Quantitative Finance and Investments Advanced Formula Sheet

Fall 2017/Spring 2018

Morning and afternoon exam booklets will include a formula package identical to the one attached to this study note. The exam committee believes that by providing many key formulas, candidates will be able to focus more of their exam preparation time on the application of the formulas and concepts to demonstrate their understanding of the syllabus material and less time on the memorization of the formulas.

The formula sheet was developed sequentially by reviewing the syllabus material for each major syllabus topic. Candidates should be able to follow the flow of the formula package easily. We recommend that candidates use the formula package concurrently with the syllabus material. Not every formula in the syllabus is in the formula package. **Candidates are responsible for all formulas on the syllabus, including those not on the formula sheet.**

Candidates should carefully observe the sometimes subtle differences in formulas and their application to slightly different situations. Candidates will be expected to recognize the correct formula to apply in a specific situation of an exam question.

Candidates will note that the formula package does not generally provide names or definitions of the formula or symbols used in the formula. With the wide variety of references and authors of the syllabus, candidates should recognize that the letter conventions and use of symbols may vary from one part of the syllabus to another and thus from one formula to another.

We trust that you will find the inclusion of the formula package to be a valuable study aide that will allow for more of your preparation time to be spent on mastering the learning objectives and learning outcomes.

Interest Rate Models - Theory and Practice, Brigo and Mercurio

Chapter 3

Table 3.1 Summary of instantaneous short rate models

Model	Dynamics	$r > 0$	$r \sim$	AB	AO
V	$dr_t = k[\theta - r_t]dt + \sigma dW_t$	N	$\mathcal{N}$	Y	Y
CIR	$dr_t = k[\theta - r_t]dt + \sigma\sqrt{r_t}dW_t$	Y	NCχ^2	Y	Y
D	$dr_t = ar_t dt + \sigma r_t dW_t$	Y	L$\mathcal{N}$	Y	N
EV	$dr_t = r_t[\eta - a\ln r_t]dt + \sigma r_t dW_t$	Y	L$\mathcal{N}$	N	N
HW	$dr_t = k[\theta_t - r_t]dt + \sigma dW_t$	N	$\mathcal{N}$	Y	Y
BK	$dr_t = r_t[\eta_t - a\ln r_t]dt + \sigma r_t dW_t$	Y	L$\mathcal{N}$	N	N
MM	$dr_t = r_t\left[\eta_t - \left(\lambda - \frac{\gamma}{1+\gamma t}\right)\ln r_t\right]dt + \sigma r_t dW_t$	Y	L$\mathcal{N}$	N	N
CIR++	$r_t = x_t + \varphi_t, \quad dx_t = k[\theta - x_t]dt + \sigma\sqrt{x_t}dW_t$	Y*	SNCχ^2	Y	Y
EEV	$r_t = x_t + \varphi_t, \quad dx_t = x_t[\eta - a\ln x_t]dt + \sigma x_t dW_t$	Y*	SL$\mathcal{N}$	N	N

*rates are positive under suitable conditions for the deterministic function φ.

(3.5) $dr(t) = k[\theta - r(t)]dt + \sigma dW(t), \qquad r(0) = r_0$

(3.6) $r(t) = r(s)e^{-k(t-s)} + \theta\left(1 - e^{-k(t-s)}\right) + \sigma\int_s^t e^{-k(t-u)}dW(u)$

(3.7) $E\{r(t)|\mathcal{F}_s\} = r(s)e^{-k(t-s)} + \theta\left(1 - e^{-k(t-s)}\right)$

$$\text{Var}\{r(t)|\mathcal{F}_s\} = \frac{\sigma^2}{2k}\left[1 - e^{-2k(t-s)}\right]$$

(3.8) $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

(3.9) $dr(t) = [k\theta - B(t,T)\sigma^2 - kr(t)]dt + \sigma dW^T(t)$

(3.11) $dr(t) = [k\theta - (k + \lambda\sigma)r(t)]dt + \sigma dW^0(t), \qquad r(0) = r_0$

(3.12) $dr(t) = [b - ar(t)]dt + \sigma dW^0(t)$

(3.13) $r(t) = r(s)e^{-a(t-s)} + \frac{b}{a}\left(1 - e^{-a(t-s)}\right) + \sigma\int_s^t e^{-a(t-u)}dW^0(u)$

(3.14) $\hat{\alpha} = \dfrac{n\sum_{i=1}^n r_i r_{i-1} - \sum_{i=1}^n r_i \sum_{i=1}^n r_{i-1}}{n\sum_{i=1}^n r_{i-1}^2 - \left(\sum_{i=1}^n r_{i-1}\right)^2}$

(3.15) $\hat{\beta} = \dfrac{\sum_{i=1}^n [r_i - \hat{\alpha}r_{i-1}]}{n(1-\hat{\alpha})}$

(3.16) $\widehat{V^2} = \dfrac{1}{n}\sum_{i=1}^n \left[r_i - \hat{\alpha}r_{i-1} - \hat{\beta}(1-\hat{\alpha})\right]^2$

(3.19) $E\{r(t)|\mathcal{F}_s\} = r(s)e^{a(t-s)}$ and $\text{Var}\{r(t)|\mathcal{F}_s\} = r^2(s)e^{2a(t-s)}\left(e^{\sigma^2(t-s)} - 1\right)$

(3.20) $P(t,T) = \dfrac{\bar{r}^p}{\pi^2}\int_0^\infty \sin(2\sqrt{\bar{r}}\sinh y)\int_0^\infty f(z)\sin(yz)dzdy + \dfrac{2}{\Gamma(2p)}\bar{r}^p K_{2p}(2\sqrt{\bar{r}})$

(3.21) $dr(t) = k(\theta - r(t))dt + \sigma\sqrt{r(t)}dW(t), \qquad r(0) = r_0$

(3.22) $dr(t) = [k\theta - (k + \lambda\sigma)r(t)]dt + \sigma\sqrt{r(t)}dW^0(t), \qquad r(0) = r_0$

(3.23) $E\{r(t)|\mathcal{F}_s\} = r(s)e^{-k(t-s)} + \theta\left(1 - e^{-k(t-s)}\right)$

$$\text{Var}\{r(t)|\mathcal{F}_s\} = r(s)\frac{\sigma^2}{k}\left(e^{-k(t-s)} - e^{-2k(t-s)}\right) + \theta\frac{\sigma^2}{2k}\left(1 - e^{-k(t-s)}\right)^2$$

(3.24) $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

(3.25) $A(t,T) = \left[\dfrac{2h\exp\{(k+h)(T-t)/2\}}{2h + (k+h)(\exp\{(T-t)h\} - 1)}\right]^{2k\theta/\sigma^2}$

$$B(t,T) = \frac{2(\exp\{(T-t)h\} - 1)}{2h + (k+h)(\exp\{(T-t)h\} - 1)}, \qquad h = \sqrt{k^2 + 2\sigma^2}$$

(3.27) $dr(t) = [k\theta - (k + B(t,T)\sigma^2)r(t)]dt + \sigma\sqrt{r(t)}dW^T(t)$

(3.28) $p^T_{r(t)|r(s)}(x) = p_{\chi^2(v,\delta(t,s))/q(t,s)}(x) = q(t,s)p_{\chi^2(v,\delta(t,s))}(q(t,s)x)$

$$q(t,s) = 2[\rho(t-s) + \psi + B(t,T)] \text{ and } \delta(t,s) = \frac{4\rho(t-s)^2 r(s)e^{h(t-s)}}{q(t,s)}$$

Page 68 $R(t,T) = \alpha(t,T) + \beta(t,T)r(t), \qquad P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

(3.29) $\sigma_f(t,T) = \dfrac{\partial B(t,T)}{\partial T}\sigma(t, r(t))$

Page 69 $dr(t) = b(t, r(t))dt + \sigma(t, r(t))dW(t)$

$$b(t,x) = \lambda(t)x + \eta(t), \qquad \sigma^2(t,x) = \gamma(t)x + \delta(t)$$

$$\frac{\partial}{\partial t}B(t,T) + \lambda(t)B(t,T) - \frac{1}{2}\gamma(t)B(t,T)^2 + 1 = 0, \qquad B(T,T) = 0$$

$$\frac{\partial}{\partial t}[\ln A(t,T)] - \eta(t)B(t,T) + \frac{1}{2}\delta(t)B(t,T)^2 = 0, \qquad A(T,T) = 1$$

Page 69/70 Vasicek $\lambda(t) = -k, \quad \eta(t) = k\theta, \quad \gamma(t) = 0, \quad \delta(t) = \sigma^2$

Page 70 CIR $\lambda(t) = -k, \quad \eta(t) = k\theta, \quad \gamma(t) = \sigma^2, \quad \delta(t) = 0$

$$b(x) = \lambda x + \eta, \qquad \sigma^2(x) = \gamma x + \delta$$

Page 71 $\lim_{t\to\infty} E\{r(t)|\mathcal{F}_s\} = \exp\left(\dfrac{\theta}{a} + \dfrac{\sigma^2}{4a}\right)$

(3.31) $\lim_{t\to\infty} \text{Var}\{r(t)|\mathcal{F}_s\} = \exp\left(\dfrac{2\theta}{a} + \dfrac{\sigma^2}{2a}\right)\left[\exp\left(\dfrac{\sigma^2}{2a}\right) - 1\right]$

(3.32) $dr(t) = [\vartheta(t) - a(t)r(t)]dt + \sigma(t)dW(t)$

(3.33) $dr(t) = [\vartheta(t) - ar(t)]dt + \sigma dW(t)$

(3.34) $\vartheta(t) = \dfrac{\partial f^M(0,t)}{\partial T} + af^M(0,t) + \dfrac{\sigma^2}{2a}(1 - e^{-2at})$

(3.35) $r(t) = r(s)e^{-a(t-s)} + \int_s^t e^{-a(t-u)}\vartheta(u)du + \sigma\int_s^t e^{-a(t-u)}dW(u)$

$$= r(s)e^{-a(t-s)} + \alpha(t) - \alpha(s)e^{-a(t-s)} + \sigma\int_s^t e^{-a(t-u)}dW(u)$$

(3.36) where $\alpha(t) = f^M(0,t) + \dfrac{\sigma^2}{2a^2}(1 - e^{-at})^2$

(3.37) $E\{r(t)|\mathcal{F}_s\} = r(s)e^{-a(t-s)} + \alpha(t) - \alpha(s)e^{-a(t-s)}$

$$\text{Var}\{r(t)|\mathcal{F}_s\} = \frac{\sigma^2}{2a}\left[1 - e^{-2a(t-s)}\right]$$

(3.38) $dx(t) = -ax(t)dt + \sigma dW(t), \qquad x(0) = 0$

Page 74 $x(t) = x(s)e^{-a(t-s)} + \sigma\int_s^t e^{-a(t-u)}dW(u)$

Page 74 $Q\{r(t) < 0\} = \Phi\left(-\frac{\alpha(t)}{\sqrt{\frac{\sigma^2}{2\alpha}[1-e^{-2\alpha t}]}}\right)$

Page 75 $\int_t^T r(u)du|\mathcal{F}_t \sim \mathcal{N}\left(B(t,T)[r(t)-\alpha(t)] + \ln\frac{P^M(0,t)}{P^M(0,T)} + \frac{1}{2}[V(0,T)-V(0,t)], V(t,T)\right)$

where $B(t,T) = \frac{1}{a}\left[1 - e^{-a(T-t)}\right]$

and $V(t,T) = \frac{\sigma^2}{a^2}\left[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right]$

(3.39) $P(t,T) = A(t,T)e^{-B(t,T)r(t)}$

where $A(t,T) = \frac{P^M(0,T)}{P^M(0,t)}\exp\left\{B(t,T)f^M(0,t) - \frac{\sigma^2}{4a}(1-e^{-2at})B(t,T)^2\right\}$

(3.40) $\mathbf{ZBC}(t,T,S,X) = P(t,S)\Phi(h) - XP(t,T)\Phi(h-\sigma_p)$

where $\sigma_p = \sigma\sqrt{\frac{1-e^{-2a(T-t)}}{2a}}B(T,S)$ and $h = \frac{1}{\sigma_p}\ln\frac{P(t,S)}{P(t,T)X} + \frac{\sigma_p}{2}$

(3.41) $\mathbf{ZBP}(t,T,S,X) = XP(t,T)\Phi(-h+\sigma_p) - P(t,S)\Phi(-h)$

(3.42) $\mathbf{Cap}(t,\mathcal{T},N,X) = N\sum_{i=1}^n(1+X\tau_i)\mathbf{ZBP}\left(t, t_{i-1}, t_i, \frac{1}{1+X\tau_i}\right)$

or $\mathbf{Cap}(t,\mathcal{T},N,X) = N\sum_{i=1}^n[P(t,t_{i-1})\Phi(-h_i+\sigma_p^i) - (1+X\tau_i)P(t,t_i)\Phi(-h_i)]$,

where $\sigma_p^i = \sigma\sqrt{\frac{1-e^{-2a(t_{i-1}-t)}}{2a}}B(t_{i-1},t_i)$ and $h_i = \frac{1}{\sigma_p^i}\ln\frac{P(t,t_i)(1+X\tau_i)}{P(t,t_{i-1})} + \frac{\sigma_p^i}{2}$

(3.43) $\mathbf{Flr}(t,\mathcal{T},N,X) = N\sum_{i=1}^n[(1+X\tau_i)P(t,t_i)\Phi(h_i) - P(t,t_{i-1})\Phi(h_i-\sigma_p^i)]$

(3.44) $\mathbf{CBO}(t,T,\mathcal{T},c,X) = \sum_{i=1}^n c_i\mathbf{ZBO}(t,T,T_i,X_i)$

(3.45) $\mathbf{PS}(t,T,\mathcal{T},N,X) = N\sum_{i=1}^n c_i\mathbf{ZBP}(t,T,t_i,X_i)$

(3.46) $\mathbf{RS}(t,T,\mathcal{T},N,X) = N\sum_{i=1}^n c_i\mathbf{ZBC}(t,T,t_i,X_i)$

(3.47) $E\{x(t_{i+1})|x(t_i) = x_{i,j}\} = x_{i,j}e^{-a\Delta t_i} =: M_{i,j}$

$$\text{Var}\{x(t_{i+1})|x(t_i) = x_{i,j}\} = \frac{\sigma^2}{2a}\left[1 - e^{-2a\Delta t_i}\right] =: V_i^2$$

(3.48) $\Delta x_i = V_{i-1}\sqrt{3} = \sigma\sqrt{\frac{3}{2a}[1 - e^{-2a\Delta t_{i-1}}}$

(3.49) $k = \text{round}\left(\frac{M_{i,j}}{\Delta x_{i+1}}\right)$

(3.50) $p_u = \frac{1}{6} + \frac{\eta_{j,k}^2}{6V_i^2} + \frac{\eta_{j,k}}{2\sqrt{3}V_i}$, $p_m = \frac{2}{3} - \frac{\eta_{j,k}^2}{3V_i^2}$, $p_d = \frac{1}{6} + \frac{\eta_{j,k}^2}{6V_i^2} - \frac{\eta_{j,k}}{2\sqrt{3}V_i}$

(3.64) $dx_t^\alpha = \mu(x_t^\alpha;\alpha)dt + \sigma(x_t^\alpha;\alpha)dW_t^x$

(3.65) $P^x(t,T) = \Pi^x(t,T,x_t^\alpha;\alpha)$

(3.66) $r_t = x_t + \varphi(t;\alpha), \qquad t \ge 0$

(3.67) $P(t,T) = \exp\left[-\int_t^T \varphi(s;\alpha)ds\right] \Pi^x(t,T,r_t - \varphi(t;\alpha);\alpha)$

(3.68) $\varphi(t;\alpha) = \varphi^*(t;\alpha) := f^M(o,t) - f^x(0,t;\alpha)$

(3.69) $\exp\left[-\int_t^T \varphi(s;\alpha)ds\right] = \Phi^*(t,T,x_0;\alpha) := \dfrac{P^M(0,T)}{\Pi^x(0,T,x_0;\alpha)} \dfrac{\Pi^x(0,t,x_0;\alpha)}{P^M(0,t)}$

(3.70) $\Pi(t,T,r_t;\alpha) = \Phi^*(t,T,x_0;\alpha)\Pi^*(t,T,r_t - \varphi^*(t;\alpha);\alpha)$

(3.71) $V^x(t,T,\tau,K) = \Psi^x(t,T,\tau,K,x_t^\alpha;\alpha)$

(3.74) $dr_t = \left[k\theta + k\varphi(t;\alpha) + \dfrac{d\varphi(t;\alpha)}{dt} - kr_t\right] dt + \sigma dW_t$

Page 100 $\varphi^{VAS}(t;\alpha) = f^M(0,t) + (e^{-kt} - 1)\dfrac{k^2\theta - \sigma^2/2}{k^2} - \dfrac{\sigma^2}{2k^2} e^{-kt}(1 - e^{-kt}) - x_0 e^{-kt}$

Page 101 $$P(t,T) = \frac{P^M(0,T)A(0,t)\exp\{-B(0,t)x_0\}}{P^M(0,t)A(0,T)\exp\{-B(0,T)x_0\}} \times A(t,T)\exp\{-B(t,T)[r_t - \varphi^{VAS}(t;\alpha)]\}$$

(3.76) $dx(t) = k(\theta - x(t))dt + \sigma\sqrt{x(t)}dW(t), \qquad x(0) = x_0, \qquad r(t) = x(t) + \varphi(t)$

(3.77) $\varphi^{CIR}(t;\alpha) = f^M(0,t) - f^{CIR}(0,t;\alpha)$

$$f^{CIR}(0,t;\alpha) = \frac{2k\theta(\exp\{th\} - 1)}{2h + (k+h)(\exp\{th\} - 1)} + x_0 \frac{4h^2\exp\{th\}}{[2h + (k+h)(\exp\{th\} - 1)]^2}$$

$$h = \sqrt{k^2 + 2\sigma^2}$$

Chapter 4

(4.4) $r_t = x(t) + y(t) + \varphi(t), \qquad r(0) = r_0$

(4.5) $dx(t) = -ax(t)dt + \sigma dW_1(t), \qquad x(0) = 0$

$$dy(t) = -by(t)dt + \eta dW_2(t), \qquad y(0) = 0$$

(4.6) $E\{r(t)|\mathcal{F}_s\} = x(s)e^{-a(t-s)} + y(s)e^{-b(t-s)} + \varphi(t)$

$$\text{Var}\{r(t)|\mathcal{F}_s\} = \frac{\sigma^2}{2a}\left[1 - e^{-2a(t-s)}\right] + \frac{\eta^2}{2b}\left[1 - e^{-2b(t-s)}\right] + 2\rho\frac{\sigma\eta}{a+b}\left[1 - e^{-(a+b)(t-s)}\right]$$

(4.7) $r(t) = \sigma\int_0^t e^{-a(t-u)}dW_1(u) + \eta\int_0^t e^{-b(t-u)}dW_2(u) + \varphi(t)$

(4.8) $dx(t) = -ax(t)dt + \sigma d\widetilde{W}_1(t) \qquad dy(t) = -by(t)dt + \eta\rho d\widetilde{W}_1(t) + \eta\sqrt{1-\rho^2}d\widetilde{W}_2(t)$

where $dW_1(t) = d\widetilde{W}_1(t)$ and $dW_2(t) = \rho d\widetilde{W}_1(t) + \sqrt{1-\rho^2}d\widetilde{W}_2(t)$

(4.9) $M(t,T) = \dfrac{1 - e^{-a(T-t)}}{a}x(t) + \dfrac{1 - e^{-b(T-t)}}{b}y(t)$

$$(4.10)\qquad V(t,T) = \frac{\sigma^2}{a^2}\left[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right]$$
$$+\frac{\eta^2}{b^2}\left[T - t + \frac{2}{b}e^{-b(T-t)} - \frac{1}{2b}e^{-2b(T-t)} - \frac{3}{2b}\right]$$
$$+2\rho\frac{\sigma\eta}{ab}\left[T - t + \frac{e^{-a(T-t)} - 1}{a} + \frac{e^{-b(T-t)} - 1}{b} - \frac{e^{-(a+b)(T-t)} - 1}{a+b}\right]$$

$$(4.11)\qquad P(t,T) = \exp\left\{-\int_t^T \varphi(u)du - \frac{1 - e^{-a(T-t)}}{a}x(t) - \frac{1 - e^{-b(T-t)}}{b}y(t) + \frac{1}{2}V(t,T)\right\}$$

$$(4.12)\qquad \varphi(T) = f^M(0,T) + \frac{\sigma^2}{2a^2}\left(1 - e^{-aT}\right)^2 + \frac{\eta^2}{2b^2}\left(1 - e^{-bT}\right)^2 + \rho\frac{\sigma\eta}{ab}(1 - e^{-aT})(1 - e^{-bT})$$

$$(4.13)\qquad \exp\left\{-\int_t^T \varphi(u)du\right\} = \frac{P^M(0,T)}{P^M(0,t)}\exp\left\{-\frac{1}{2}[V(0,T) - V(0,t)]\right\}$$

$$(4.14)\qquad P(t,T) = \frac{P^M(0,T)}{P^M(0,t)}\exp\left\{\mathcal{A}(t,T)\right\}$$

$$\mathcal{A}(t,T) := \frac{1}{2}[V(t,T) - V(0,T) + V(0,t)] - \frac{1 - e^{-a(T-t)}}{a}x(t) - \frac{1 - e^{-b(T-t)}}{b}y(t)$$

$$(4.15)\qquad P(t,T) = A(t,T)\exp\{-B(a,t,T)x(t) - B(b,t,T)y(t)\}$$

$$(4.16)\qquad \sigma_f(t,T) = \sqrt{\sigma^2 e^{-2a(T-t)} + \eta^2 e^{-2b(T-t)} + 2\rho\sigma\eta e^{-(a+b)(T-t)}}$$

Page 152
$$\frac{Cov(df(t,T_1), df(t,T_2))}{dt}$$
$$= \sigma^2\frac{\partial B}{\partial T}(a,t,T_1)\frac{\partial B}{\partial T}(a,t,T_2) + \eta^2\frac{\partial B}{\partial T}(b,t,T_1)\frac{\partial B}{\partial T}(b,t,T_2)$$
$$+\rho\sigma\eta\left[\frac{\partial B}{\partial T}(a,t,T_1)\frac{\partial B}{\partial T}(b,t,T_2) + \frac{\partial B}{\partial T}(a,t,T_2)\frac{\partial B}{\partial T}(b,t,T_1)\right]$$
$$= \sigma^2 e^{-a(T_1+T_2-2t)} + \eta^2 e^{-b(T_1+T_2-2t)}$$
$$+\rho\sigma\eta\left[e^{-aT_1-bT_2+(a+b)t} + e^{-aT_2-bT_1+(a+b)t}\right]$$

$$Corr(df(t,T_1), df(t,T_2)) = \frac{\sigma^2 e^{-a(T_1+T_2-2t)} + \eta^2 e^{-b(T_1+T_2-2t)}}{\sigma_f(t,T_1)\sigma_f(t,T_2)}$$
$$+\frac{\rho\sigma\eta\left[e^{-aT_1-bT_2+(a+b)t} + e^{-aT_2-bT_1+(a+b)t}\right]}{\sigma_f(t,T_1)\sigma_f(t,T_2)}$$

Page 153 $$f(t, T_1 T_2) = \frac{\ln P(t,T_1) - \ln P(t,T_2)}{T_2 - T_1}$$

$$df(t,T_1,T_2) = ...dt + \frac{B(a,t,T_2) - B(a,t,T_1)}{T_2 - T_1}\sigma dW_1(t)$$
$$+\frac{B(b,t,T_2) - B(b,t,T_1)}{T_2 - T_1}\eta dW_2(t)$$

$$\sigma_f(t,T_1,T_2) = \sqrt{\sigma^2 \beta(a,t,T_1,T_2)^2 + \eta^2 \beta(b,t,T_1,T_2)^2 + 2\rho\sigma\eta\beta(a,t,T_1,T_2)\beta(b,t,T_1,T_2)}$$

where $$\beta(z,t,T_1,T_2) = \frac{B(z,t,T_2) - B(z,t,T_1)}{T_2 - T_1}$$

$$\frac{Cov(df(t,T_1,T_2), df(t,T_3,T_4))}{dt}$$
$$\sigma^2 \frac{B(a,t,T_2) - B(a,t,T_1)}{T_2 - T_1} \frac{B(a,t,T_4) - B(a,t,T_3)}{T_4 - T_3}$$
$$+\eta^2 \frac{B(b,t,T_2) - B(b,t,T_1)}{T_2 - T_1} \frac{B(b,t,T_4) - B(b,t,T_3)}{T_4 - T_3}$$
$$+\rho\sigma\eta \left[\frac{B(a,t,T_2) - B(a,t,T_1)}{T_2 - T_1} \frac{B(b,t,T_4) - B(b,t,T_3)}{T_4 - T_3} \right.$$
$$\left. + \frac{B(a,t,T_4) - B(a,t,T_3)}{T_4 - T_3} \frac{B(b,t,T_2) - B(b,t,T_1)}{T_2 - T_1} \right]$$

Page 160 $$\sigma_3 = \sqrt{\sigma_1^2 + \frac{\sigma_2^2}{(\bar{a} - \bar{b})^2} + 2\bar{\rho}\frac{\sigma_1 \sigma_2}{\bar{b} - \bar{a}}}$$

$$dZ_3(t) = \frac{\sigma_1 dZ_1(t) - \frac{\sigma_2}{\bar{a} - \bar{b}} dZ_2(t)}{\sigma_3}, \qquad \sigma_4 = \frac{\sigma_2}{\bar{a} - \bar{b}}$$

Page 161 $$a = \bar{a}, \quad b = \bar{b}, \quad \sigma = \sigma_3, \quad \eta = \sigma_4, \quad \rho = \frac{\sigma_1 \bar{\rho} - \sigma_4}{\sigma_3}$$

$$\varphi(t) = r_0 e^{-\bar{a}t} + \int_0^t \theta(v) e^{-\bar{a}(t-v)} dv$$
$$\bar{a} = a, \quad \bar{b} = b, \quad \sigma_1 = \sqrt{\sigma^2 + \eta^2 + 2\rho\sigma\eta}, \quad \sigma_2 = \eta(a - b)$$
$$\bar{\rho} = \frac{\sigma\rho + \eta}{\sqrt{\sigma^2 + \eta^2 + 2\rho\sigma\eta}}, \quad \theta(t) = \frac{d\varphi(t)}{dt} + a\varphi(t)$$

Managing Credit Risk: The Great Challenge for Global Financial Markets, Caouette, et. al.

Chapter 20

(20.2) $$R_p = \sum_{i=1}^{N} X_i EAR$$

(20.3) $$V_p = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij}$$

(20.5) $$UAL_p = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \sigma_i \sigma_j \rho_{ij} 1$$

Page 403 $$CVaR(CL) = EAD \bullet LGD \bullet \left(\Phi \left(\frac{\sqrt{\rho}\Phi^{-1}(CL) + \Phi^{-1}(PD)}{\sqrt{1-\rho}} \right) - PD \right) \times \frac{1 + (M - 2.5) \bullet b(PD)}{1 - 1.5b(PD)}$$

Bond-CDS Basis Handbook: Measuring, Trading and Analysing Basis Trades, Elizalde, Doctor, and Saltuk

Page 13, Equation 1 $$S = PD \times (1 - R)$$

Page 15, Equation 2 $$FR = \frac{U - AI}{RA} + FC$$

Page 18, Equation 3 $$SS = \frac{PV[c + p] - BP}{RFA}$$

Page 25, Equation 4 $$BTP1 = CN \times (100 - R - U - CP - FC) + BN \times (R + CR - BP - FC)$$

Page 25, Equation 5 $$BTP2 = BN \times (100 + CR - BP - FC) - CN \times (U + CP + FC)$$

Page 43, Equation 7 $$CN = \frac{BP - R}{100 - R - U} \times BN$$

A Survey of Behavioral Finance, Barberis and Thaler

(1) $$(x, p : y, q) = \pi(p)v(x) + \pi(q)v(y)$$

(2) $$\sum_i \pi_i v(x_i) \text{ where } v = \begin{matrix} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\alpha & \text{if } x < 0 \end{matrix} \text{ and } \pi_i = w(P_i) - w(P_i^*),$$

$$w(P) = \frac{P^\gamma}{(P^\gamma + (1-P)^\gamma)^{1/\gamma}}$$

(3) $$\frac{D_{t+1}}{D_t} = e^{g_D + \sigma_D \varepsilon_{t+1}}$$

(4) $$\frac{C_{t+1}}{C_t} = e^{g_C + \sigma_C \eta_{t+1}}$$

(5) $$\begin{pmatrix} \varepsilon_t \\ \eta_t \end{pmatrix} \tilde{} N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & w \\ w & 1 \end{pmatrix} \right), \text{ i.i.d.over time}$$

(6) $$E_0 \sum_{t=0}^{\infty} \rho^t \frac{C_t^{1-\gamma}}{1-\gamma}$$

(7) $$1 = \rho E_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{t+1} \right]$$

(8) $$R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t} = \frac{1 + P_{t+1}/D_{t+1}}{P_t/D_t} \frac{D_{t+1}}{D_t}$$

(9) $$r_{t+1} = \Delta d_{t+1} + \text{const.} \equiv d_{t+1} - d_t + \text{const.}$$

(10) $E_\pi v[(1-w)R_{f,t+1} + wR_{t+1} - 1]$

(11) $E_0 \sum_{t=0}^{\infty} \left[\rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + b_0 \overline{C}_t^{-\gamma} \hat{v}(X_{t+1}) \right]$

(13) $R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$

(14) $p_t - d_t = E_t \sum_{j=0}^{\infty} \rho^t \Delta d_{t+1+j} - E_t \sum_{j=0}^{\infty} \rho^t r_{t+1+j} + E_t \lim_{j\to\infty} \rho^j (p_{t+j} - d_{t+j}) + \text{const.}$

(15) $E_0 \sum_{t=0}^{\infty} \left[\rho^t \frac{C_t^{1-\gamma}}{1-\gamma} + b_0 \overline{C}_t^{-\gamma} \tilde{v}(X_{t+1}, z_t) \right]$

(16) $\overline{r}_i - r_f = \beta_{i.1}(\overline{F}_1 - r_f) + \ldots + \beta_{i,K}(\overline{F}_K - r_f)$

(17) $r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,1}(F_{1,t} - r_{f,t}) + \ldots + \beta_{i,K}(F_{K,t} - r_{f,t}) + \varepsilon_{i,t}$

(18) $R_f = \frac{1}{\rho} e^{\gamma g_C + 0.5\gamma^2 \sigma_C^2}$

(19) $1 = \rho \frac{1+f}{f} e^{g_D - \gamma G_C + 0.5(\sigma_D^2 + \gamma^2 \sigma_C^2 - 2\gamma\sigma_C\sigma_D w)}$

(20) $R_{t+1} = \frac{D_{t+1} + P_{t+1}}{P_t} = \frac{1 + P_{t+1}/D_{t+1}}{P_t/D_t} \frac{D_{t+1}}{D_t} = \frac{1+f}{f} e^{g_D + \sigma_D \varepsilon_{t+1}}$

CAIA Level II: Advanced Core Topics in Alternative Investments, Black, Chambers, Kazemi

Chapter 16

(16.1) $P_t^{\text{reported}} = \alpha + \beta_0 P_t^{\text{true}} + \beta_1 P_{t-1}^{\text{true}} + \beta_2 P_{t-2}^{\text{true}} + \cdots$

(16.2) $P_t^{\text{reported}} = \alpha P_t^{\text{true}} + \alpha(1-\alpha) P_{t-1}^{\text{true}} + \alpha(1-\alpha)^2 P_{t-2}^{\text{true}} + \cdots$

(16.3) $P_t^{\text{true}} = (1/\alpha) \times P_t^{\text{reported}} - [(1-\alpha)/\alpha] \times P_{t-1}^{\text{reported}}$

(16.4) $P_t^{\text{true}} = P_{t-1}^{\text{reported}} + [(1/\alpha) \times (P_t^{\text{reported}} - P_{t-1}^{\text{reported}})]$

(16.5) $R_{t,\text{reported}} \approx \beta_0 R_{t,\text{true}} + \beta_1 R_{t-1,\text{true}} + \beta_2 R_{t-2,\text{true}} + \cdots$

(16.6) $P_t^{\text{reported}} = (1-\rho) P_t^{\text{true}} + \rho P_{t-1}^{\text{reported}}$

(16.7) $\Delta P_t^{\text{reported}} = (1-\rho) \Delta P_t^{\text{true}} + \rho \Delta P_{t-1}^{\text{reported}}$

(16.8) $R_{t,\text{reported}} \approx (1-\rho) R_{t,\text{true}} + \rho R_{t-1,\text{reported}}$

(16.9) $R_{t,\text{true}} = (R_{t,\text{reported}} - \rho R_{t-1,\text{reported}})/(1-\rho)$

(16.10) $\hat{\rho} = \text{corr}(R_{t,\text{reported}} - R_{t-1,\text{reported}})$

(16.11) $\rho_{i,j} = \sigma_{ij}/(\sigma_i \sigma_j)$

(16.12) $R_t^{\text{reported}} = \alpha + \beta_1 R_{t-1}^{\text{reported}} + \beta_2 R_{t-2}^{\text{reported}} + \cdots + \beta_k R_{t-k}^{\text{reported}} + \varepsilon_t$

Managing Investment Portfolio: A Dynamic Process, Maginn, Tuttle, Pinto, McLeavey

Chapter 8

Page 523 $TRCI = CR + RR + SR$

Page 553 $RR_{n,t} = (R_t + R_{t-1} + R_{t-2} + \ldots + R_{t-n})/n$

Page 554 $DD = \sqrt{\frac{\sum_{i=1}^{n}[\min(r_i - r^*, 0)]^2}{n-1}}$

Page 555 $\text{Sharpe Ratio} = \frac{ARR - rf}{SD}$

Page 556 $\text{Sortino Ratio} = \frac{ARR - rf}{DD}$

The Secular and Cyclic Determinants of Capitalization Rates: The Role of Property Fundamentals, Macroeconic Factors, and "Structural Changes," Chervachidze, Costello, Wheaton

(1) $$Log(C_{j,t}) = a_0 + a_1 \log(C_{j,t-1}) + a_2 \log(C_{j,t-4}) + a_3 \log(RRI_{j,t}) + a_4 RTB_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j$$

(1.1) $$RRI_{j,t-s} = RR_{j,t}/MRR_j$$

(2) $$Log(C_{j,t}) = a_0 + a_1 \log(C_{j,t-1}) + a_2 \log(C_{j,t-4}) + a_3 \log(RRI_{j,t-s}) + a_4 RTB_t + a_5 SPREAD_t + a_6 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j$$

(2.1) $$DEBTFLOW_t = TNBL_t/GDP_t$$

(3) $$Log(C_{j,t}) = a_0 + a_1 \log(C_{j,t-1}) + a_2 \log(C_{j,t-4}) + a_3 \log(RRI_{j,t-s}) + a_4 RTB_t + a_5 SPREAD_t + a_6 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t$$

(4) $$Log(C_{j,t}) = a_0 + a_1 yearq + a_2 \log(C_{j,t-1}) + a_3 \log(C_{j,t-4}) + a_4 \log(RRI_{j,t-s}) + a_5 RTB_t + a_6 SPREAD_t + a_7 DEBTFLOW_t + a_7 Q2_t + a_8 Q3_t + a_9 Q4_t + a_{10} D_j$$

Analysis of Financial Time Series, Tsay

Chapter 9

(9.1) $r_{it} = \alpha_i + \beta_{i1} f_{1t} + \cdots + \beta_{im} f_{mt} + \epsilon_{it}, \; t = 1, \ldots, T, \; i = 1, \ldots, k$

(9.2) $\mathbf{r}_t = \alpha + \beta \mathbf{f}_t + \epsilon_t, \; t = 1, \ldots, T$

(9.3) $\mathbf{R}_i = \alpha_i \mathbf{1}_T + \mathbf{F}\beta_i' + \mathbf{E}_i$

(9.4) $\mathbf{R} = \mathbf{G}\xi' + \mathbf{E}$

(9.5) $r_{it} = \alpha_i + \beta_i r_{mt} + \epsilon_{it}, \; i = 1, \ldots, k \; \; t = 1, \ldots, T$

(9.11) $\text{Var}(y_i) = \mathbf{w}_i' \mathbf{\Sigma}_r \mathbf{w}_i, \; i = 1, \ldots, k$

(9.12) $\text{Cov}(y_i, y_j) = \mathbf{w}_i' \mathbf{\Sigma}_r \mathbf{w}_j, \; i, j = 1, \ldots, k$

(9.13) $\sum_{i=1}^{k}\text{Var}(r_i) = \text{tr}(\mathbf{\Sigma}_r) = \sum_{i=1}^{k}\lambda_i = \sum_{i=1}^{k}\text{Var}(y_i)$

(9.14) $\hat{\mathbf{\Sigma}}_r \equiv [\hat{\sigma}_{ij,r}] = \frac{1}{T-1}\sum_{t=1}^{T}(\mathbf{r}_t - \bar{\mathbf{r}})(\mathbf{r}_t - \bar{\mathbf{r}})', \;\; \bar{\mathbf{r}} = \frac{1}{T}\sum_{t=1}^{T}\mathbf{r}_t$

(9.15) $\hat{\rho}_r = \hat{\mathbf{S}}^{-1}\hat{\mathbf{\Sigma}}_r\hat{\mathbf{S}}^{-1}$

(9.16) $\mathbf{r}_t - \mu = \beta\mathbf{f}_t + \epsilon_t$

(9.17) $\mathbf{\Sigma}_r = \text{Cov}(\mathbf{r}_t) = E[(\mathbf{r}_t - \mu)(\mathbf{r}_t - \mu)'] = E[(\beta\mathbf{f}_t + \epsilon_t)(\beta\mathbf{f}_t + \epsilon_t)'] = \beta\beta' + \mathbf{D}$

(9.18) $\text{Cov}(\mathbf{r}_t, \mathbf{f}_t) = E[(\mathbf{r}_t - \mu)\mathbf{f}_t'] = \beta E(\mathbf{f}_t\mathbf{f}_t') + E(\epsilon_t\mathbf{f}_t') = \beta$

(9.19) $\hat{\beta} \equiv [\hat{\beta}_{ij}] = \left[\sqrt{\hat{\lambda}_1}\hat{\mathbf{e}}_1 | \sqrt{\hat{\lambda}_2}\hat{\mathbf{e}}_2 | \cdots | \sqrt{\hat{\lambda}_m}\hat{\mathbf{e}}_m\right]$

(9.20) $\text{LR}(m) = -\left[T - 1 - \frac{1}{6}(2k+5) - \frac{2}{3}m\right]\left(\ln|\hat{\mathbf{\Sigma}}_r| - \ln|\hat{\beta}\hat{\beta}' + \hat{\mathbf{D}}|\right)$

Handbook of Fixed Income Securities, Fabozzi

Chapter 69

(69 – 4) Asset Allocation $\sum_s (w_s^P - w_s^B) \cdot R_s^B$

(69 – 5) Security Selection $\sum_s w_s^P \cdot (R_s^P - R_s^B)$

(69 – 12) $\alpha_k^P f_k^P - \alpha_k^B f_k^B = \sum_s \alpha_{k,s}^P f_{k,s}^P - \sum_s \alpha_{k,s}^B f_{k,s}^B$

Chapter 70

(70 – 1) Asset Allocation $w^P \cdot \sum_s \left(\frac{w_s^P}{w^P} - \frac{w_s^B}{w^B}\right) \cdot (TR_s^B - TR^B)$

(70 – 2) Sector Management $\sum_s w_s^P \cdot (TR_s^P - TR_s^B)$

(70 – 3) Top-Level Exposure $(w^P - w^B) \cdot TR^B$

(70 – 4) Asset Allocation $w^P \cdot \sum_s \left(\frac{w_s^P}{w^P} - \frac{w_s^B}{w^B}\right) \cdot (ER_s^B - ER^B)$

(70 – 5) Sector Management $\sum_s w_s^P \cdot (ER_s^P - ER_s^B)$

(70 – 6) Top-Level Exposure $(w^P - w^B) \cdot ER^B$

(70 – 7) Outperformance from average carry $\left(y_{avg}^P - y_{avg}^B\right) \cdot \Delta t$

(70 – 8) Key rate contributions $\sum_j \left(\omega_j^P\left(y_j - y_{avg}^P\right) - \omega_j^B\left(y_j - y_{avg}^B\right)\right) \cdot \Delta t$

(70 – 9) Outperformance from avg. parallel shifts $-\left(OAD^P - OAD^B\right) \cdot \Delta y_{avg}$

(70 – 10) Outperformance from reshaping $-\sum_j \left(KRD_j^P - KRD_j^B\right) \cdot (\Delta y_j - \Delta y_{avg})$

(70 − 11) Asset Allocation

$$-OASD^P \cdot \sum_s \left(\frac{w_s^P OASD_s^P}{OASD^P} - \frac{w_s^B OASD_s^B}{OASD^B} \right) \cdot \left(\Delta OAS_s^B - \Delta OAS^B \right)$$

(70 − 12) Security Selection $-\sum_s w_s^P OASD_s^P \cdot \left(\Delta OAS_s^P - \Delta OAS_s^B \right)$

(70 − 13) Spread Duration Mismatch $-(OASD^P - OASD^B) \cdot \Delta OAS^B$

(70 − 14) Asset Allocation $-\sum_s \left(w_s^P OASD_s^P - w_s^B OASD_s^B \right) \cdot \Delta OAS_s^B$

(70 − 15) Security Selection $-\sum_s w_s^P OASD_s^P \cdot \left(\Delta OAS_s^P - \Delta OAS_s^B \right)$

Introduction to Credit Risk Modeling, 2nd ed., Bluhm, Overbeck, Wagner

Chapter 6

Page 237

$M_n = M_1^n$

Guarantees and Target Volatility Funds, Morrison and Tadrowski

Page 4

$$w_t^{equity} = \min \left(\frac{\sigma_{target}}{\hat{\sigma}_t^{equity}}, 100\% \right)$$

$$\left(\hat{\sigma}_t^{equity} \right)^2 = \lambda \left(\hat{\sigma}_{t-\Delta t}^{equity} \right)^2 + (1-\lambda) \frac{1}{\Delta t} \left(\ln \left(\frac{S_t}{S_{t-\Delta t}} \right) \right)^2$$

Proxy Functions for the Projection of Variable Annuity Greeks, Clayton, Morrison, Turnbull, and Vysniasuskas

Page 4

$$\sum_i (\hat{V}_{i,t} - V_t^{proxy}(S_{i,t}, R_{i,t}\sigma_{i,t}))^2$$

$$\Delta_t^{proxy}(S, R, \sigma) = \frac{\partial}{\partial S} V_t^{proxy}(S, R, \sigma)$$

$$\rho_t^{proxy}(S, R, \sigma) = \frac{\partial}{\partial R} V_t^{proxy}(S, R, \sigma)$$

$$\mathcal{V}_t^{proxy}(S, R, \sigma) = \frac{\partial}{\partial \sigma} V_t^{proxy}(S, R, \sigma)$$

Page 5

$$\begin{pmatrix} S_{i,t}^{stress1} - S_{i,t}^{base} & R_{i,t}^{stress1} - R_{i,t}^{base} & \sigma_{i,t}^{stress1} - \sigma_{i,t}^{base} \\ S_{i,t}^{stress2} - S_{i,t}^{base} & R_{i,t}^{stress2} - R_{i,t}^{base} & \sigma_{i,t}^{stress2} - \sigma_{i,t}^{base} \\ S_{i,t}^{stress3} - S_{i,t}^{base} & R_{i,t}^{stress3} - R_{i,t}^{base} & \sigma_{i,t}^{stress3} - \sigma_{i,t}^{base} \end{pmatrix} \begin{pmatrix} \hat{\Delta}_{i,t} \\ \hat{\rho}_{i,t} \\ \hat{\mathcal{V}}_{i,t} \end{pmatrix} = \begin{pmatrix} \hat{V}_{i,t}^{stress1} - \hat{V}_{i,t}^{base} \\ \hat{V}_{i,t}^{stress2} - \hat{V}_{i,t}^{base} \\ \hat{V}_{i,t}^{stress3} - \hat{V}_{i,t}^{base} \end{pmatrix}$$

$$\sum_i (\hat{\Delta}_{i,t} - \Delta_t^{proxy}(S_{i,t}, R_{i,t}, \sigma_{i,t}))^2$$

$$\sum_i (\hat{\rho}_{i,t} - \rho_t^{proxy}(S_{i,t}, R_{i,t}, \sigma_{i,t}))^2$$

$$\sum_i (\hat{\mathcal{V}}_{i,t} - \mathcal{V}_t^{proxy}(S_{i,t}, R_{i,t}, \sigma_{i,t}))^2$$

$$\sum_i (\hat{V}_{i,t}^{base} - V_t^{proxy}(S_{i,t}, R_{i,t}, \sigma_{i,t}))^2$$

Page 6

$$\sum_i w\left(\frac{S_{i,t} - S}{h^S}, \frac{R_{i,t} - R}{h^R}, \frac{\sigma_{i,t} - \sigma}{h^\sigma}\right) \left(\hat{V}_{i,t}^{base} - V_t^{proxy}(S_{i,t}, R_{i,t}, \sigma_{i,t})\right)^2$$

$$\sum_{i,t} w\left(\frac{t - s}{h^S}, \frac{S_{i,t} - S}{h^S}, \frac{R_{i,t} - R}{h^R}, \frac{\sigma_{i,t} - \sigma}{h^\sigma}\right) \left(\hat{V}_{i,t}^{base} - V^{proxy}(t, S_{i,t}, R_{i,t}, \sigma_{i,t})\right)^2$$

Recent Advances in Credit Risk Modeling, Capuano, Chan-Lau, Gasha, Medeiros, Santos, and Souto

(II.1) $E = \max(0, V - D)$

(II.2) $DD_T = \dfrac{\ln \dfrac{V}{D} + \left(\mu - \dfrac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}}$

(II.3) $x_i = a_i M + \sqrt{1 - a_i^2} Z_i$

(II.4) $\text{Prob}\{x_i < \bar{x}_i | M\} = q_i(t|M) = \Phi\left(\dfrac{\bar{x}_i - a_i M}{\sqrt{1 - a_i^2}}\right)$

(II.5) $p^{K+1}(0, t|M) = p^K(0, t|M)(1 - q_{K+1}(t|M))$

(II.6) $p^{K+1}(l, t|M) = p^K(l, t|M)(1 - q_{K+1}(t|M)) + p^K(l - 1, t|M) q_{K+1}(t|M),$
$l = 1, \ldots, K$

(II.7) $p^{K+1}(K + 1, t|M) = p^K(K, t|M) q_{K+1}(t|M)$

(II.8) $p(l, t) = \int_{-\infty}^{\infty} p^N(l, t|M) \phi(M) dM$

(III.1) $\tau = \inf\{t \geq 0 | V_t \leq K\}$

Market Models: A Guide to Financial Data Analysis, Chapter 6, Aledander

(6.1) $\mathbf{P} = \mathbf{XW}$

(6.2) $X_i = w_{i1}P_1 + w_{i2}P_2 + \cdots + w_{ik}P_k$

(6.3) $\sigma_K - \sigma_{ATM} = -b(K - S)$

(6.4) $\Delta(\sigma_K - \sigma_{ATM}) = w_{K1}P_1 + w_{K2}P_2 + w_{K3}P_3$

(6.5) $P_{i,t} = \gamma_{i,t}\Delta S + \varepsilon_{i,t}$

(6.6) $\Delta\sigma_{ATM} = \alpha + \beta\Delta S + \varepsilon$

(6.7) $\beta_{K,t} = \beta_t + \Sigma w_{Ki}\gamma_{i,t}$

(6.8) $\mathbf{y} = \mathbf{a} + \mathbf{Pb} + \mathbf{e}$

(6.9) $\mathbf{y} = \mathbf{a} + \mathbf{X}^*\mathbf{b}^* + \mathbf{e}$

(6.10) $\mathbf{y} = \mathbf{c} + \mathbf{Xd} + \mathbf{e}$

(6.11) $\text{rcac} = \underset{(1.45)}{0.0003} + \underset{(14.71)}{0.1943}\ \text{rparibas} + \underset{(17.21)}{0.2135}\ \text{rsocgen} + \underset{(20.55)}{0.2995}\ \text{rdan}$

(6.12) $0.85867P_1 + 0.047495P_2 + 0.091244P_3 + 0.35181P_4$

Stochastic Modeling, Theory and Reality from an Actuarial Perspective

(I.B-1) $dS = \mu S dt + \sigma S dz$

(I.B-2) $\ln S_T \sim N(\ln S_0 + (r - \sigma^2/2)T, \sigma\sqrt{T})$

(I.B-3) $\mu^* = \ln S_0 + (r - \sigma^2/2)T, \quad \sigma^* = \sigma\sqrt{T}$

(I.B-4) $\hat{c} = \frac{1}{N}\sum_{i=1}^{N} c_i$

(I.B-5) $c = S_0 N(d_1) - Ke^{-rT}N(d_2)$

(I.B-6) $d_1 = \frac{\ln(S_0/K) + (r + \sigma^2/2)T}{\sigma\sqrt{T}}, \quad d_2 = d_1 - \sigma\sqrt{T}$

(I.B-7) $\text{MC sampling error} = \frac{1}{\sqrt{N}} Stdev(c_i)$

(I.B-8) $\tilde{f} = \frac{1}{2}(f(u_1) + f(u_2))$

(I.B-9) $\frac{1}{\sqrt{N}} Stdev(\tilde{f})$

(I.B-10) $f^*(u) - g^*(u) + g(u)$

(I.B-11) $\frac{1}{\sqrt{N}} Stdev(f(u) - g(u))$

(I.B-12) $h_i = \frac{1}{n_i} \sum_{j=1}^{N} f(v_i^{(j)})$

(I.B-13) $\hat{f} = \sum_{i=1}^{k} (x_{i+1} - x_i) h_i$

(I.B-14) $\sum_{i=1}^{k} (x_{i+1} - x_i) \frac{Stdev(h_i^{(j)})}{\sqrt{n_i}}$ (there is an error in the book formula, the upper limit of the sum should be k)

(I.B-15) $\frac{1}{N} \sum_{i=1}^{N} \frac{f(z_i)}{g(z_i)}$

(I.B-16) $\sum_{i=1}^{N} Stdev\left(\frac{f(z_i)}{g(z_i)}\right)$

(I.B-17) $S_0 = e^{-r\Delta t}[pS_0 u + (1-p)S_0 d]$

(I.B-18) $p = \frac{e^{r\Delta t} - d}{u - d}$

(I.B-19) $u = e^{\sigma\sqrt{\Delta t}}$ and $d = u^{-1}$

(I.B-20) $C_0 = e^{-r\Delta t}[pC_u + (1-p)C_d]$

(I.B-21) $S_m = S_0 u^n d^{m-n}, \quad n = 0, 1, \ldots, m$

(there is an error in the book formula, it is n that goes from 0 to m)

(I.B-22) $S_m = S_0(1-\eta) u^n d^{m-n}, \quad n = 0, 1, \ldots, m$ (same error)

(I.B-23) $p_- = -\sqrt{\frac{\Delta t}{12\sigma^2}}\left(r - \frac{1}{2}\sigma^2\right) + \frac{1}{6}, \quad p_0 = \frac{2}{3}, \quad p_+ = p_- + \frac{2}{6},$

$u = e^{\sigma\sqrt{3\Delta t}}, \quad d = u^{-1}$

(I.B-24) $\log\left(\frac{S_t}{S_r}\right) \sim N(\mu(t-r), \sigma^2(t-r))$

(I.B-25) $\log\left(\frac{S_{t+1}}{S_t}\right) |\rho(t) \sim N(\mu_{\rho(t)}, \sigma^2_{\rho(t)})$

(I.B-26) $p_{ij} = \Pr(\rho(t+1) = j | \rho(t) = i), \quad i = 1, 2, \ldots, K, \quad j = 1, 2, \ldots, K$ (there is an error in the book formula, $y+1$ should be $t+1$)

(I.B-27) $L(\Theta) = f(y_1|\Theta) f(y_2|\Theta, y_1) f(y_3|\Theta, y_1, y_2) \cdots f(y_n|\Theta, y_1, y_2, \ldots, y_{n-1})$

(I.B-28) $f(\rho(t), \rho(t-1), y_t|\Theta, y_1, y_2, \ldots, y_{t-1})$ for $\rho(t) = 1, 2$ and $\rho(t-1) = 1, 2$

(I.B-29) $\pi_{i,t-1} \equiv p(\rho(t-1) = i|\Theta, y_1, y_2, \ldots, y_{t-1})$

(I.B-30) $p_{ij} = p(\rho(t) = j|\rho(t-1) = i, \Theta)$

(I.B-31) $g_{j,t} = f(y_t|\rho(t) = j, \Theta) = \phi\left(\frac{y_t - \mu_j}{\sigma_j}\right) = \frac{1}{\sigma_j\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{y_t - \mu_j}{\sigma_j}\right)^2\right]$

(I.B-32) $$\pi_{i,t} = \frac{\sum_{k=1}^{2} \pi_{k,t-1} \times p_{ki} \times g_{i,t}}{\sum_{j=1}^{2}\sum_{i=1}^{2} \pi_{i,t-1} \times p_{ij} \times g_{j,t}}$$

(I.B-33) $$\pi_{1,0} = \frac{p_{21}}{p_{12}+p_{21}}, \quad \pi_{2,0} = \frac{p_{12}}{p_{12}+p_{21}}$$

(I.B-34) $$f(y_1|\Theta) = f(\rho(0)=1, y_1|\Theta) + f(\rho(0)=2, y_1|\Theta)$$
$$= \pi_{1,0}\phi\left(\frac{y_1-\mu_1}{\sigma_1}\right) + \pi_{2,0}\phi\left(\frac{y_1-\mu_2}{\sigma_2}\right)$$

(II.A-1) $$S(0,1) = -\ln\{1/[1+C(0,1)]\}$$
$$S(0,2) = -(1/2)\ln\{[1-C(0,2)\exp(-S(0,1))]/[1+C(0,2)]\}$$
$$S(0,3) = -(1/3)\ln\{[1-C(0,3)\exp(-S(0,1)-C(0,3)\exp(-2S(0,2))]/[1+C(0,3)]\}$$

(II.A-2) $$\Delta r = \sigma(r/\bar{r})^{\gamma}\phi\Delta t$$

(II.A-3) $$F_t(t+T) = F_{t-1}(t+T) + \sum_q \Lambda_{q,T+1}\phi_{q,t}$$

(II.A-4) $$F_t(t+T) = F_{t-1}(t+T) + \sum_q \left[\Lambda_{g,T+1}\phi_{q,t} + \Lambda_{q,T+1}\left(-\Lambda_{q,T+1}/2 + \sum_{i=1}^{T+1}\Lambda_{q,i}\right)\right]$$

(II.A-5) $$E[\exp(-F_0(0) - F_1(1) - F_2(2) - \cdots - F_N(N))]$$
$$= \exp(-F_0(0) - F_0(1) - F_0(2) - \cdots - F_0(N))$$

(II.A-6) $$PV = E[\exp(-F_0(0) - F_1(1) - F_2(2) - \cdots - F_N(N))CF_N]$$

(II.A-7) $$F_t(t+T) = F_{t-1}(t+T) + [\sum_g \Lambda_{q,T+1}\phi_{q,t}]$$
$$+[\delta_{t\le t_{\text{target}}}(1/t_{\text{target}})(F_\infty(t_{\text{target}}+T) - F_0(t_{\text{target}}+T))]$$
$$+[(1-\delta_{t\le t_{\text{target}}})(F_\infty(t_{\text{target}}+T) - F_\infty(t_{\text{target}}+T+1)]$$

(II.A-8) $$F_t(t+T) = F_{t-1}(t+T) + k_T\sum_q \left[\Lambda_{q.T+1}\phi_{q,t} + \Lambda_{q,T+1}\left(-\Lambda_{q,T+1}/2 + \sum_{i=1}^{T+1}\Lambda_{q,i}\right)\right]$$

(II.A-9) $$\Lambda_{1,j} = \Lambda_{1,1}\exp[-a(j-1)], \quad \Lambda_{i>1,j} = 0$$

(II.A-10) $$Factor1 = Factor1^{(0)} + \rho_{12}Factor2^{(0)} + \rho_{13}Factor3^{(0)}$$
$$Factor2 = (1-\rho_{12}^2)^{1/2}Factor2^{(0)} + \rho_{13}Factor3^{(0)}$$
$$Factor3 = (1-\rho_{13}^2-\rho_{13}^2)^{1/2}Factor3^{(0)}$$

(II.A-11) $$S_t = S_{t-1}\exp[F_{t-1}(t-1) + \sigma_{t-1}\phi_{t-1}^{(e)} - \sigma_{t-1}^2/2]$$

(II.A-12) $$X(t+1) = X(t) * \exp(RFF - RFD)$$

(II.A-13) $$X(t+\Delta t) = X(t) * \exp((RFF - RFD - vol^2/2) * \Delta t + sqrt(\Delta t) * vol * Z)$$

(II.A-14) $$dS_t = \mu_t S_t dt + \sigma_t S_t dW_t$$

(II.A-15) $$dS_t = S_t\exp\{(\mu_t - \sigma_t^2/2)dt + \sigma_t dW_t\}$$

(II.A-16) $$S_t = S_{t-1}\exp\{(\mu_t - \sigma_t^2/2)dt + \sigma_t\phi_t\}$$

(II.A-17) $S_t = S_{t-1} \exp\{(F_t(t+dt) - \sigma_t^2/2)dt + \sigma_t \phi_t\}$

(II.A-18) $S_t = S_{t-1} \exp\{(F_t(t+dt) - q - \sigma_t^2/2)dt + \sigma_t \phi_t\}$

(II.A-19) $dS_t = \mu_t S_t dt + \sigma(S_t, t) dW_t$

(II.A-20) $dS_t = \mu_t S_t dt + \sigma(t) S_t^{\alpha} dW_t$

(II.A-21) $dS_t = \mu_t S_t dt + \sqrt{V_t} S_t dW_t, \quad dV_t = \kappa(\theta - V_t)dt + \upsilon\sqrt{V_t} dZ_t, \quad d[S,V] = \rho dt$

(II.A-22) $\mu_t = F_t(t+dt) + rp$

(II.A-23) $\sigma_t^F = sqrt\left[\frac{(\sigma_i^S)^2 * t_i - \sigma_{i-1}^S * t_{i-1}}{t_i - t_{i-1}}\right]$

(II.A-24) $\min \sum_{i=1}^{n} w_i \frac{(\sigma_i^{\text{model}} - \sigma_i^{\text{market}})^2}{\sigma_i^{\text{model}}}$

(II.A-25) $E(\textit{Value of Equity}) = A(\textit{Value of Assets}) * N(d_1)$

$$-F(\textit{Face Value of Debt}) * e^{-rT} * N(d_2)$$

$$d_1 = \frac{\log(A/F) + (r + \sigma_A^2/2) * T}{\sigma_A * \sqrt{T}}$$

$$d_2 = d_1 - \sigma_A * \sqrt{T}$$

$$\sigma_E = \sigma_A * N(d_1) * \frac{A}{E}$$

$$\textit{Risk Neutral Probability of Default} = N(-d_2)$$

$$\textit{Recovery Rate} = \frac{A * N(-d_1)}{N(-d_2)}$$

(II.A-26) $Threshold = \Phi^{-1}(D)$

(II.A-27) $q(t) = h(t) * \exp\left[-\int_0^t h(\tau)d\tau\right]$

(II.A-28) $\pi(t) - 1 - \int_{\tau=0}^{t} q(\tau)d\tau$

(II.A-29) $Spread = \frac{\int_{t=0}^{T}[1 - R - A(t) * r]q(t)\nu(t)dt}{\int_{t=0}^{T} q(t) * \{u(t) + e(t)]dt + \pi(T) * u(T)}$

(II.A-30) $d\ln(h_t) = \alpha(\beta - \ln(h_t))dt + \gamma dz_t$