

**LTAM Exam**  
**Spring 2021**  
**Solutions For Form 2**

**LTAM Exam**  
**Spring 2021**  
**Solutions to Multiple Choice Questions**

## Spring 2021, Multiple Choice Solutions, Form 2

### MC1: Answer B

### MC2: Answer A

$${}_1p_0 = e^{-\mu} \quad \text{and} \quad {}_1p_2 = e^{-3\mu}$$

For time  $t$  between 1 and 2,  $\mu_t = (2-t)\mu + (t-1)3\mu = \mu(2t-1)$ .

$$\text{Therefore, } {}_1p_1 = e^{-\int_1^2 (2t-1)\mu dt} = e^{-2\mu}$$

$$E[N] = (20,000)_3p_0 = (20,000)_1p_0 \cdot {}_1p_1 \cdot {}_1p_2$$

$$= (20,000)(e^{-\mu})(e^{-2\mu})(e^{-3\mu}) = (20,000)e^{-6\mu}$$

$${}_1p_0 = e^{-\mu} = 1 - 0.1 = 0.9 \Rightarrow E[N] = (20,000)(0.9)^6 = 10,628$$

### MC3: Answer D

$$f_{70}(15) = {}_{15}p_{70} \mu_{85}; \quad f_{80}(5) = {}_5p_{80} \mu_{85}$$

$${}_{10}q_{70} = 1 - {}_{10}p_{70}; \quad {}_{10}p_{70} = \frac{{}_{15}p_{70}}{{}_5p_{80}} = \frac{{}_{15}p_{70} \mu_{85}}{{}_5p_{80} \mu_{85}} = \frac{f_{70}(15)}{f_{80}(5)} = \frac{0.05}{0.08} = 0.625$$

$$\Rightarrow {}_{10}q_{70} = 0.375$$

**MC4: Answer B**

$$S(4) = \left(\frac{9}{10}\right) \left(\frac{8}{9}\right) \frac{(8+N-1)(8+N-2)}{(8+N)(8+N-1)} = 0.72$$

$$\Rightarrow \left(\frac{8}{10}\right) \frac{(6+N)}{(8+N)} = 0.72$$

$$\Rightarrow \frac{6+N}{8+N} = 0.90$$

$$\Rightarrow 6+N = (0.9)(8+N)$$

$$\Rightarrow N(1-0.9) = 8 \times 0.9 - 6$$

$$\Rightarrow N = 12$$

**MC5: Answer B**

$$lm(80,1) = \alpha_{80} + \beta_{80} \cdot K_1 = \alpha_{80} + \beta_{80} (K_0 + c + \sigma_k \cdot Z_1) = -3.26 + (0.3)(1 + (-0.5) + 0.2 \cdot Z_1)$$

Then 90% quantile of  $lm(80,1)$  is

$$Q_{90} = -3.26 + (0.3)(1 + (-0.5) + 0.2 \times 1.2816) = -3.03310$$

And 90% quantile of  $m(80,1)$  is  $e^{Q_{90}} = e^{-3.03310} = 0.0482$

**MC6: Answer E**

Let  $PV$  denote the  $PV$  for discrete payout,  
and  $\overline{PV}$  denote the  $PV$  for payment at the moment of death.

$$E(PV) = 1000v_1q_5 + 1000v^2_{11}q_5 + 1000v^3_{12}q_5$$

$$= 1000(1.04)^{-1} \left( \frac{24}{488} \right) + 1000(1.04)^{-2} \left( \frac{37}{488} \right) + 1000(1.04)^{-3} \left( \frac{43}{488} \right) = 195.72$$

$$E(PV^2) = (1000v)^2_1 q_5 + (1000v^2)^2_{11} q_5 + (1000v^3)^2_{12} q_5$$

$$= (1000(1.04)^{-1})^2 \left( \frac{24}{488} \right) + (1000(1.04)^{-2})^2 \left( \frac{37}{488} \right) + (1000(1.04)^{-3})^2 \left( \frac{43}{488} \right) = 179,919$$

$$E[PV] = 195.72 \Rightarrow E[\overline{PV}] = \left( \frac{i}{\delta} \right) 195.72 = 199.61$$

$$E[PV^2] = 179,919 \Rightarrow E[\overline{PV}^2] = \frac{(1+i)^2 - 1}{2\delta} \times 179,919 = \frac{(1.04)^2 - 1}{2(\ln(1.04))} = 187,164$$

$$\Rightarrow SD[\overline{PV}] = \sqrt{187,164 - 199.61^2} = 383.8$$

**MC7: Answer E**

$$P = {}_{30}E_{30} \ddot{a}_{60} = {}_{10}E_{30} \cdot {}_{20}E_{40} \cdot \ddot{a}_{60} = (0.61152)(0.36663)(14.9041) = 3.3415$$

**MC8: Answer C**

$$\bar{a}_{50:\overline{20}|}^{11} = \bar{a}_{50}^{11} - {}_{20}p_{50}^{11} \times v^{20} \times \bar{a}_{70}^{11} - {}_{20}p_{50}^{10} \times v^{20} \times \bar{a}_{70}^{01}$$

$${}_{20}p_{50}^{11} = {}_{10}p_{50}^{11} \times {}_{10}p_{60}^{11} + {}_{10}p_{50}^{10} \times {}_{10}p_{60}^{01} = (0.81210)(0.71551) + (0.06063)(0.19589) = 0.59294$$

$${}_{20}p_{50}^{10} = 0.04525 \text{ is a given.}$$

$$\Rightarrow \bar{a}_{50:\overline{20}|}^{11} = 12.3919 - (0.59294)(1.05)^{-20}(7.3744) - (0.04525)(1.05)^{-20}(3.0177) = 10.6925$$

**MC9: Answer D**

$$EPV = (2400)\ddot{a}_{55:\overline{55}|}^{(12)} = 2400\left(2\ddot{a}_{55:\overline{15}|}^{(12)} - \ddot{a}_{55:\overline{55}|}^{(12)}\right)$$

$$\ddot{a}_{55:\overline{15}|}^{(12)} = \ddot{a}_{55}^{(12)} - v^{15} \cdot {}_{15}p_{55} \cdot \ddot{a}_{70}^{(12)} = 16.54 - (1.045)^{-15}(0.9310)(12.03) = 10.753$$

$$\ddot{a}_{55:\overline{55}|}^{(12)} = \ddot{a}_{55:\overline{55}}^{(12)} - v^{15} \cdot \left({}_{15}p_{55}\right)^2 \cdot \ddot{a}_{70:\overline{70}}^{(12)} = 14.93 - (1.045)^{-15}(0.9310)^2(9.84) = 10.523$$

$$\Rightarrow EPV = 2400[(2)(10.753) - 10.523] = 26,359$$

**MC10: Answer A**

Present Value of Premiums = Present Value of Benefits

$$P\ddot{a}_{40:\overline{10}|} + 4P {}_{10}E_{40} \ddot{a}_{50:\overline{10}|} = 1000A_{40:\overline{10}|}^1 + 4000 {}_{10}E_{40} \cdot A_{50}$$

$$P[8.0863 + 4(0.60920)(8.0550)] = 1000(0.61494 - 0.60920) + 4000(0.60920)(0.18931)$$

$$\Rightarrow P = \frac{467.05}{27.7147} = 16.85$$

**MC11: Answer E**

For the  $j$ th policy the loss is

$$L_j = 14v^{K_0+1} - 10 \text{ with expected value } EPV = 14(0.66667) - 10 = -0.66662$$

$$\text{and with variance: } V[L_j] = V[14v^{K_0+1} - 10] = 14^2 ({}^2A_0 - A_0^2) = 9.67870 = 3.1111^2$$

$$\text{For the aggregate loss, } L = \sum_{j=1}^{100} L_j \text{ we have } E[L] = 100(-0.66662) = -66.662$$

$$\text{and } V[L] = 100(9.67870) = 31.111^2$$

$$\Pr[L < 0] \approx \Phi\left(\frac{0 - (-66.662)}{31.111}\right) = \Phi(2.14) = 0.9838$$

**MC12: Answer A**

$$L = (100,000 + 1000)v^{K_{45}+1} - 0.95(1050)\ddot{a}_{\overline{K_{45}+1}|} + (100 + 0.45)(1050)$$

$$= \left(101000 + \frac{997.5}{d}\right)v^{K_{45}+1} - \frac{997.5}{d} + 572.5$$

$$SD[L] = \left(101,000 + \frac{997.5}{d}\right)({}^2A_{45} - A_{45}^2)^{0.5}$$

$$= \left(101,000 + \frac{997.5}{(0.05/1.05)}\right)(0.03463 - (0.15161)^2)^{0.5} = 13,159$$

**MC13: Answer C**

Present Value of costs = Present Value of fees

$$(12)(2600)(\ddot{a}_{70}^{(12)00}) + (12)(5000)(\ddot{a}_{70}^{(12)01}) + (12)(14,000)(\ddot{a}_{70}^{(12)02}) = 100,000 + 12F(\ddot{a}_{70}^{(12)00} + \ddot{a}_{70}^{(12)01} + 1.3\ddot{a}_{70}^{(12)02})$$

$$\Rightarrow F = \frac{(12)(2600)(10.0554) + (12)(5000)(0.74720) + (12)(14,000)(0.30944) - 100,000}{12[10.0554 + 0.74720 + (1.3)(0.30944)]}$$

$$= \frac{410,546 - 100,000}{134.458} = 2309.6$$

**MC14: Answer C**

The recursion formula is the recursion for a paid up (premium = 0) whole life policy with sum insured 2500.

Therefore, the reserve is Present Value of Future Benefits – Present Value of Future Premiums

$$= {}_5V = 2500A_{75} - 0 = 1272.0$$

**MC15: Answer C**

$$P = 35.66 \text{ and } {}_{10}V = 1000A_{80} - P\ddot{a}_{80} = 288.09$$

$${}_{10}V + P = 323.75$$

$$({}_{10}V + P)(1+i)^{0.33} = {}_{0.33}q_{80} \times 1000 \times v^{0.67} + {}_{0.33}p_{80} {}_{10.33}V$$

$${}_{0.33}q_{80} = 0.33q_{80} = (0.33)(0.32658) = 0.01077714$$

$$\Rightarrow {}_{10.33}V = \frac{(323.75)(1.05)^{0.33} - (0.01077714)(1000)(1.05)^{-0.67}}{1 - 0.01077714} = \frac{329.00 - 10.43}{0.98923} = 322.04$$

**MC16: Answer E**

$$P = \frac{10,000\bar{A}_{45}^{01} + 5000\bar{A}_{45}^{02}}{\bar{a}_{45}^{00}} = \frac{(10)(1.7086) + (5)(327.9484)}{17.0915} = 96.94$$

$${}_{10}V = 10,000\bar{A}_{55}^{01} + 5,000\bar{A}_{55}^{02} - P\bar{a}_{55}^{00} = (10)(1.4835) + (5)(439.5236) - (96.94)(14.2540) = 830.66$$

**MC17: Answer D**

$${}_{15}V^{(0)} = 1000\bar{a}_{65}^{01} + 10,000\bar{A}_{65}^{02} - 450\bar{a}_{65}^{00}$$

$$= (1000)(2.8851) + (10,000)(0.53559) - (450)(6.6338) = 5255.8$$



**MC18: Answer D**

$$\begin{aligned} NPV(5) &= NPV(4) + {}_4p_x^{00} \cdot v^5 \cdot Pr_5^{(0)} + {}_4p_x^{01} \cdot v^5 \cdot Pr_5^{(1)} \\ &= 25.40 + (0.65)(1.10)^{-5}(70) + (0.20)(1.10)^{-5}(10) = 54.9 \end{aligned}$$

**MC19: Answer B**

$$\text{Final Ave Salary} = FAS = (70,000) \left( \frac{1.015^8 + 1.015^9}{2} \right) = 79,446$$

$$NC = \alpha \cdot FAS \cdot v^{10} \cdot {}_{10}p_{55} \cdot \ddot{a}_{65} = (0.02)(79,446)(1.05)^{-10} \left( \frac{94,579.7}{97,846.2} \right) (13.5498) = 12,776$$

**MC20: Answer C**

$$1+i^* = \frac{1+i}{(1+j)(c)} = \frac{1.05}{(1.02)(1.04)} = 0.98982 \implies i^* = -1.018\%$$

$$EPVTB = B(65,0) \ddot{a}_B(65) {}_{10}p_{55} (1.02)^{10} v^{10} \quad \text{where } \ddot{a}_B(65) = \ddot{a}_{65} \text{ at } i^* = -1.018\%$$

$$= (2338)(26.607) \left( \frac{94,579.7}{97,846.2} \right) (1.02)^{10} (1.05)^{-10} = 44,999$$

**LTAM Exam**

**Spring 2021**

**Solutions to Written Answer Questions**

## Question 1 Model Solution

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a)

$$\begin{aligned} \text{i) } {}_t p_x &= \exp\left\{-\int_0^t \mu_{x+s} ds\right\} = \exp\left\{-\int_0^t A + Bc^{x+s} ds\right\} \\ &= \exp\left\{-At - \frac{B}{\ln c} c^x (c^t - 1)\right\} \quad \text{Given in the tables.} \\ {}_{0.5} p_{64} &= \exp\left\{-0.00022(0.5) - \frac{2.7 \times 10^{-6}}{\ln 1.124} 1.124^{64} (1.124^{0.5} - 1)\right\} \\ &= \exp\{-0.00257678\} = 0.9974265 \end{aligned}$$

$$\Leftrightarrow 100,000 {}_{0.5} q_{64} = 100,000 (1 - {}_{0.5} p_{64}) = 257.35$$

*Comments: Candidates generally did well in this part and most of them received full credit. Mistakes include:*

- *Using force of mortality as a constant force of mortality; and*
- *Using constant force of mortality assumption and the survival probability from the table for Part c while assuming uniform distribution of deaths.*

$$\begin{aligned} \text{ii) } {}_4 V &= EPV(\text{Benefit}) - EPV(\text{premiums}) \\ &= 100,000 [{}_{0.5} q_{64} v^{0.5} + {}_{0.5|0.5} q_{64} v] - (0.9)(270)[1 + {}_{0.5} p_{64} v^{0.5}] \end{aligned}$$

where

$$\begin{aligned} {}_{0.5|0.5} q_{64} &= {}_{0.5} p_{64} {}_{0.5} q_{64.5} \\ &= {}_{0.5} p_{64} \left[ 1 - \exp\{-0.00022(0.5) \right. \\ &\quad \left. - \frac{2.7 \times 10^{-6}}{\ln 1.124} 1.124^{64.5} (1.124^{0.5} - 1)\right\} \\ &= (0.9974265) (1 - 0.9972785) = 0.0027145 \end{aligned}$$

or

$${}_{0.5|0.5} q_{64} = q_{64} - {}_{0.5} q_{64} = 0.005288 \text{ (from SULT)} - 0.0025735 = 0.0027145$$

or

$$\begin{aligned} {}_{0.5|0.5} q_{64} &= {}_{0.5} p_{64} - p_{64} \\ &= 0.9974265 - (1 - q_{64}) \end{aligned}$$

$$= 0.9974265 - (1 - 0.005288) = 0.0027145$$

$$\Rightarrow {}_4V = 100,000[(0.0025735)1.05^{-0.5} + (0.0027145) 1.05^{-1}] - (0.9)(270)[1 + (0.9974265) 1.05^{-0.5}]$$

$$= (100,000)(0.0025115 + 0.0025852) - (243)(1.9733886)$$

$$= 509.67 - 479.533 = 30.14$$

Alternatively, by recursion,

$$\begin{aligned} [{}_{4.5}V + (0.9)(270)]1.05^{0.5} &= 100,000 {}_{0.5}q_{64.5} + {}_{0.5}p_{64.5} {}_5V \\ &= 100,000(1 - 0.9972785) + 0 = 272.15 \end{aligned}$$

$${}_{4.5}V = 272.15(1.05)^{-0.5} - (0.9)(270) = 22.5912$$

$$[{}_4V + (0.9)(270)] 1.05^{0.5} = 100,000 {}_{0.5}q_{64} + {}_{0.5}p_{64} {}_{4.5}V$$

$${}_4V = (100,000 {}_{0.5}q_{64} + {}_{0.5}p_{64} {}_{4.5}V)1.05^{-0.5} - (0.9)(2.70) = 30.14$$

*Comments: Generally, most candidates knew to use either EPV or recursive formula to compute the reserve. Common errors made by candidates include:*

- *Not properly handling the semi-annual premiums;*
- *Approximating semi-annual annuity using annual annuity; and*
- *Assuming uniform distribution of death or constant force of mortality.*

**b)**

$$\begin{aligned} {}_1p_{64}^{\overline{00}} &= \exp\left\{-\int_0^1 \mu_{64+t}^{01} + \mu_{64+t}^{03} dt\right\} \\ &= \exp\{-0.06\} * p_{64}^{SULT} \\ &= e^{-0.06}(0.994712) = 0.9367845 \end{aligned}$$

*Comments: Most candidates received full credit for this part.*

c)

$${}_4V^{(0)} = 100,000 \left[ \underbrace{0.5p_{64}^{03} v^{0.5}}_{\text{i.e. EPV mid-yr benefit}} + \underbrace{(p_{64}^{03} - 0.5p_{64}^{03}) v^1}_{\text{EPV yr-end benefit}} \right] - (0.9)(270) \underbrace{[1 + (1 - 0.5p_{64}^{03})v^{0.5}]}_{\text{EPV mid-yr premium}}$$

$$1 - 0.5p_{64}^{03} = 1 - 0.00264 = 0.99736,$$

$$\text{or } 1 - 0.5p_{64}^{03} = 0.5p_{64}^{00} + 0.5p_{64}^{01} + 0.5p_{64}^{02} = 0.98661 + 0.00059 + 0.01016 = 0.99736$$

$$\begin{aligned} p_{64}^{03} - 0.5p_{64}^{03} &= 0.5p_{64}^{00} \cdot 0.5p_{64.5}^{03} + 0.5p_{64}^{01} \cdot 0.5p_{64.5}^{13} + 0.5p_{64}^{02} \cdot 0.5p_{64.5}^{23} \\ &= (0.98661)(0.00279) + (0.00059)(0.0064) + (0.01016)(0.0074) \\ &= 0.0027526 + 0.0000038 + 0.0000752 = 0.0028316 \end{aligned}$$

$$\begin{aligned} {}_4V^{(0)} &= 100,000[(0.00264) 1.05^{-0.5} + (0.0028316) 1.05^{-1}] \\ &\quad - (0.9)(270)[1 + 0.99736 \times 1.05^{-0.5}] \\ &= 100,000[0.0025764 + 0.00269676] - (243)(1.9733237) \\ &= 527.3140 - 479.5177 = 47.80 \end{aligned}$$

Alternatively, recursively,

$$\begin{aligned} [{}_4V^{(0)} + (0.9)(270)] 1.05^{0.5} \\ = 100,000 \cdot 0.5p_{64}^{03} + 0.5p_{64}^{00} {}_{4.5}V^{(0)} + 0.5p_{64}^{01} {}_{4.5}V^{(1)} + 0.5p_{64}^{02} {}_{4.5}V^{(2)} \end{aligned}$$

where

$$[{}_{4.5}V^{(0)} + (0.9)(270)] 1.05^{0.5} = 100,000 \cdot 0.5p_{64.5}^{03} \Rightarrow {}_{4.5}V^{(0)} = 29.2761$$

$$[{}_{4.5}V^{(1)} + (0.9)(270)] 1.05^{0.5} = 100,000 \cdot 0.5p_{64.5}^{13} \Rightarrow {}_{4.5}V^{(1)} = 381.5760$$

$$[{}_{4.5}V^{(2)} + (0.9)(270)] 1.05^{0.5} = 100,000 \cdot 0.5p_{64.5}^{23} \Rightarrow {}_{4.5}V^{(2)} = 479.1661$$

$$\Rightarrow [{}_4V^{(0)} + 243] 1.05^{0.5} = 264 + 0.98661(29.2761) + 0.00059(381.5760) + 0.01016(479.1661)$$

$$\Rightarrow {}_4V^{(0)} = 297.9776/1.05^{0.5} - 243 = 47.80$$

Comments: Candidates generally earned some credit. Typical mistakes included:

- Did not consider all possible transitions when calculating the probability for the year-end benefit;
- Did not realize premium is paid in State 0, 1, and 2 or improperly calculated the EPV; and

- *Did not consider state dependent reserves when using the recursive formula in a multiple state model.*

**d)**

The expected time of each sojourn in “At risk” is

$$\int_0^{\infty} {}_tP_x^{\bar{11}} dt = \int_0^{\infty} e^{-(\mu^{10} + \mu^{12})t} dt = \int_0^{\infty} e^{-(20+80)t} dt = 0.01 \text{ year}$$

The expected sojourn in “At Risk” is 3.65 days.

*Comment: Candidates did not do as well this part. Many candidates skipped this question. Most that attempted the question did not understand the expected time of sojourn.*

## Question 2 Model Solution

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a)

EPV(benefit and expenses) = EPV(premiums)

$$\begin{aligned}200 + 10a_{50:\overline{20}|} + 1,000,000 A_{50:\overline{10}|}^1 + 360,000 {}_{10}E_{50} A_{60:\overline{10}|}^1 &= 0.95G a_{50:\overline{20}|} + 0.5G \\200 + 10a_{50:\overline{20}|} + 360,000 A_{50:\overline{20}|}^1 + 640,000 A_{50:\overline{10}|}^1 &= 0.95G a_{50:\overline{20}|} + 0.5G \\190 + 10\ddot{a}_{50:\overline{20}|} + 360,000 A_{50:\overline{20}|}^1 + 640,000 A_{50:\overline{10}|}^1 &= 0.95G \ddot{a}_{50:\overline{20}|} - 0.45G\end{aligned}$$

$$A_{50:\overline{20}|}^1 = A_{50} - {}_{20}E_{50} A_{70} = A_{50:\overline{20}|} - {}_{20}E_{50} = 0.38844 - 0.34824 = 0.04020$$

$$A_{50:\overline{10}|}^1 = A_{50} - {}_{10}E_{50} A_{60} = A_{50:\overline{10}|} - {}_{10}E_{50} = 0.61643 - 0.60182 = 0.01461$$

$$\Rightarrow 190 + 10(12.8428) + (360,000)(0.04020) + (640,000)(0.01461) = G (0.95(12.8428) - 0.45)$$

$$G = (190 + 128.428 + 14,472 + 9350.4) / 11.75066 = 2054.42$$

Comments: Candidates did well with most getting full credit.

b)

$$\begin{aligned}\text{i) } {}_{10}V^g &= 360,000 A_{60:\overline{10}|}^1 + 10\ddot{a}_{60:\overline{10}|} - (0.95)G\ddot{a}_{60:\overline{10}|} \\&= 360,000(0.62116 - 0.57864) + 10(7.9555) - \\&\quad (0.95)(2054.42)(7.9555) \\&= 15,307.2 + 79.555 - 15,526.764 = -140.01\end{aligned}$$

- ii) If experience is close to the valuation assumptions, on average, a negative reserve at time 10 means that the value of the benefits and expenses paid during the first 10 years exceeds the value of the premiums received during that period. Therefore, the insurer will incur a loss if the policyholder lapses at time 10. This also means that the policyholder will need to pay premiums in excess of benefits during the last ten years which could incent lapses.

Comments: Candidates did well on part (i). Some mixed up the signs (considered some expenses as inflow). Many candidates struggled with part (ii). Most explained why the reserve was negative instead of explaining why it was problematic. Almost all

candidates failed to mention that the problem would occur 'on average' or 'if experience emerged as priced', which was a requirement for full credit. Many candidates indicated the policy would be at risk for lapse, but did not mention the lapse would be a loss for the insurer.

c)

$$360,000 A_{50:\overline{20}|}^1 + 640,000 A_{50:\overline{10}|}^1 = 1,000,000 v q_{50} + P^{FPT} v p_{50} \ddot{a}_{51:\overline{19}|}$$

$$v p_{50} \ddot{a}_{51:\overline{19}|} = \ddot{a}_{50:\overline{20}|} - 1 = 11.8428$$

$$\Rightarrow 23,822.4 \text{ (from Part A)} = 1,000,000 (0.001209)/1.05 + P^{FPT} (11.8428)$$

$$P^{FPT} = (23,822.4 - 1151.4286)/11.8428 = 1914.325$$

$$\begin{aligned} {}_{10}V^{FPT} &= 360,000 A_{60:\overline{10}|}^1 - P^{FPT} \ddot{a}_{60:\overline{10}|} \\ &= 15,307.2 \text{ (from Part B)} - 1914.325 (7.9555) = 77.79 \end{aligned}$$

Alternatively,

$$640,000 A_{51:\overline{9}|}^1 + 360,000 A_{51:\overline{19}|}^1 = P^{FPT} \ddot{a}_{51:\overline{19}|}$$

where

$$\begin{aligned} A_{51:\overline{9}|}^1 &= A_{51} - {}_9E_{51} A_{60} \\ &= 0.1978 - (1.05^{-9}) \left( \frac{96,634.1}{98,457.2} \right) (0.29028) = 0.1978 - 0.183652 = 0.014148 \end{aligned}$$

$$\begin{aligned} A_{51:\overline{19}|}^1 &= A_{51} - {}_{19}E_{51} A_{70} \\ &= 0.1978 - (1.05^{-19}) \left( \frac{91,082.4}{98,457.2} \right) (0.42818) = 0.1978 - 0.156753 = 0.041047 \end{aligned}$$

$$\ddot{a}_{51:\overline{19}|} = \ddot{a}_{51} - {}_{19}E_{51} \ddot{a}_{70} = 16.8461 - (1.05^{-19}) \left( \frac{91,082.4}{98,457.2} \right) (12.0083) = 12.44996$$

$$P^{FPT} = \frac{640,000(0.014148) + 360,000(0.041047)}{12.44996} = \frac{23,831.64}{12.44996} = 1914.19$$

$${}_{10}V^{FPT} = 360,000 A_{60:\overline{10}|}^1 - P^{FPT} \ddot{a}_{60:\overline{10}|} = 77.79$$

*Comments: Candidates did poorly on this part of the question. Many candidates incorrectly included expenses in the FPT reserve. In general candidates need to spend more time building understanding of this material.*



**d)**

The problem of the negative (gross premium) reserve in b) (ii) is not solved by changing the reserve method. The value of the benefits paid is still larger than that of the premiums

received during the first 10 years, if experience is close to the assumptions, which would result in a loss if the policyholder lapses the policy.

*Comments: Many candidates indicated that change in reserve would solve the problem. For those that responded the change in reserve would not solve the problem, many lacked any insightful commentary to explain why.*

### Question 3 Model Solution

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a)

Direct-marketed policies generally offer relatively low benefits with little or no medical evidence except for a standard questionnaire. Because of the potential for adverse selection, insurers assume higher (first year) mortality for these policies.

or

Broker-issued policies have lower (first year) mortality as a result of the selection through underwriting which eliminates the least healthy lives.

*Comments: Many candidates were able to get at least some partial credit, especially if they mentioned underwriting/adverse selection. Candidates who simply repeated the question received no credit.*

b)

$${}^2A_{[50]:\overline{20}|} = v^2 q_{[50]} + v^2 p_{[50]} {}^2A_{51:\overline{19}|}$$

$${}^2A_{50:\overline{20}|} = v^2 q_{50} + v^2 p_{50} {}^2A_{51:\overline{19}|}$$

$$0.15415 = 0.001209/1.05^2 + (0.998791/1.05^2) {}^2A_{51:\overline{19}|}$$

$${}^2A_{51:\overline{19}|} = (0.15415 - 0.0010966)/0.9059329 = 0.1689456$$

$$\begin{aligned} {}^2A_{[50]:\overline{20}|} &= (0.6)(0.001209)/1.05^2 + {}^2A_{51:\overline{19}|} (1 - (0.6)(0.001209))/1.05^2 \\ &= 0.000658 + (0.1689456)(0.9063715) = 0.1537855 \end{aligned}$$

$$10,000 {}^2A_{[50]:\overline{20}|} = 1537.86$$

*Comments: Candidates did well on this part with many candidates earning full credit.*

c)

$$i) \quad {}_0L = \begin{cases} 100,000 v^{K_{[50]}+1} - P \cdot \ddot{a}_{\overline{K_{[50]}+1}|} & K_{[50]} = 0, 1, \dots, 19 \\ 100,000 v^{20} - P \cdot \ddot{a}_{\overline{20}|} & K_{[50]} = 20, 21, \dots \end{cases}$$

or

$${}_0L = 100,000 v^{\min(K_{[50]}+1, 20)} - P \cdot \ddot{a}_{\overline{\min(K_{[50]}+1, 20)|}}$$

$$ii) \quad P = \frac{100,000 A_{[50]:\overline{20}|}}{\ddot{a}_{[50]:\overline{20}|}}$$

$$\begin{aligned} \ddot{a}_{[50]:\overline{20}|} &= \frac{1 - A_{[50]:\overline{20}|}}{d} \\ &= \frac{1 - 0.38817}{0.05/1.05} = 12.84843 \end{aligned}$$

$$P = 100,000 \left( \frac{0.38817}{12.84843} \right) = 3021.15$$

*Comments: Candidates had some difficulty writing the future loss at issue random variable with many earning only partial credit. On the other hand, most candidates did very well on the second part when asked to calculate the premium.*

d)

$$\begin{aligned} \text{i)} \quad V[{}_0L] &= V \left[ 100,000 v^{\min(K_{[50]+1}, 20)} - P \cdot \ddot{a}_{\min(K_{[50]+1}, 20)} \right] \\ &= V \left[ 100,000 v^{\min(K_{[50]+1}, 20)} - P \cdot \frac{1 - v^{\min(K_{[50]+1}, 20)}}{d} \right] \\ &= V \left[ \left( 100,000 + \frac{P}{d} \right) v^{\min(K_{[50]+1}, 20)} \right] \\ &= \left( 100,000 + \frac{P}{d} \right)^2 \left( {}^2A_{[50]:\overline{20}|} - (A_{[50]:\overline{20}|})^2 \right) \\ &= \left( 100,000 + \frac{3021.15}{0.05/1.05} \right)^2 (0.1537855 - (0.38817)^2) \\ &= (163,444.15)^2 (0.0031096) = 83,069,824 \end{aligned}$$

$$\Rightarrow \text{SD} = 9114.26$$

ii) L: Aggregate loss

$$L \sim N(1000 \cdot 0, 1000 \cdot 9114.26^2) = L \sim N(0, 288,218^2)$$

$$P(L > 200,000) = 1 - \Phi((200,000 - 0) / 288,218)$$

$$= 1 - \Phi(0.694) \cong 0.2451$$

*Comments: Most candidates did very well and were able to correctly apply the variance formula for an endowment insurance in part i. A few stumbled on the normal approximation calculation in part ii. Common errors were:*

- *Not realizing that the expected value of the aggregate loss was zero;*
- *Incorrectly multiplying the standard deviation of the individual loss by 100 or 10 instead of the square-root of 1000; and*
- *Not appropriately taking the complement of the z-score for the final answer.*

## Question 4 Model Solution

(a)

**Cohort Effects:** In some population it has been observed that systemic mortality improvement depends not only on age and calendar year, but also on cohort. The mortality improvements for a given cohort will be connected over time. That is, the mortality improvement for age  $x$  and year of birth (cohort)  $B$  will be connected to the past mortality improvements for ages  $x-t$  for that same cohort with a year of birth of  $B$ .

**Age Effects:** The mortality improvement factors in year  $Y$  depend on primarily on age. For example, larger mortality improvement usually occurs at younger ages with smaller mortality improvement at older ages.

*Comments: Candidates did understand the differences between cohort effects and age effects and generally got some credit. Many candidates used examples to illustrate the two effects which were useful in demonstrating their understanding of the concepts.*

(b)

$$C_c(46 + t, 2020 + t) = \tilde{a} t^3 + \tilde{b} t^2 + \tilde{c} t + \tilde{d}$$

$$C_c(46, 2020) = \tilde{a} 0^3 + \tilde{b} 0^2 + \tilde{c} 0 + \tilde{d} \Rightarrow \tilde{d} = \varphi(46, 2020) = 0.023$$

$$C'_c(46 + t, 2020 + t) = 3\tilde{a} t^2 + 2\tilde{b} t + \tilde{c}$$

$$\begin{aligned} C'_c(46, 2020) &= 3\tilde{a} 0^2 + 2\tilde{b} 0 + \tilde{c} \Rightarrow \tilde{c} = \varphi(46, 2020) - \varphi(45, 2019) \\ &= 0.023 - 0.021 = 0.002 \end{aligned}$$

*Comment: Most students correctly determined  $\tilde{d}$  but many candidates erroneously determined  $\tilde{c}$ . The most common mistakes were:*

- $C'_c(46, 2020) = \tilde{c} = \varphi(47, 2020) - \varphi(46, 2019) = 0.001$
- $C'_c(46, 2020) = \tilde{c} = \varphi(46, 2020) - \varphi(45, 2020) = 0.001$

(c)

$$C_a(50, 2020 + t) = a t^3 + b t^2 + c t + d$$

$$C_a(50, 2020) = d = \varphi(50, 2020) = 0.025$$

$$C'_a(50, 2020 + t) = 3a t^2 + 2b t + c$$

$$C'_a(50, 2020) = c = \varphi(50, 2020) - \varphi(50, 2019) = 0.025 - 0.024 = 0.001$$

$$C'_a(50, 2040) = 1200a + 40b + c = \varphi(50, 2041) - \varphi(50, 2040) = 0$$

$$\Rightarrow 40b = -1200a - c = -1200(6.25 \times 10^{-6}) - 0.001$$

$$\Rightarrow b = -0.0002125 = -2.125 \times 10^{-4}$$

Or

$$C_a(50, 2040) = 8000a + 400b + 20c + d = \varphi(50, 2040) = 0.01$$

$$400b = -8000a - 20c - d = -(8000)(6.25 \times 10^{-6}) - (20)(0.001) - 0.025 + 0.01 = -0.085$$

$$b = \frac{-0.085}{400} - 0.0002125 = -2.125 \times 10^{-4}$$

*Comment: Most candidates correctly determined  $c$  and  $d$  but many candidates had difficulty calculating  $b$ . Many candidates did not even attempt calculation of  $b$ .*

(d)

$$\begin{aligned} \text{i)} \quad C_a(48, 2020 + t) &= 6x10^{-6} t^3 - 2.05x10^{-4} t^2 + 0.001 t + 0.024 \\ C_a(48, 2022) &= 6x10^{-6} (2^3) - 2.05x10^{-4} (2^2) + 0.001 (2) + 0.024 \\ &= 0.025228 \end{aligned}$$

$$\begin{aligned} \text{ii)} \quad C_c(48, 2022) &= C_c(46 + 2, 2020 + 2) = \tilde{a} (2^3) + \tilde{b} (2^2) + \tilde{c}(2) + \tilde{d} \\ &= 8.25x10^{-6} (2^3) - 2.98x10^{-4} (2^2) + 0.002 (2) + 0.023 \\ & \\ &= 0.025874 \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad q(48, 2022) &= q(48, 2021) \times (1 - \varphi(48, 2022)) \\ \varphi(48, 2022) &= \frac{C_a(48, 2022) + C_c(48, 2022)}{2} = \frac{0.025228 + 0.025874}{2} = 0.025551 \end{aligned}$$

$$\Rightarrow q(48, 2022) = (0.00511)(1 - 0.025551) = 0.0049794$$

*Comment: Candidates did well on this part. Many candidates received full credit. It should be noted that candidates can earn full credit on Part d even if they did not get Part b and Part c correct. If they correctly used the values that they determined in Parts b and c in Part d, they still earned full credit.*

## Question 5 Model Solution

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(a)

$$i) \quad AL_0 = \alpha (TPS)_{63} \times \left\{ \frac{r_{63}}{l_{63}} v^{0.5} \ddot{a}_{63.5}^{(12)} + \frac{r_{64}}{l_{63}} v^{1.5} \ddot{a}_{64.5}^{(12)} + \frac{r_{65}}{l_{63}} v^2 \ddot{a}_{65}^{(12)} \right\}$$

$$= (0.02) (2,400,000) \cdot \left\{ \frac{4515}{47,579} 1.05^{-0.5} (13.5139) + \right.$$

$$\left. + \frac{4061}{47,579} 1.05^{-1.5} (13.2312) + \frac{38,488}{47,579} 1.05^{-2} (13.087) \right\}$$

$$= 48,000 \{1.2515 + 1.0496 + 9.6022\} = 571,358.40$$

$$ii) \quad NC = \alpha S_{63} \cdot \left\{ 0.5 \frac{r_{63}}{l_{63}} v^{0.5} \ddot{a}_{63.5}^{(12)} + \frac{r_{64}}{l_{63}} v^{1.5} \ddot{a}_{64.5}^{(12)} + \frac{r_{65}}{l_{63}} v^2 \ddot{a}_{65}^{(12)} \right\}$$

$$= (0.02) (170,000) \left\{ \begin{array}{l} 0.5 \frac{4515}{47,579} 1.05^{-0.5} (13.5139) + \frac{4061}{47,579} 1.05^{-1.5} (13.2312) \\ + \frac{38,488}{47,579} 1.05^{-2} (13.087) \end{array} \right\}$$

$$= 3,400 \left\{ \frac{1.2515}{2} + 1.0496 + 9.6022 \right\} = 38,344$$



Alternatively,

$$NC = vp_{63}AL_1 - AL_0 + EPV(\text{mid-year retirements})$$
$$= \left(\frac{1}{1.05}\right) \left(\frac{42,805}{47,579}\right) (638,995.53) - 571,358.40 + 62,199.55 = 38,346$$

where

$$AL_1 = \alpha [(TPS)_{63} + S_{63}] \cdot \left\{ \frac{r_{64}}{l_{64}} v^{0.5} \ddot{a}_{64.5}^{(12)} + \frac{r_{65}}{l_{64}} v^1 \ddot{a}_{65}^{(12)} \right\}$$
$$= (0.02)(2,570,000) \left\{ \frac{4061}{42,805} \left(\frac{13.2312}{1.05^{0.5}}\right) + \frac{38,488}{42,805} \left(\frac{13.087}{1.05}\right) \right\}$$
$$= 51,400 \{1.22502 + 11.2068\} = 638,995.53$$

$$EPV(\text{mid-year retirements}) = \alpha [(TPS)_{63} + 0.5 S_{63}] \cdot \left\{ \frac{r_{63}}{l_{63}} v^{0.5} \ddot{a}_{63.5}^{(12)} \right\}$$
$$= (0.02) [2,485,000] \{1.2515\} = 62,199.55$$

*Comments: Candidates always struggle with the pension related question on the exam and the results on this question was no different. Many students did not correctly calculate the actuarial liability, incorrectly handled the mid-year retirements, or just left this question blank.*

(b)

$$\begin{aligned} \text{i) } AL_0 &= 4 \alpha (TPS)_{63} x \left\{ \frac{d_{63}}{l_{63}} v^{0.5} + \frac{d_{64}}{l_{63}} v^{1.5} \right\} \\ &= 4(0.02) (2,400,000) x \left\{ \frac{213.9}{47,579} 1.05^{-0.5} + \frac{215.1}{47,579} 1.05^{-1.5} \right\} \\ &= 192,000 \{0.004387 + 0.004202\} = 1649.09 \end{aligned}$$

$$\begin{aligned} \text{ii) } NC &= 4\alpha S_{63} x \left\{ 0.5 \frac{d_{63}}{l_{63}} v^{0.5} + \frac{d_{64}}{l_{63}} v^{1.5} \right\} \\ &= 4(0.02) (170,000) x \left\{ 0.5 \frac{213.9}{47,579} 1.05^{-0.5} + \frac{215.1}{47,579} 1.05^{-1.5} \right\} \\ &= 13,600 \left\{ \frac{0.004387}{2} + 0.004202 \right\} = 86.98 \end{aligned}$$

Alternatively,

$$NC = vp_{63}AL_1 - AL_0 + EPV(\text{mid-year deaths})$$

$$= \left( \frac{1}{1.05} \right) \left( \frac{42,805}{47,579} \right) (1008.26) - 1649.09 + 872.20 = 86.95$$

where

$$\begin{aligned} AL_1 &= 4\alpha [(TPS)_{63} + s_{63}] \cdot \left\{ \frac{d_{64}}{l_{64}} v^{0.5} \right\} \\ &= 4(0.02)(2,570,000) \left\{ \frac{215.1}{42,805} v^{0.5} \right\} \\ &= 1008.26 \end{aligned}$$

$$\begin{aligned} EPV(\text{mid-year deaths}) &= 4\alpha [(TPS)_{63} + 0.5 S_{63}] \cdot \left\{ \frac{d_{63}}{l_{63}} v^{0.5} \right\} \\ &= 4(0.02)(2,485,000) \{ 0.004387 \} = 872.14 \end{aligned}$$

*Comments: Candidates did even worse on this portion of the question as many candidates did not understand that benefit provided upon death was calculated using the same approach as for the retirement benefit.*

## Question 6 Model Solutions

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(a)

$\bar{a}_{xy}^{01}$  is the actuarial value of a (whole-life) annuity paid to (y) starting on the death of (x), payable continuously at a rate of 1 per year.

Or

A reversionary annuity paid to (y) starting on the death of (x), payable continuously at a rate of 1 per year.

Comments: Most candidates did very well. Some candidates didn't mention "continuous", and a few candidates only provided a description that was not specific to the given joint life model (e.g., "a continuous annuity payable at a rate of 1 per year while in state 1 given that it is issued in state 0")

(b)

$$\frac{d}{dt} {}_t p_{xy}^{00} = -{}_t p_{xy}^{00} (\mu_{x+t}^{01} + \mu_{y+t}^{02}); \quad {}_0 p_{xy}^{00} = 1$$

And

$$\frac{d}{dt} {}_t p_{xy}^{01} = {}_t p_{xy}^{00} \mu_{x+t}^{01} - {}_t p_{xy}^{01} \mu_{y+t}^{13}; \quad {}_0 p_{xy}^{01} = 0$$

Comments: Candidates did well on this part. A common error was incorrect or missing subscripts (i.e., ages). A few candidates didn't provide the boundary conditions.

(c)

We have

$$\bar{a}_{xy}^{00} = \int_0^{\infty} {}_t p_{xy}^{00} e^{-\delta t} dt = \int_0^{\infty} g(t) dt$$

and

$$\ddot{a}_{xy}^{00} = \sum_{k=0}^{\infty} {}_k p_{xy}^{00} e^{-\delta k} = \sum_{k=0}^{\infty} g(k)$$

} i.e.  $\bar{a}$  is the integral,  $\ddot{a}$  the sum

where

$$g(t) = {}_t p_{xy}^{00} e^{-\delta t}$$

$$g(t) = e^{-\int_0^t \mu_{x+s}^{01} + \mu_{x+s}^{02} ds} e^{-\delta t} = e^{-\int_0^t \delta + \mu_{x+s}^{01} + \mu_{x+s}^{02} ds}$$

$$\text{So, } g(0) = 1.$$

Using the chain rule,

$$\begin{aligned} g'(t) &= e^{-\int_0^t \delta + \mu_{x+s}^{01} + \mu_{x+s}^{02} ds} \frac{d}{dt} \left( -\int_0^t \delta + \mu_{x+s}^{01} + \mu_{x+s}^{02} ds \right) \\ &= e^{-\int_0^t \delta + \mu_{x+s}^{01} + \mu_{x+s}^{02} ds} (-\delta - \mu_{x+t}^{01} - \mu_{x+t}^{02}) \end{aligned}$$

$$\text{So, } g'(0) = e^{-0} (-\delta - \mu_{x+t}^{01} - \mu_{x+t}^{02}) = -(\delta + \mu_x^{01} + \mu_y^{02}).$$

Alternatively, the derivative of  $g(t)$  using the product rule is

$$\begin{aligned} g'(t) &= \left( \frac{d}{dt} {}_t p_{xy}^{00} \right) e^{-\delta t} + {}_t p_{xy}^{00} \left( \frac{d}{dt} e^{-\delta t} \right) \\ &= -{}_t p_{xy}^{00} (\mu_{x+t}^{01} + \mu_{y+t}^{02}) e^{-\delta t} + {}_t p_{xy}^{00} (-\delta e^{-\delta t}). \end{aligned}$$

$$\text{So, } g'(0) = -1 \cdot (\mu_x^{01} + \mu_y^{02}) \cdot 1 + 1 \cdot (-\delta \cdot 1) = -(\mu_x^{01} + \mu_y^{02} + \delta)$$

The approximation

$$\int_0^{\infty} g(t) dt \approx \sum_{k=0}^{\infty} g(k) - \frac{1}{2} g(0) + \frac{1}{12} g'(0)$$

then gives

$$\bar{a}_{xy}^{00} \approx \ddot{a}_{xy}^{00} - \frac{1}{2} - \frac{1}{12} (\mu_x^{01} + \mu_y^{02} + \delta).$$

Comments: Candidates did fairly well, but not many received full credit. The most common issue was to derive  $g'(t)$ . A lot of students didn't apply the chain rule or product rule properly. Besides, a number of students mistakenly assumed "constant force of mortality" and attempted the question in a significantly simplified version.

(d)

Let  $B$  = annual annuity payment rate

$$1,000,000 = B \bar{a}_{70:\overline{70}|} = B(\bar{a}_{70:70}^{00} + \bar{a}_{70:70}^{01} + \bar{a}_{70:70}^{02})$$

$$\text{Since } \bar{a}_{70:70}^{01} = \bar{a}_{70:70}^{02} = \bar{a}_{70|70} = 2.0317$$

$$1,000,000 = B \bar{a}_{70:\overline{70}|} = B(\bar{a}_{70:70}^{00} + 2\bar{a}_{70|70})$$

Using the approximation in (c),

$$\begin{aligned} \bar{a}_{70:70}^{00} &\approx \ddot{a}_{70:70}^{00} - \frac{1}{2} - \frac{1}{12}(2\mu_{70} + \delta) = \ddot{a}_{70:70}^{SULT} - \frac{1}{2} - \frac{1}{12}(2(0.009881) + 0.04879) \\ &= 9.9774 - 0.5057 = 9.4717 \end{aligned}$$

$$\Rightarrow B = \frac{1,000,000}{9.4717 + 2(2.0317)} = \frac{1,000,000}{13.5351} = 73,881.98$$

*Comments: Candidates did fair on this part. A few candidates mistakenly applied the EMW approximation in Part c) for a last-survivor annuity. Note that the formula for  $g(t)$  is different for different cases, the EMW approximation for a last-survivor annuity would be different from c).*