analyzed in a pairwise manner, we can conduct an additional scenario of putting together observation coming from three countries. To illustrate the idea, we will use the results for the Cancer mortality rates as shown in the Table 2. We see that for males, the countries 1 (FR), 2 (IT) and 5 (EW) as well as 1 (FR), 4 (SP) and 5 (EW) can built two groups of three countries each in which every two countries have cointegrated country-specific time trends: 1&2&5 and 1&4&5. Simultaneous modelling of the cause-specific mortality rates for these countries is then justified by the fact that every pair of the country-specific time trends shares some stochastic trends, and so, there may exist a trend shared together by all three countries. We do not analyze the combination 2&3&5, for example, because the countries 2 and 3 do not have cointegrated time trends, and so, there is less reason to believe that modelling three countries together can bring an additional benefit in comparison with the two-country model already built. It is even more so for a combination like 1&2&3 in which only the countries 1 and 2 have cointegrated time trends. The cointegration relation present between them will still exist in the three-variable system, but the three-country modelling will hardly bring any additional benefit in comparison with the two-country case. For the females, there is only one three-country combination for Cancer in which every two countries have cointegrated country-specific time trends (2&4&5). For the sake of completeness, we apply the cointegration analysis to the identified three-country combinations and show

Table 3: Country-specific explanation ratios by cause, males

Countries	I&P	Canc	Circ	Resp	Ext
1 & 2	0.9663	0.7383	0.9612	0.9502	0.9242
	0.9547	0.8188	0.9701	0.9552	0.9271
1 & 3	0.9762	0.8702	0.9610	0.9588	0.9386
	0.7939	0.7271	0.7795	0.7051	0.8530
1 & 4	0.9645	0.8602	0.9623	0.9506	0.8687
	0.9567	0.8492	0.9550	0.9256	0.5690
1 & 5	0.9587	0.8418	0.9545	0.9518	0.8613
	0.9181	0.8739	0.9647	0.9322	0.9495
2 & 3	0.9709	0.9343	0.9737	0.9617	0.9507
	0.7617	0.7737	0.7945	0.7052	0.8404
2 & 4	0.9568	0.9328	0.9623	0.9220	0.8942
	0.9493	0.8561	0.9422	0.9204	0.5597
2 & 5	0.9522	0.9379	0.9628	0.9308	0.9305
	0.9463	0.9208	0.9580	0.9189	0.9407
3 & 4	0.7944	0.7930	0.7249	0.7055	0.8720
	0.9665	0.8759	0.9554	0.9401	0.5469
3 & 5	0.8002	0.7801	0.8126	0.7285	0.8320
	0.9534	0.9469	0.9717	0.9509	0.9622
4 & 5	0.9391	0.8839	0.9236	0.9196	0.4792
	0.9180	0.8945	0.9479	0.9276	0.9568

Upper number in the cell corresponds to the explanation ratio for the left country in the pair.

the results in the Table 4. Unsurprisingly, the three-country combinations of the country-specific time trends remain cointegrated.

Table 4: p values for the null hypotheses of no autocorrelation and normality of the residuals of the VECM fitted to the country-specific time trends, Cancer.

			Autocorrelation			Normality		
Sex	Countries	Model	15 lags	25 lags	35 lags	Skewness	Kurtosis	Both
Males	1 & 2 & 5	l=1, NT, 1 CR	0.8139	0.4927	0.3739	0.2952	0.5610	0.4506
Males	1 & 4 & 5	l=0, NT, 2 CR	0.4338	0.3434	0.1086	0.9902	0.7094	0.9597
Females	2 & 4 & 5	l=1, TC, 1 CR	0.3614	0.8633	0.9841	0.0453	0.1735	0.0428

A null hypothesis is accepted at a α% significance level when the p value is higher than α%.

4.2 Cause-specific forecasts for countries having similar experiences

In cases where the country- and cause-specific time trends are cointegrated and assuming that the observed coherence between the experiences of two countries continues in the future, we expect that the Li-Lee model built for this pair of countries will deliver improved forecasts of the cause-specific mortality rates in comparison with the basic Lee-Carter approach. To check this, we use the data for 1952-2004 to estimate the parameters of the Li-Lee and the Lee-Carter models, project the time trends using the ARIMA framework for the 2005-2014, retrieve the projected cause-specific mortality rates and compare with values observed in 2005-2014 on the basis of the mean absolute average percentage error (MAPE). First, the country-, sex- and cause-specific absolute percentage error values (APE) were calculated for each age group x and the projection year t :

$$APE(x,t) = \frac{abs(ln(m_{x,t}^{observed}) - ln(m_{x,t}^{projected}))}{ln(m_{x,t}^{observed})} \tag{8}$$

At the second stage, the individual $APE(x,t)$ corresponding to the countries included in the pair were averaged across the pair taking into account the population numbers of each country and then again averaged for all x and t. In this way, we obtained the MAPE values averaged over two countries. The results of these calculations are shown in the Table 5. We observe that indeed, for the male rates, the Li-Lee model allows obtaining better forecasts for five pairs of countries out of seven pairs that have cointegrated time trends for the Infectious&Parasitic diseases, for every pair of countries that has cointegrated time trends for the Cancer diseases, for six out of nine pairs of countries for the Circulatory diseases, for five out of six pairs of countries for the Respiratory diseases, and for two out of five pairs of countries for the External causes.

Similar observations hold for the female mortality rates: for the Infectious&Parasitic diseases, the Li-Lee model permits to obtain more precise forecasts for seven out of eight pairs of countries that have the cointegrated time trends; for four out of five pairs of countries for the Cancer diseases; for every pair of countries that has the cointegrated time trend for the Circulatory diseases; for three out of five pairs of countries for the Respiratory diseases, and for two out of three pairs of countries for the External causes.

Both the Lee-Carter and Li-Lee model give age-specific forecasts for each year in 2005-2014. These forecasts are in fact point estimates. The incertainty related to the estimates is best described using the confidence intervals, but comparing the intervals is a more challenging task that would probably not deliver clear-cut results. For this reason, we will limit our analysis to comparing the point estimates produced by both models.

Table 5: Cause-specific MAPE averaged over two countries.

		Males					Females		
Cause	Countries	LC	Li-Lee	diff	Cause	Countries	LC	Li-Lee	diff
IP	1 & 2	0.0281	0.0297	-0.0016	IP	1 & 2	0.0411	0.0409	0.0002
IP	1 & 3	0.0251	0.0216	0.0034	IP	1 & 3	0.0249	0.0207	0.0042
IP	1 & 5	0.0258	0.0383	-0.0125	IP	1 & 4	0.0360	0.0336	0.0024
IP	2 & 3	0.0383	0.0350	0.0033	IP	1 & 5	0.0297	0.0326	-0.0029
IP	3 & 4	0.0644	0.0437	0.0207	IP	2 & 4	0.0506	0.0453	0.0053
IP	3 & 5	0.0369	0.0299	0.0070	IP	3 & 4	0.0542	0.0396	0.0146
IP	4 & 5	0.0407	0.0209	0.0198	IP	3 & 5	0.0454	0.0376	0.0078
					IP	4 & 5	0.0381	0.0175	0.0205
Canc	1 & 2	0.0440	0.0159	0.0281	Canc	1 & 5	0.0121	0.0132	-0.0010
Canc	1 & 4	0.0397	0.0233	0.0165	Canc	2 & 4	0.0205	0.0158	0.0047
Canc	1 & 5	0.0264	0.0156	0.0108	Canc	2 & 5	0.0145	0.0117	0.0028
Canc	2 & 5	0.0339	0.0117	0.0222	Canc	3 & 5	0.0113	0.0106	0.0007
Canc	3 & 5	0.0220	0.0126	0.0094	Canc	4 & 5	0.0218	0.0169	0.0050
Canc	4 & 5	0.0279	0.0151	0.0128					
Circ	1 & 3	0.0126	0.0110	0.0016	Circ	1 & 3	0.0174	0.0124	0.0050
Circ	1 & 4	0.0202	0.0168	0.0034	Circ	2 & 3	0.0339	0.0189	0.0149
Circ	1 & 5	0.0219	0.0194	0.0025	Circ	2 & 4	0.0361	0.0186	0.0175
Circ	2 & 3	0.0147	0.0158	-0.0011	Circ	2 & 5	0.0344	0.0222	0.0122
Circ	2 & 4	0.0164	0.0131	0.0033	Circ	4 & 5	0.0382	0.0286	0.0096
Circ	2 & 5	0.0181	0.0170	0.0011					
Circ	3 & 4	0.0214	0.0269	-0.0055					
Circ	3 & 5	0.0245	0.0275	-0.0029					
Circ	4 & 5	0.0234	0.0185	0.0049					
Resp	1 & 2	0.0276	0.0231	0.0046	Resp	1 & 2	0.0245	0.0273	-0.0027
Resp	1 & 3	0.0129	0.0144	-0.0015	Resp	1 & 3	0.0162	0.0201	-0.0039
Resp	2 & 3	0.0391	0.0295	0.0096	Resp	2 & 4	0.0288	0.0250	0.0038
Resp	2 & 4	0.0336	0.0254	0.0082	Resp	3 & 4	0.0325	0.0243	0.0083
Resp	3 & 4	0.0282	0.0264	0.0018	Resp	4 & 5	0.0359	0.0197	0.0161
Resp	3 & 5	0.0282	0.0273	0.0009					
Ext	1 & 2	0.0205	0.0160	0.0045	Ext	1 & 2	0.0172	0.0132	0.0040
Ext	1 & 5	0.0216	0.0285	-0.0069	Ext	2 & 5	0.0165	0.0182	-0.0018
Ext	2 & 3	0.0182	0.0163	0.0019	Ext	4 & 5	0.0336	0.0307	0.0029
Ext	2 & 5	0.0178	0.0244	-0.0066					
Ext	4 & 5	0.0335	0.0375	-0.0040					

Forecasts for the set of three countries

The comparison of the forecasts for the Lee-Carter and the Li-Lee model built for three countries is shown in the Table 6. We can see that the Li-Lee model substantially improves the quality of the cause-specific forecast for the males whereas the improvement is less pronounced for the females.

Table 6: Cause-specific MAPE averaged over three countries, Cancer.

	Males				Females		
Countries	LC model	Li-Lee model	diff	Countries	LC model	Li-Lee model	diff
1 & 2 & 5	0.0350	0.0184	0.0166	2 & 4 & 5	0.0189	0.0143	0.0046
1 & 4 & 5	0.0313	0.0156	0.0158				

5 Discussion and conclusion

We live in a world that becomes more and more interconnected, globalized, and in many regards less diversified. For some time now, this trend found its reflection in the converging mortality levels around the world (Wilson, 2011). So it seems less and less adequate to forecast mortality rates for individual countries without considering their future development in a larger picture. Also, it has been noted that individual application of the Lee-Carter model to the G7 countries leads to an increase of the largest gap in the life expectancy from about 4 to 8 years over a 50 year forecast horizon (Tuljapurkar et al., 2000). Such results enter in contradiction with the converging pattern of the mortality rates around the world. These considerations have lead Li and Lee (2005) to propose a model that takes into account the membership of the countries in a group by identifying the central tendencies proper to all countries and letting the weight of each country's particularities diminish in the long run.

There is no reason why what is true for the all-cause mortality would not be true for the cause-specific mortality rates. Even more so: as it may be easier to identify the driving factors of the cause-specific mortality than those of the all-cause mortality, it may also be easier to establish the coherence on the cause-specific level (Lyu et al., 2020).

Then the question arises: how to "measure" the coherence of the experiences of several countries, when each experience is contained in a large-scale matrix of observations by age and year? We suggest using the cointegration analysis that allows us precisely to say if two (or more) nonstationary vectors remain close enough to each other over a long period of time to build a stationary linear combination. To reduce the dimensionality of the mortality data we propose to apply the cointegration analysis to the mortality trends extracted by the Lee-Carter model.

We chose five most populated Western European countries to increase our chances to find the coherence between their respective cause-specific experiences. And indeed, looking at the countries in a pairwise manner, we see that very often their cause-specific time trends are cointegrated. At the same time, one needs to be cautious because not in all cases the cointegration was found. This means that even such similar countries may not have coherent experience for all considered causes of death. At the same time, should

one apply the approaches proposed by Lyu et al. (2020) and Li and Lee (2005), this would lead to the conclusion that the county- and cause-specific mortality experiences are to a large extent comparable and so, the corresponding countries should be modelled together. Hence, the cointegration analysis delivers a more nuanced answer.

Once the countries having the cointegrated cause-specific time trends were included together in a augmented common factor model proposed by Li and Lee (2005), in many cases this allowed improving the forecasting results in comparison with the basic cause-specific Lee-Carter approach. Additionally to ensuring the convergence of the forecasts, the Li-Lee model helps to enrich the experience of one country with the observations from another which can be beneficial in case of limited or volatile data as the country-specific noise is levelled out by the information from the similar countries. In cases when for some causes and combinations of countries no improvement was found, this can probably be explained by the fact that the coherence stated in the past did not continue during the forecast horizon. This is particularly true for such an independent cause as the External causes of death. Indeed, as this cause represents such random events as transport and other accidents (falls, poisoning, accidental fire, drowning), suicides, homicides, and war injuries, there are less reasons to expect that the experiences of any two countries have been following a similar path in the past. At the same time, should this have been the case, it is less probable that the observed similarity of experiences will be stable enough to continue into the future.

If one takes into account the proper character of each cause, the cointegration analysis proves to be a useful tool to assess the similarities between the experiences of two countries and so, helps building more accurate forecasts for the cause-specific mortality rates.

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References

Arnold, S. and Sherris, M. (2015) Causes-of-Death Mortality: What Do We Know on their Dependence? *North American Actuarial Journal*, **19**(2), 116–128.

Arnold, S. and Sherris, M. (2016) International Cause-Specific Mortality Rates: New Insights from a Cointegration Analysis. *ASTIN Bulletin*, **46**(1), 9–38.

Arnold, S. and Glushko, V. (2021a) Short- and long-term dynamics of cause-specific mortality rates using cointegration analysis. *North American Actuarial Journal*, URL: `https://doi.org/10.1080/10920277.2021.1874421`.

Arnold, S. and Glushko, V. (2021b) Cause-specific mortality rates: common trends and differences. *Insurance: Mathematics and Economics*, **99**, 294–308. URL: `https://doi.org/10.1016/j.insmatheco.2021.03.027`.

Booth, H. and Tickle, L. (2008) Mortality Modelling and Forecasting: A Review of Methods. *Annals of Actuarial Science*, **3**, 3–43.

Brouhns, N., Denuit, M., and Vermunt, J. K. (2002) A Poisson log-bilinear regression approach to the construction of projected lifetables. *Insurance: Mathematics and Economics*, **31**(3), 373–393.

Cairns, A. (2013) "Modeling and management of longevity risk". *Recreating Sustainable Retirement: Resilience, Solvency, and Tail Risk*. Ed. by P. B. Hammond, R. Maurer, and O. S. Mitchell. Oxford, UK: Oxford University Press, 71–88.

Engle, R. and Granger, C. (1987) Co-integration and Error Correction: Representation, Estimation and Testing. *Econometrica*, **55**, 251–276.

Gutterman, S. and Vanderhoof, I. T. (1998) Forecasting Changes in Mortality: A Search for a Law of Causes and Effects. *North American Actuarial Journal*, **2**(4), 135–138.

Hamilton, J. D. (1994) *Time Series Analysis*. Princeton: Princeton University Press.

Johansen, S. (1995) *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. NewYork: Oxford University Press.

Lee, R. D. and Carter, L. R. (1992) Modeling and Forecasting U.S. Mortality. *J. American Statistical Association*, **87**, 659–671.

Li, N. and Lee, R. (2005) Coherent mortality forecasts for a group of populations: An extension of the lee-carter method. *Demography*, **42**(3), 575–594.

Lütkepohl, H. (2005) *New Introduction to Multiple Time Series Analysis*. Crown Publishing Group.

Lyu, P., De Waegenaere, A., and Melenberg, B. (2020) A Multi-population Approach to Forecasting All-Cause Mortality Using Cause-of-Death Mortality Data. *North American Actuarial Journal*, 1–36.

Osterwald-Lenum, M. (1992) A note with quantiles of the asymptotic distribution of the maximum likelihood cointegration rank test statistics. *Oxford Bulletin of Economics and Statistics*, **54**(3), 461–472.

Tabeau, E., Ekamper, P., Huisman, C., and Bosch, A. (1999) Improving Overall Mortality Forecasts by Analysing Cause-of-Death, Period and Cohort Effects in Trends. *European Journal of Population*, **15**. Available at `http://actuaries.asn.au/Library/FSF10_Paper_burgess.pdf`, 153–183.

Tuljapurkar, S., Li, N., and Boe, C. (2000) A Universal Pattern of Mortality Decline in the G7 Countries. *Nature*, **405**, 789–792.

Villegas, A., Haberman, S., Kaishev, V. K., and Millossovich, P. (2017) A comparative study of two population models for the assessment of basis risk in longevity hedges. *ASTIN Bulletin*, **47**(3), 631–679.

Wilmoth, J. R. (1995) Are Mortality Projections Always More Pessimistic When Disaggregated by Cause of Death? *Mathematical Population Studies*, **5**(4), 293–319.

Wilson, C. (2011) Understanding Global Demographic Convergence since 1950. *Population and Development Review*, **37**(2), 375–388.

World Health Organization (2020) *WHO Mortality Database*. `http://www.who.int/whosis/mort/download/en/index.html`.

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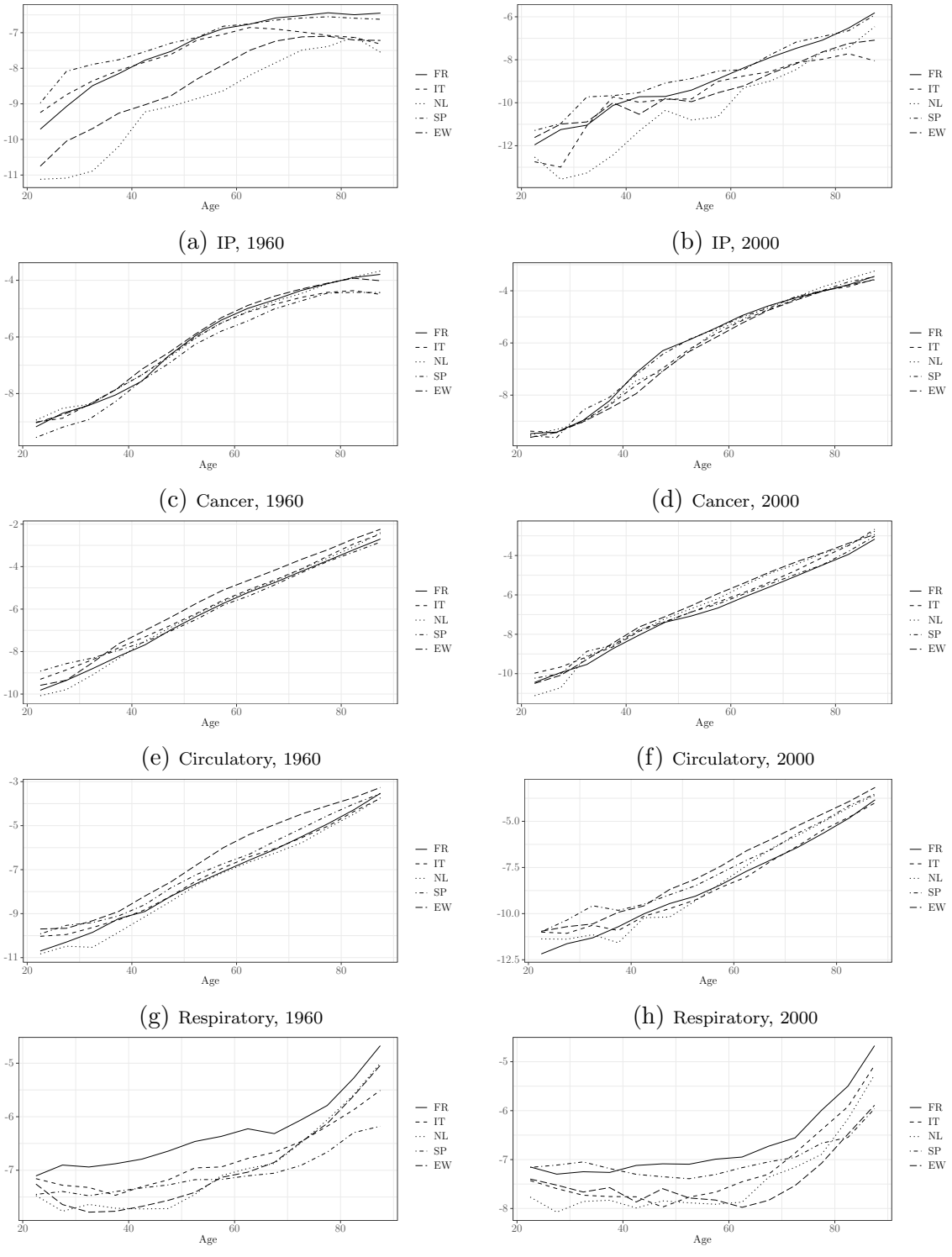
APPENDIX



(a) IP, 1960 (b) IP, 2000

(c) Cancer, 1960 (d) Cancer, 2000

(e) Circulatory, 1960 (f) Circulatory, 2000

(g) Respiratory, 1960 (h) Respiratory, 2000

(i) External, 1960 (j) External, 2000

Figure A1: Log-death cause-specific rates by cause and year, males.

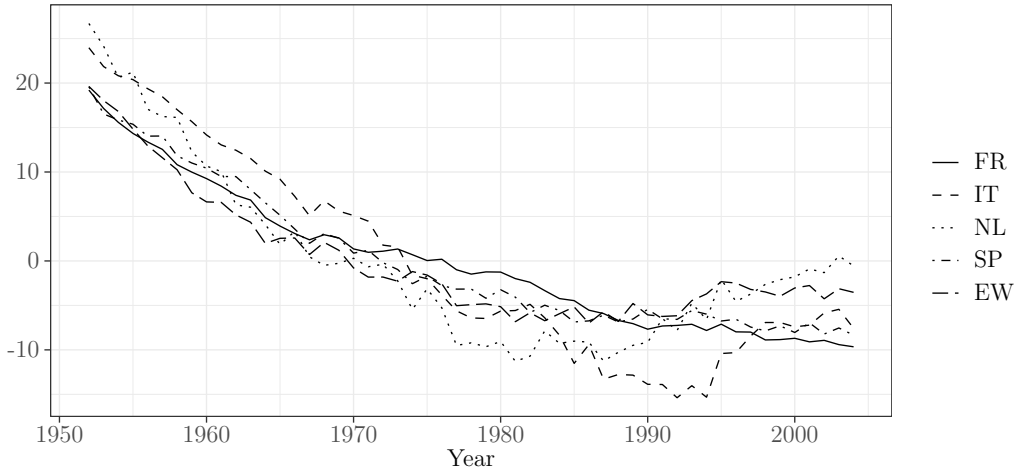


Figure A2: Time trends by country for the IP diseases, females.

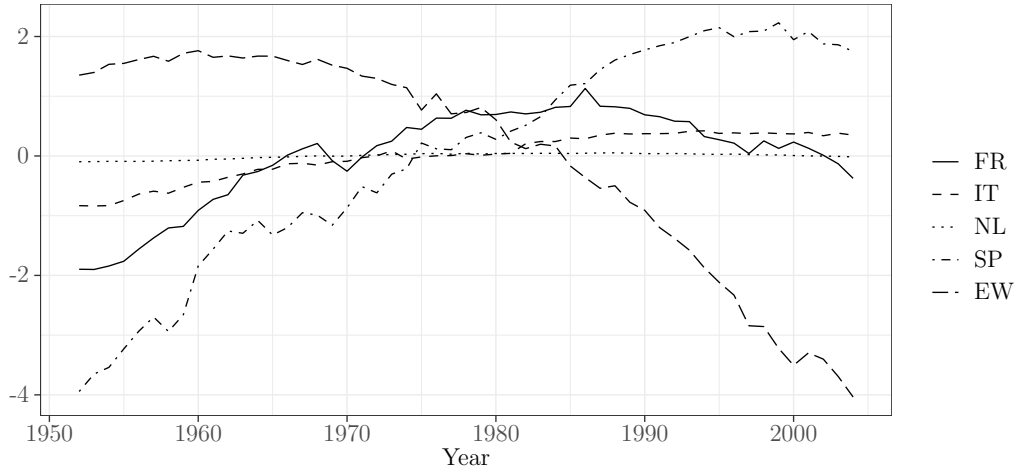


Figure A3: Time trends by country for the Cancer diseases, males.

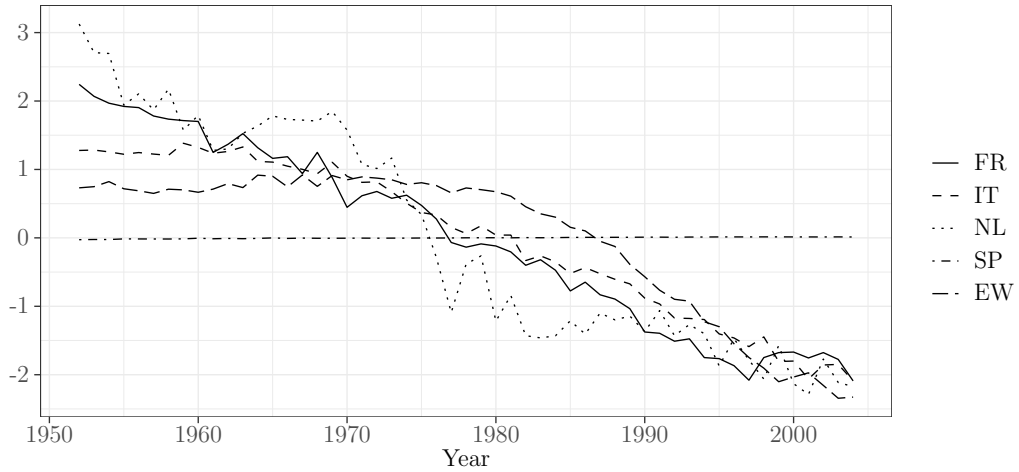


Figure A4: Time trends by country for the Cancer diseases, females.

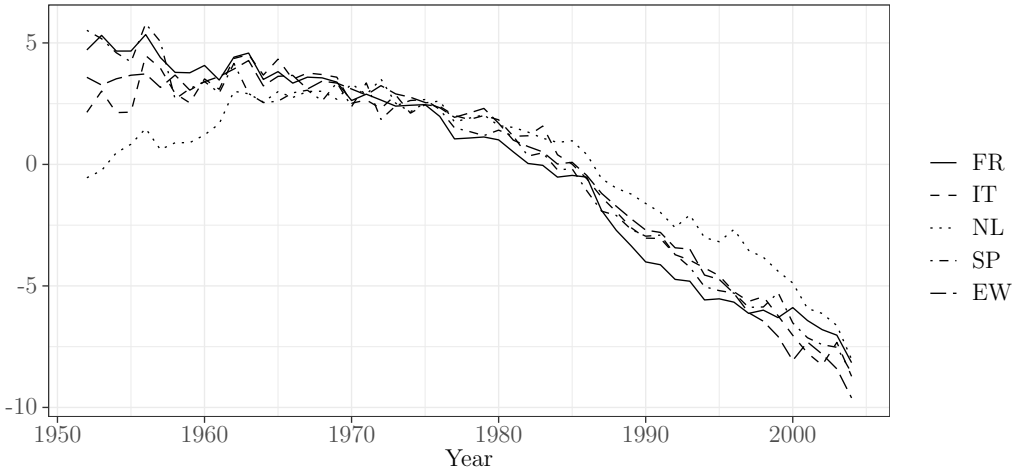


Figure A5: Time trends by country for the Circulatory diseases, males.

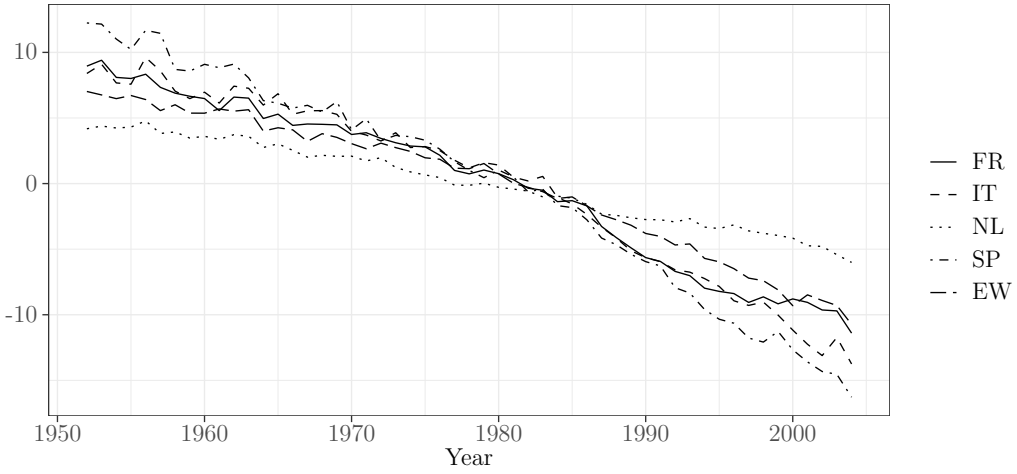


Figure A6: Time trends by country for the Circulatory diseases, females.

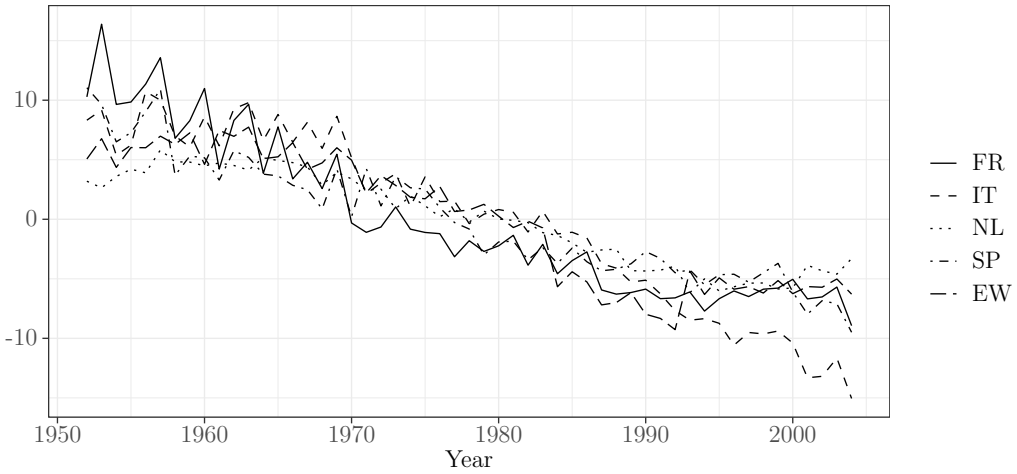


Figure A7: Time trends by country for the Respiratory diseases, males.

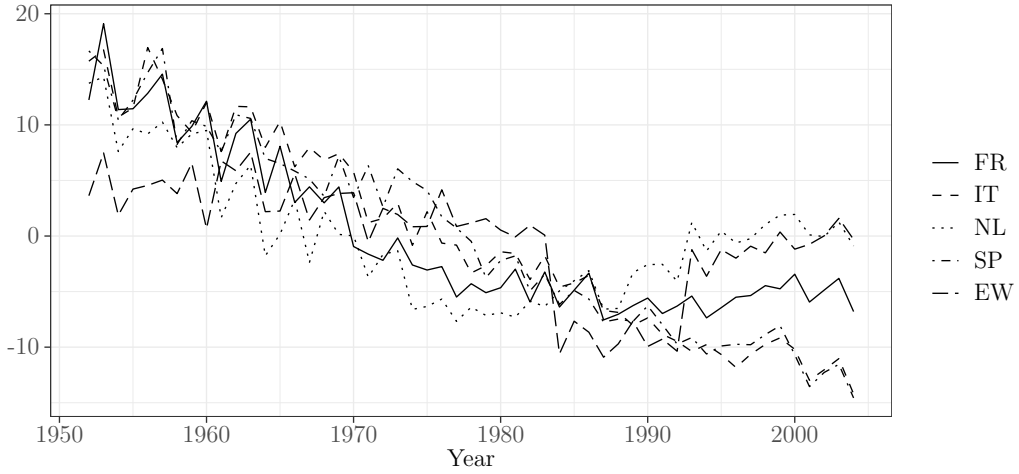


Figure A8: Time trends by country for the Respiratory diseases, females.

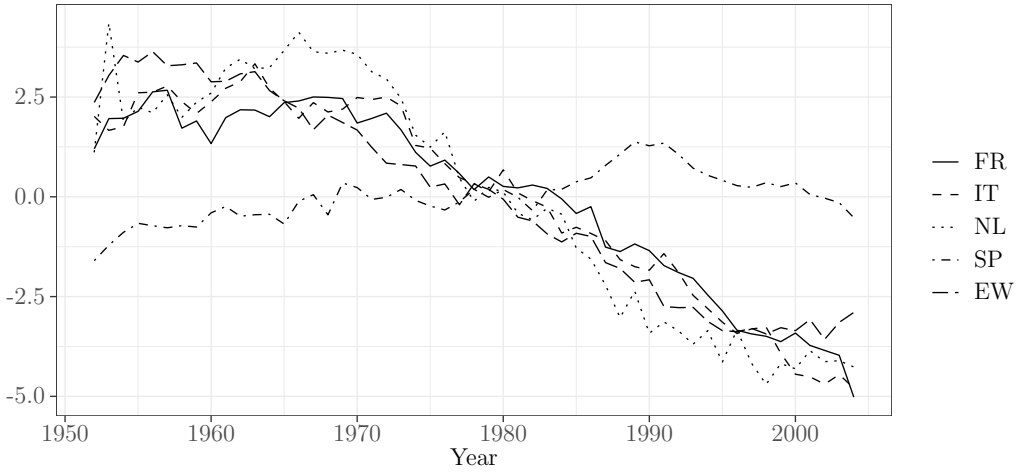


Figure A9: Time trends by country for the External causes, males.

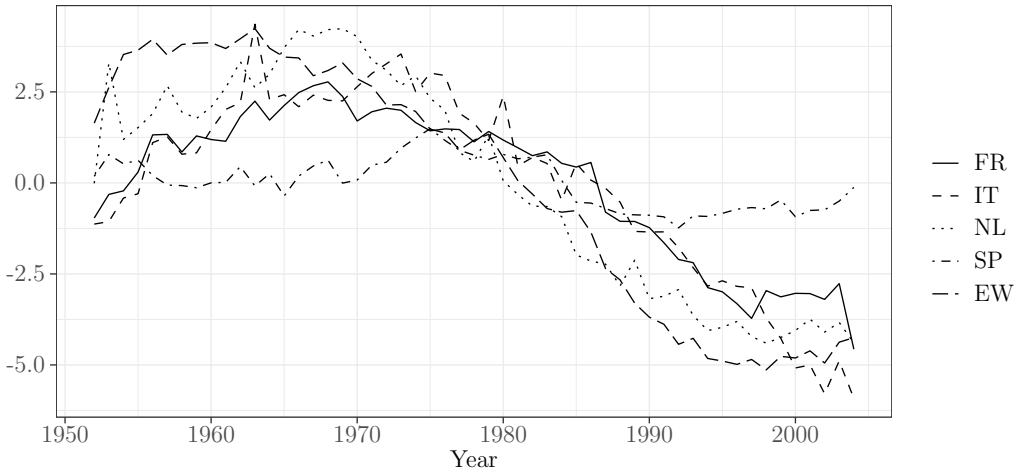


Figure A10: Time trends by country for the External causes, females.

Table A1: Number of cointegration relations and the form of the VECM, if any, describing the relation between the country-specific time trends, females.

Countries	IP	Canc	Circ	Resp	Ext
1 & 2	1 CR, NT l=1	0 CR	0 CR	1 CR, QT l=1	1 CR, TC l=1
1 & 3	1 CR, NT l=1	0 CR	1 CR, TC l=1	1 CR, TC l=0	0 CR
1 & 4	1 CR, NT l=1	0 CR	0 CR	0 CR	0 CR
1 & 5	1 CR, NT l=0	1 CR, TC l=0	0 CR	0 CR	0 CR
2 & 3	0 CR	0 CR	1 CR, TC l=0	0 CR	0 CR
2 & 4	1 CR, TC l=1	1 CR, TC l=0	1 CR, NT l=0	1 CR, TC l=1	0 CR
2 & 5	0 CR	1 CR, NT l=1	1 CR, NT l=0	0 CR	1 CR, TC l=0
3 & 4	1 CR, QT l=0	0 CR	0 CR	1 CR, TC l=0	0 CR
3 & 5	1 CR, NT l=1	1 CR, NT l=1	0 CR	0 CR	0 CR
4 & 5	1 CR, NT l=1	1 CR, NT l=1	1 CR, QT l=1	1 CR, TC l=0	1 CR, TC l=0

Note: CR = cointegration relation; QT = quadratic trend in the levels of the variables; TC = linear trend in the cointegration relation; NT = no trend; l = number of lags.

Table A2: p values for the null hypotheses of no autocorrelation and normality of the residuals of the VECM fitted to the country-specific time trends, males.

			Autocorrelation			Normality		
Cause	Countries	Model	15 lags	25 lags	35 lags	Skewness	Kurtosis	Both
IP	1 & 2	l=1, QT, 1 CR	0.2052	0.3892	0.3008	0.8439	0.2416	0.5281
IP	1 & 3	l=0, QT, 1 CR	0.0669	0.1547	0.1042	0.7046	0.3638	0.6053
IP	1 & 5	l=1, NT, 1 CR	0.4658	0.2906	0.3003	0.4232	0.6015	0.6028
IP	2 & 3	l=1, QT, 1 CR	0.0473	0.3001	0.1290	0.4920	0.1830	0.3068
IP	3 & 4	l=0, NT, 1 CR	0.3759	0.3559	0.4575	0.2122	0.1155	0.1154
IP	3 & 5	l=1, QT, 1 CR	0.3760	0.4781	0.4088	0.0228	0.0987	0.0159
IP	4 & 5	l=0, NT, 1 CR	0.1868	0.3197	0.5076	0.0547	0.6239	0.1495
Canc	1 & 2	l=1, NT, 1 CR	0.2768	0.2467	0.2858	0.5839	0.1612	0.3165
Canc	1 & 4	l=1, NT, 1 CR	0.1564	0.1765	0.6300	0.9285	0.5085	0.8264
Canc	1 & 5	l=1, QT, 1 CR	0.9500	0.8273	0.6646	0.4136	0.7251	0.6611
Canc	2 & 5	l=1, NT, 1 CR	0.7074	0.8131	0.6609	0.2328	0.1005	0.1113
Canc	3 & 5	l=1, QT, 1 CR	0.5439	0.8769	0.8575	0.9339	0.1563	0.4269
Canc	4 & 5	l=1, NT, 1 CR	0.8490	0.7211	0.0938	0.3727	0.5019	0.5007
Circ	1 & 3	l=0, TC, 1 CR	0.1088	0.2020	0.2628	0.4921	0.3433	0.4693
Circ	1 & 4	l=0, NT, 1 CR	0.5027	0.7622	0.6161	0.1131	0.7013	0.2804
Circ	1 & 5	l=1, NT, 1 CR	0.1926	0.4667	0.3768	0.9295	0.5091	0.8272
Circ	2 & 3	l=0, QT, 1 CR	0.7329	0.8532	0.8850	0.1252	0.9719	0.3781
Circ	2 & 4	l=0, NT, 1 CR	0.3959	0.6746	0.8418	0.1476	0.7491	0.3541
Circ	2 & 5	l=1, QT, 1 CR	0.4580	0.8213	0.6247	0.3805	0.7720	0.6536
Circ	3 & 4	l=0, QT, 1 CR	0.6637	0.6956	0.5042	0.8488	0.7402	0.9203
Circ	3 & 5	l=1, NT, 1 CR	0.7050	0.8905	0.6391	0.0412	0.0586	0.0170
Circ	4 & 5	l=0, QT, 1 CR	0.7879	0.9135	0.9558	0.6435	0.7234	0.8214
Resp	1 & 2	l=0, TC, 1 CR	0.2234	0.3956	0.8512	0.8326	0.7200	0.9062
Resp	1 & 3	l=1, TC, 1 CR	0.4658	0.4705	0.1625	0.9059	0.3891	0.7201
Resp	2 & 3	l=0, TC, 1 CR	0.0928	0.1988	0.1980	0.7266	0.6841	0.8445
Resp	2 & 4	l=1, TC, 1 CR	0.6321	0.7747	0.8087	0.1622	0.1049	0.0864
Resp	3 & 4	l=0, TC, 1 CR	0.5709	0.1923	0.2564	0.4238	0.8656	0.7348
Resp	3 & 5	l=0, NT, 1 CR	0.1114	0.0228	0.0163	0.4340	0.5995	0.6105
Ext	1 & 2	l=0, NT, 1 CR	0.6169	0.9758	0.9817	0.1146	0.7351	0.2926
Ext	1 & 5	l=0, QT, 1 CR	0.4720	0.4658	0.7119	0.0480	0.5109	0.1154
Ext	2 & 3	l=1, TC, 1 CR	0.2407	0.2370	0.2027	0.6203	0.1282	0.2809
Ext	2 & 5	l=0, NT, 1 CR	0.1402	0.2740	0.4255	0.7726	0.2176	0.4679
Ext	4 & 5	l=0, TC, 1 CR	0.3539	0.3563	0.6735	0.1618	0.3120	0.2012

A null hypothesis is accepted at a α% significance level when the p value is higher than α%.

Table A3: p values for the null hypotheses of no autocorrelation and normality of the residuals of the VECM fitted to the country-specific time trends, females.

			Autocorrelation			Normality		
Cause	Countries	Model	15 lags	25 lags	35 lags	Skewness	Kurtosis	Both
IP	1 & 2	l=1, NT, 1 CR	0.5333	0.6791	0.5972	0.9612	0.5131	0.8418
IP	1 & 3	l=1, NT, 1 CR	0.0399	0.3974	0.4287	0.8135	0.5242	0.7899
IP	1 & 4	l=1, NT, 1 CR	0.3530	0.1626	0.3574	0.7525	0.1424	0.3465
IP	1 & 5	l=0, NT, 1 CR	0.4699	0.4436	0.6921	0.9372	0.1842	0.4758
IP	2 & 4	l=1, TC, 1 CR	0.1839	0.1525	0.3068	0.7851	0.1887	0.4311
IP	3 & 4	l=0, QT, 1 CR	0.2559	0.1389	0.1941	0.7811	0.2086	0.4586
IP	3 & 5	l=1, NT, 1 CR	0.7933	0.8437	0.8806	0.9556	0.2385	0.5649
IP	4 & 5	l=1, NT, 1 CR	0.5775	0.2677	0.3877	0.9402	0.0798	0.2694
Canc	1 & 5	l=0, TC, 1 CR	0.5170	0.7723	0.6811	0.9433	0.2615	0.5919
Canc	2 & 4	l=0, TC, 1 CR	0.5663	0.4804	0.5546	0.0180	0.1735	0.0212
Canc	2 & 5	l=1, NT, 1 CR	0.6308	0.6110	0.8195	0.5690	0.2874	0.4596
Canc	3 & 5	l=1, NT, 1 CR	0.6758	0.3139	0.0809	0.1784	0.3895	0.2548
Canc	4 & 5	l=1, NT, 1 CR	0.1455	0.5899	0.8862	0.0148	0.0225	0.0030
Circ	1 & 3	l=1, TC, 1 CR	0.3867	0.5658	0.7723	0.8762	0.7271	0.9243
Circ	2 & 3	l=0, TC, 1 CR	0.4389	0.3220	0.2340	0.1342	0.6330	0.2944
Circ	2 & 4	l=0, NT, 1 CR	0.9101	0.9757	0.9749	0.4050	0.2932	0.3718
Circ	2 & 5	l=0, NT, 1 CR	0.2280	0.7746	0.7418	0.2993	0.6775	0.5264
Circ	4 & 5	l=1, QT, 1 CR	0.7526	0.8769	0.9453	0.8221	0.7101	0.8980
Resp	1 & 2	l=1, QT, 1 CR	0.0376	0.1869	0.5414	0.8165	0.8554	0.9491
Resp	1 & 3	l=0, TC, 1 CR	0.0011	0.0140	0.0411	0.6019	0.7309	0.8012
Resp	2 & 4	l=1, TC, 1 CR	0.8012	0.4841	0.6196	0.1026	0.2862	0.1330
Resp	3 & 4	l=0, TC, 1 CR	0.1099	0.0362	0.1729	0.0198	0.5736	0.0623
Resp	4 & 5	l=0, TC, 1 CR	0.1805	0.0316	0.0353	0.9974	0.3537	0.7203
Ext	1 & 2	l=1, TC, 1 CR	0.1989	0.3484	0.0206	0.1574	0.6173	0.3237
Ext	2 & 5	l=0, TC, 1 CR	0.1562	0.1251	0.1811	0.6831	0.2083	0.4197
Ext	4 & 5	l=0, TC, 1 CR	0.3593	0.4089	0.4226	0.5324	0.4532	0.5844

A null hypothesis is accepted at a α% significance level when the p value is higher than α%.

Table A4: Country-specific explanation ratios by cause, females.

Countries	I&P	Canc	Circ	Resp	Ext
1 & 2	0.9487	0.8176	0.9710	0.9386	0.8788
	0.9378	0.9146	0.9851	0.9653	0.8726
1 & 3	0.9632	0.8817	0.9615	0.9447	0.8901
	0.6074	0.8630	0.7723	0.6747	0.7933
1 & 4	0.9550	0.8387	0.9670	0.9340	0.8767
	0.9415	0.7440	0.9781	0.9579	0.5017
1 & 5	0.9542	0.8081	0.9529	0.9033	0.7996
	0.9151	0.8402	0.9621	0.9103	0.9130
2 & 3	0.9640	0.9239	0.9830	0.9692	0.9071
	0.6371	0.8349	0.7865	0.6785	0.7656
2 & 4	0.9355	0.8977	0.9794	0.9649	0.8372
	0.9387	0.6621	0.9816	0.9607	0.5117
2 & 5	0.9307	0.9123	0.9684	0.9647	0.8820
	0.9003	0.8427	0.9273	0.9047	0.9427
3 & 4	0.6275	0.8430	0.7928	0.6544	0.7034
	0.9523	0.7760	0.9760	0.9617	0.6082
3 & 5	0.6701	0.8381	0.7887	0.6471	0.8156
	0.9328	0.8815	0.9706	0.9269	0.9645
4 & 5	0.9347	0.6697	0.9776	0.9416	0.5685
	0.9155	0.8422	0.9246	0.9097	0.9378

Upper number in the cell corresponds to the explanation ratio for the left country in the pair.

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