SOCIETY OF ACTUARIES

Advanced Topics in General Insurance

Exam GIADV

Date: Friday, November 3, 2023

INSTRUCTIONS TO CANDIDATES

General Instructions

1. This examination has 13 questions numbered 1 through 13 with a total of 60 points.

   The points for each question are indicated at the beginning of the question.

2. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions provided in this document.

Written-Answer Instructions

1. Each question part or subpart should be answered either in the Word document or the Excel file as directed. Graders will only look at work in the indicated file.

   a) In the Word document, answers should be entered in the box marked ANSWER. The box will expand as lines of text are added. There is no need to use special characters or subscripts (though they may be used). For example, β_1 can be typed as beta_1 and σ^2 can be typed as sigma^2.

   b) Calculations should be done in Excel and entered as formulas. Performing calculations on scratch paper or with a calculator and then entering the answer in the cell will not earn full credit. Formatting of cells or rounding is not required for credit. Rows can be inserted to the answer input area as required to provide space for your answer.

   c) Individual exams may provide additional directions that apply throughout the exam or to individual items.

2. The answer should be confined to the question as set.

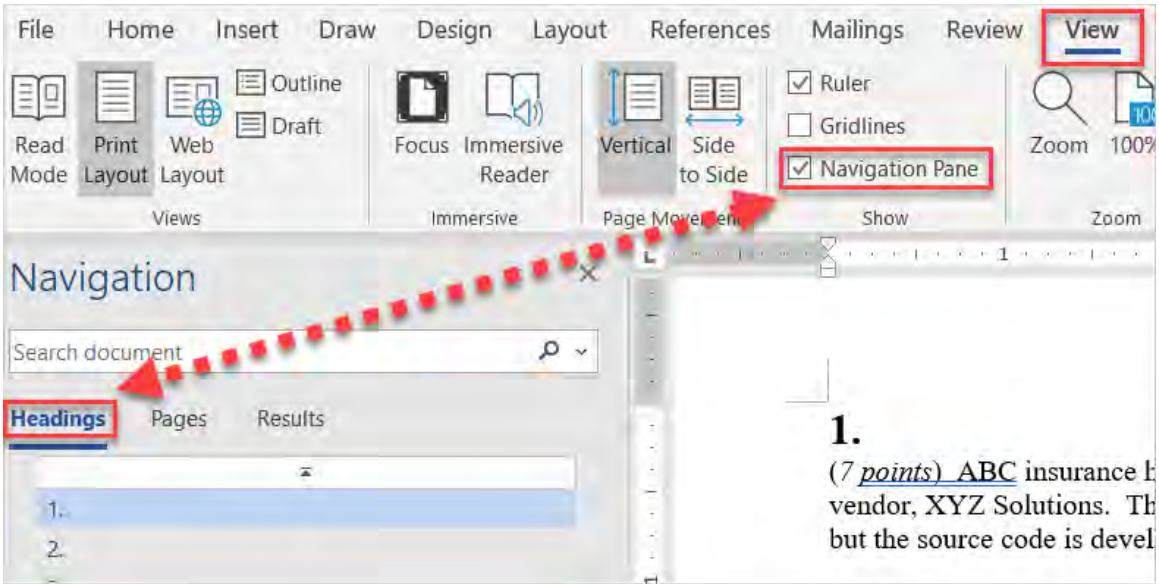
3. Prior to uploading your Word and Excel files, each file should be saved and renamed with your five-digit candidate number in the filename.

4. The Word and Excel files that contain your answers must be uploaded before the five-minute upload period expires.

Navigation Instructions

Open the Navigation Pane to jump to questions.

Press Ctrl+F, or click View > Navigation Pane:



Provide the response for this question in the Excel spreadsheet.

1.

(*5 points*) A reinsurer is pricing a property per risk excess treaty for accident year 2024 covering the layer 3,000,000 excess of 1,000,000.

You are given the following information:

Loss ID	Accident Year	Untrended Loss Evaluated as of 12/31/2022
1	2019	900,000
2	2019	2,900,000
3	2020	1,600,000
4	2020	800,000
5	2021	700,000
6	2021	2,000,000
7	2022	1,800,000
8	2022	1,400,000

Accident Year	On Level Subject Premium
2019	15,000,000
2020	6,000,000
2021	6,500,000
2022	7,500,000

- All losses of at least 500,000 are shown.
- Loss trend is 7% per year.
- Accident year development factors applicable to losses in the layer 3,000,000 excess of 1,000,000:

12-Ultimate	2.00
24-Ultimate	1.50
36-Ultimate	1.25
48-Ultimate	1.00

(a) (*3 points*) Calculate the annual experience rating loss cost for each year in 2019-2022.

1. Continued

Changing weather patterns are creating uncertainty in the projection of future losses. To allow for this, the 2019-2022 losses are to be adjusted as follows:

Increase in Losses	Probability
0%	50%
10%	30%
20%	20%

(b) (*1.5 points*) Calculate the revised expected loss cost for each year in 2019-2022.

(c) (*0.5 points*) Explain why using the average of all years may not be appropriate for pricing the 2024 treaty.

2.

(*4 points*)

(a) (*1.5 points*) Explain how certain features included in prospective experience rating plans promote equity among insureds regarding the determination of premiums.

ANSWER:

(b) (*1 point*) Describe split rating as it pertains to the NCCI experience rating plan.

ANSWER:

(c) (*1 point*) Explain why the use of prospective experience rating for an insured does not eliminate the need for schedule rating of that insured.

ANSWER:

(d) (*0.5 points*) Identify two examples of risk characteristics used in schedule rating plans.

ANSWER:

Provide the response for this question in the Excel spreadsheet.

3.

(*6 points*) You are given the following data extracted from a triangle of cumulative paid losses:

Accident Year	From (months)	To (months)	Increment	Diagonal Age	Accident Year Total
2019	0	12	2,500	48	5,000
2019	12	24	1,800	48	5,000
2019	24	36	500	48	5,000
2019	36	48	200	48	5,000
2020	0	12	4,100	36	7,000
2020	12	24	2,000	36	7,000
2020	24	36	900	36	7,000
2021	0	12	4,600	24	6,800
2021	12	24	2,200	24	6,800
2022	0	12	5,300	12	5,300

You apply Clark's stochastic reserving model using the LDF method and a loglogistic distribution with cumulative distribution function $G(x) = \frac{x^{\omega}}{x^{\omega} + \theta^{\omega}}$ where x is in months.

Clark states "The Cape Cod method … will usually produce a significantly smaller estimation error [than the LDF method]."

(a) (*1 point*) State two reasons why this is the case.

The maximum likelihood estimate (MLE) of θ is 6.8410 and of ω is 0.9804.

(b) (*2 points*) Calculate the maximum likelihood estimates of *ULT* for each of the four accident years.

(c) (*1 point*) Calculate $\hat{\sigma}^2$, the estimate of the scale factor.

(d) (*1 point*) Estimate the process standard deviation of the loss reserve for all accident years combined.

A likelihood ratio test indicates that $\omega = 1$ is a plausible value. Using this value and re-estimating the other parameters leads to a significant reduction in the estimated scale factor.

(e) (*1 point*) Explain why this reduction is to be expected.

Provide the response for this question in the Excel spreadsheet.

4.

(*8 points*) You are interested in determining a model for loss development. The triangle of incremental loss data you are working with, by accident year (AY) and development year, is:

	Development Year							
AY	1	2	3	4	5	6	7	8
1	2,011	2,736	1,116	2,850	4,799	5,006	2,071	854
2	1,900	4,142	6,108	9,472	12,482	8,153	2,355	
3	2,185	2,251	4,263	5,215	5,991	1,196		
4	1,957	1,562	2,728	3,552	1,024			
5	2,065	1,798	4,427	2,385				
6	1,896	2,731	3,815					
7	1,698	3,795						
8	1,923							

You wish to consider the following four alternative models:

(i) Mack chain ladder

(ii) Chain ladder with unweighted average development factors

(iii) Additive chain ladder

(iv) Bornhuetter Ferguson (BF)

(a) (*6 points*) Construct the fitted incremental triangle and compute the sum of squared errors (SSE) for each of the four alternative models. (Omit the first column from the fitted triangles and the SSE calculations.)

(b) (*1.5 points*) Compute one test statistic, based on the SSE, for each model.

(c) (*0.5 points*) Identify the best model based on the value of this test statistic for each model.

Provide the response for this question in the Excel spreadsheet.

5.

(*4 points*) ABC Reinsurance Company has three property catastrophe accounts, X, Y and Z.

You are given the following information:

Account	**X**	**Y**	**Z**
Expected Losses	5,000	4,000	2,500
Coefficient of Variation (CoV)	45%	25%	30%

Correlation of Losses Between Accounts			
	X	**Y**	**Z**
X	1.0	0.4	0.7
Y	0.4	1.0	0.2
Z	0.7	0.2	1.0

- ABC uses the Marginal Surplus method to calculate risk loads.
- The required return on marginal surplus is 10% and the z-score is 1.5.

(a) (*1.5 points*) Calculate the renewal risk loads for each of the three accounts (X, Y and Z) using the Marginal Surplus method.

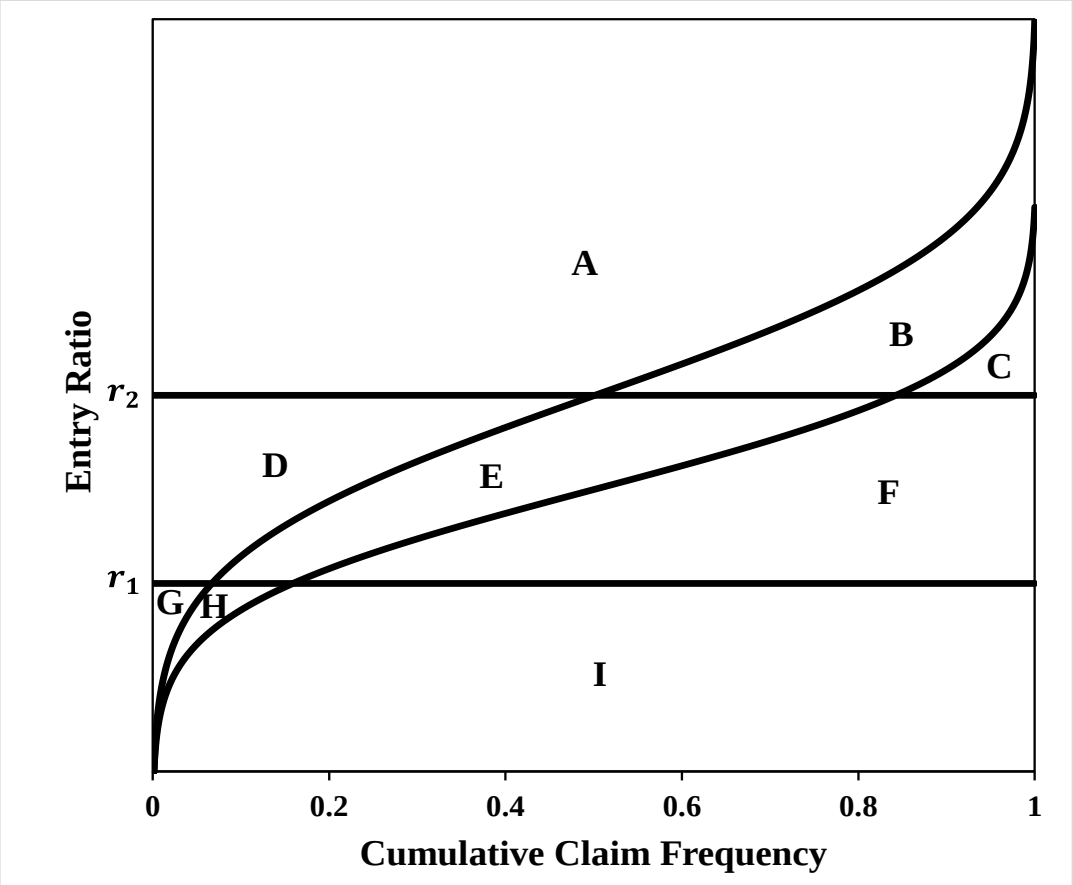
ABC is considering using the Marginal Variance method to allocate risk loads. The risk load for the combined portfolio of the three accounts is the same as that calculated using the Marginal Surplus method.

(b) (*1.5 points*) Calculate the renewal risk loads for each of the three accounts using the Marginal Variance method.

(c) (*1 point*) Demonstrate that the renewal risk loads for accounts X, Y and Z, as calculated in both parts (a) and (b), are not renewal additive.

6.

(*4 points*) You are given the following graph of the loss distribution for a risk with a retrospective rating plan, both with and without a per accident loss limitation:



You are also given the following quantities defined by areas on the graph:

area_1 = B + C

area_2 = B + C + E + F + H + I

area_3 = B + C + E + H

area_4 = B + E + H

area_5 = D + E + F + G + H + I

area_6 = G

area_7 = G + H

area_8 = G + H + I

6. Continued

Identify which of the quantities above is equal to each of the following:

(i) 1

(ii) r_1, the entry ratio associated with the minimum premium

(iii) r_2, the entry ratio associated with the maximum premium

(iv) k, the loss elimination ratio

(v) $\psi(r_1)$, the Table M savings

(vi) $\psi^*(r_1)$, the Table L savings

(vii) $\phi(r_2)$, the Table M charge

(viii) $\phi^*(r_2)$, the Table L charge

ANSWER:

(i)

(ii)

(iii)

(iv)

(v)

(vi)

(vii)

(viii)

Provide the response for this question in the Excel spreadsheet.

7.

(*4 points*) You are given the following information for a general insurance company that writes retrospectively rated policies:

Retrospective Rating Parameters	
Basic premium factor (BPF)	0.225
Expected ultimate loss divided by standard premium (ELR)	65%
Loss conversion factor (LCF)	1.255
Tax multiplier (TM)	1.035

Retrospective Adjustment	Cumulative Expected Percentage of Loss Emerged	Incremental Loss Capping Ratio
First	79%	87.5%
Second	92%	61.4%
Third	98%	20.2%
Fourth	100%	18.8%

Policy Year	Ultimate Losses	Losses Reported at Prior Retro Adjustment	Premium Booked from Prior Adjustment	Premium Booked as of 12/31/22	Completed Retro Adjustments as of 12/31/22
2019	234,150	199,950	345,790	351,120	2
2020	228,660	172,530	339,570	345,680	1
2021	289,800	0	0	385,644	0

(a) (*2 points*) Calculate the implied Cumulative Premium Development to Loss Development (CPDLD) ratios for the first to fourth retrospective rating adjustments using the formula approach.

(b) (*1 point*) Provide two situations in which one would favor the formula approach to estimating PDLD ratios over the empirical approach assuming there is sufficient data to use the empirical approach.

(c) (*1 point*) Calculate the premium asset on retrospectively rated policies as of December 31, 2022.

Provide the response for this question in the Excel spreadsheet.

8.

(*4 points*) "A Framework for Assessing Risk Margins," by Marshall et al. (Marshall), described various methods for assessing correlation effects with respect to risk margins.

(a) (*1 point*) Identify two reasons that quantitative methods should not be used to assess these correlation effects.

You are calculating a risk margin for claim liabilities in a long-tailed line of business. You are using Marshall's balanced scorecard approach to measure the internal systemic risk coefficient of variation (CoV).

Your balanced scorecard approach has the following features:

- Each of the three main sources of internal systemic risk has only two risk indicators.
- Risk indicators are given equal weight within each source of internal systemic risk.
- You assign a score of 1 or 5 to each risk indicator. A score of 1 represents poor practice, while a score of 5 represents good practice.

You are given the following information for this line of business:

- Actuaries meet regularly with the relevant managers to discuss business and claim process changes.
- Two different models were used to estimate the claim liabilities. The first model has 8 predictors and produces a result of 12 million. The second model has 6 predictors and produces a result of 23 million.
- Claim level data is available to analyze key predictors.
- The claims data used for modeling was reconciled to the general ledger. There were no material differences.

8. Continued

(b) (*2.5 points*) Complete the following internal systemic risk balanced scorecard:

Complete in the Excel spreadsheet.

Risk Source	Risk Indicator	Score (1 or 5)	Reason for receiving the score	Weight
Parameter selection error	Ability to identify and use best predictors	5		30%
	Best predictors are stable over time	1		
Specification error				40%
Data error				30%

You are provided with the following CoV scale:

Score from balanced scorecard assessment	Internal Systemic Risk CoV
1.0-2.0	12.5%
2.0-3.0	8.5%
3.0-4.0	6.5%
4.0-5.0	5.5%

(c) (*0.5 points*) Select the internal systemic risk CoV using the completed internal systemic risk balanced scorecard from part (b).

Provide the response for this question in the Excel spreadsheet.

9.

(*4 points*)

(a) (*0.5 points*) Explain why the "10% - 10% rule" is often not considered appropriate for determining the existence of sufficient risk transfer in a reinsurance agreement.

(b) (*1 point*) Define the expected reinsurer deficit (ERD) metric as used for determining the existence of sufficient risk transfer in a reinsurance agreement.

You are given the following information regarding an annual aggregate excess of loss reinsurance agreement for UVW Insurance by X-Re:

- The aggregate excess loss layer is 800 million excess of 200 million.
- The annual after-tax investment yield is 3.5%.
- The reinsurance with X-Re is for 85% of the aggregate loss layer (i.e., participation by UVW of 15% in the aggregate excess layer).
- Reinsured losses are assumed to be settled two years after expiration of the agreement.

UVW Direct Losses (millions)	**Probability**
100	91.35%
225	1.85%
275	1.60%
350	1.35%
450	1.25%
550	0.70%
650	0.55%
750	0.40%
850	0.35%
950	0.60%

The reinsurance premium paid by UVW to X-Re results in an ERD of 5%.

(c) (*1.5 points*) Determine the reinsurance premium. [Using Excel's Goal Seek function is an acceptable method for determining this amount.]

(d) (*1 point*) Explain why UVW would likely not need to test for risk transfer with respect to this reinsurance agreement.

Provide the response for this question in the Excel spreadsheet.

10.

(*4 points*) You are estimating ultimate claims for the layer of 500,000 excess of 250,000. You are given the following information including estimated cumulative development factors (CDFs):

Accident Year (AY)	Reported Claims (000) at 12/31/2022			Estimated CDF	Severity Relativity (R_t)	
	250,000 Limits	750,000 Limits	Total Limits	Total Limits	250,000 to Unlimited	750,000 to Unlimited
2016	4,978	5,693	6,170	1.000	0.730	0.901
2017	4,332	5,040	5,616	1.005	0.733	0.903
2018	5,088	5,785	6,167	1.013	0.739	0.909
2019	4,334	5,192	5,699	1.033	0.747	0.913
2020	3,704	4,298	5,029	1.085	0.751	0.917
2021	4,222	4,640	4,721	1.248	0.762	0.951
2022	3,721	3,978	3,999	1.747	0.780	0.975

There is no development beyond 84 months.

(a) (*1.5 points*) Calculate total IBNR for the layer as of December 31, 2022 using Siewert's formula.

(b) (*1 point*) Describe a peculiarity with the CDFs derived from Siewert's formula in part (a).

You are considering using the increased limit factor (ILF) method to estimate ultimate claims for the layer. You have the following additional information:

- Ultimate claims at 250,000 limits for AY 2022 using actual development factors are 5,019,000.
- The estimated ILF at 750,000 relative to 250,000 at the January 1, 2020 cost level is 1.19.
- The selected annual trend factor for a 750,000 limit is 2.2%.
- The selected annual trend factor for a 250,000 limit is 1.0%.

(c) (*1.5 points*) Calculate the layer IBNR for AY 2022 as of December 31, 2022 using the ILF method.

Provide the response for this question in the Excel spreadsheet.

11.

(*3 points*)

(a) (*0.5 points*) Identify two advantages of claims-made coverage.

(b) (*0.5 points*) Identify two advantages of occurrence coverage.

A regional association of professionals is considering a malpractice coverage program for its members. The members were experiencing availability and affordability issues in the insurance market. It is thought that better terms can be had in the market by negotiating as a group.

All of the association members have been practicing for at least four years. Some members have purchased claims-made coverage in the past, but premiums were considered too high.

You are provided with the following historical information for the association:

- Report year 2021 ultimate claims are comprised of the following:
  - 40% from accident year lag 0,
  - 25% from accident year lag 1,
  - 20% from accident year lag 2, and
  - 15% from accident year lag 3.
- The annual claim trend is 8.5%.

(c) (*1 point*) Compare the size of expected ultimate claims for report year 2024 to expected ultimate claims for accident year 2024.

The association has decided upon obtaining occurrence coverage policies for its members. All members that obtained claims-made coverage policies had effective dates of January 1 with a policy term of one year.

(d) (*1 point*) Explain why members with claims-made policies for prior years will have a coverage gap if they decide to get coverage with the association on January 1, 2024.

Provide the response for this question in the Excel spreadsheet.

12.

(*4 points*)

(a) (*1.5 points*) Explain how insurance policy deductibles assist in reducing both moral and morale hazard.

(b) (*1 point*) Define the following terms:

(i) Franchise deductible

(ii) Disappearing deductible

You are given the following information for four property insurance policies:

Policy	Property Value	Insured Limit	Straight Deductible	Coinsurance Requirement
A	600,000	500,000	1,000	85% of property value
B	600,000	500,000	1,000	100% of property value
C	600,000	500,000	2,000	None
D	600,000	500,000	2,000	90% of property value

The policy terms are applied in the following order:

- coinsurance requirement
- limit
- deductible

(c) (*1.5 points*) Determine the total amount paid by the insurance company if the following loss amounts occurred on <u>each</u> of policies A to D:

(i) 3,500

(ii) 350,000

Provide the response for this question in the Excel spreadsheet.

13.

(*6 points*) You are using a collective risk model to model catastrophe risks. The annual number of catastrophe losses has a Poisson distribution with mean 1. The loss size distribution for catastrophes is as follows:

Loss Size (billions)	Probability
1	0.4
2	0.3
3	0.2
4	0.1

Loss sizes are independent of one another and independent of the number of losses.

(a) (*1.5 points*) Demonstrate that the mean and coefficient of variation of aggregate losses are 2 billion and 1.118, respectively.

(b) (*3 points*) Complete the following aggregate loss probability table:

Aggregate Losses (billion)	Probability
0	
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

You decide to approximate aggregate losses with a lognormal distribution.

(c) (*1.5 points*) Calculate the method of moments estimates for μ and σ^2.

****END OF EXAMINATION****