Using expert opinion in actuarial science

Why am I the last one?

Michel Jacques Mathieu Pigeon

École d'actuariat
Université Laval

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Outline

Motivation

The big picture

Toy example

Quick review of multiple experts opinion models

Calibrated models

Rehabilitation costs for mine tailings sites

- Regulators require a rehabilitation plan
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Third party liability insurance for aviation

- “State” airlines are allowed to self insure
- Cases settled off court: confidentiality

What can be done?

- Apply rules of thumb
- Do not consider: risk is noninsurable
- Use partial knowledge hidden in someone's head

How to access experts opinion?

- Choice of questions, formulation of questions
- Choice of experts
- Psychological assessment of probabilities
- Multiple experts: aggregation or consensus?

What is the probability that I speak during time slot *i* at ARC 2006?

- Time slots $\{1, \ldots, 67\}$
- Experts: 1 and 2

Simple models

Linear combination of experts' data with

- equal weights
- weights linked to the experts quality

Caveat

- Arbitrariness
- No dependence between experts

Bayesian models

- Error between true value and expert assessment modelled by a (multidimensional) distribution
- Choose a priori distribution for true value
- Use Bayes Theorem

Caveat

- Arbitrariness in choice of parameters

Why calibration?

- Actuaries would use experts in domains outside their field of knowledge
- How to assess their quality?

Idea

- Ask experts opinion on things you know, but they don't
- Use their answers to assess quality

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  - Number of emails with Louis Doray last 2 months

Mendel-Sheridan setting

- Calibrating variables: $X_j,\ j = 1, \ldots, n$
- Objective variable: X_{n+1}
- k experts give R quantiles (the same for all experts)

Example

- $k = 2, \; R = 3$

	10%	50%	90%
Expert 1	1	4	7
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- $X_j \in J_{2,3}$: Joint intersection of interquantile intervals
- or $S_{2,3} = 1$
- Generalized to $J_{m_1,\ldots,m_k}$ and $S_{m_1,\ldots,m_k}$

- Let

$$p_{m_1,m_2,\ldots,m_k} = \mathrm{P}[X_j \in J_{m_1,\ldots,m_k}]$$

measure the (joint) quality of experts (identical for all calibrating variables)

Philosophy of model

- Assume a priori distribution for $p_{m_1,m_2,\ldots,m_k}$ (Dirichlet with parameters $(a_1,\ldots,a_M)$ where $M=(R+1)^k$): $f(p)$

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- Use Bayes Theorem to incorporate calibration: $f(p|S)$ (again Dirichlet, with parameters $(a_1 + S_1, \ldots, a_M + S_M)$)
- Get a predictive distribution for X_{n+1}, $f(x_{n+1}|S)$ by conditioning on the $p_{m_1,m_2,\ldots,m_k}$:

$$\mathrm{P}[X_{n+1} \in J_{m_1,\ldots,m_k}|S] = \frac{S_{m_1,\ldots,m_k} + a_{m_1,\ldots,m_k}}{n + \sum_i a_i}$$

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  X_{n+1} can lie in J_{21}, but cannot lie in J_{23}
- Given the experts' quantiles for X_{n+1}, revise distribution to $f(x_{n+1}|S, q)$:

$$P[X_{n+1} \in J_{m_1,\ldots,m_k}|S, q] = \frac{P[X_{n+1} \in J_{m_1,\ldots,m_k}|S]}{\sum_{\text{possible } r\text{'s}} P[X_{n+1} \in J_{r_1,\ldots,r_k}|S]}$$

if $\neq 0$

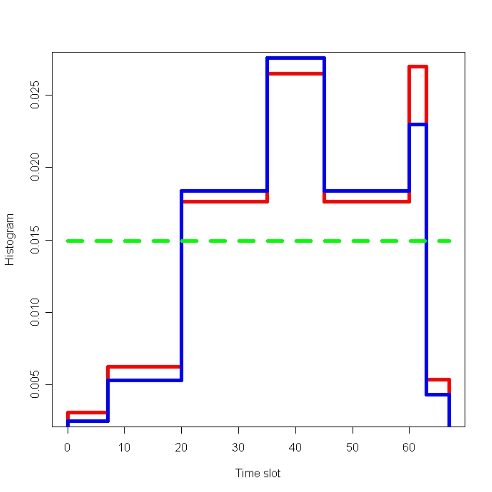
Results of toy example

Stays	10%	50%	90%
Expert 1	1	4	7
Expert 2	1	2	4

Emails	10%	50%	90%
Expert 1	2	4	10
Expert 2	1	3	5

Time slot	10%	50%	90%
Expert 1	7	35	60
Expert 2	20	45	63

True values: Stays: 3, Emails: 4 ($S_{2,3} = 2$)


Histogram
0.005
0.010
0.015
0.020
0.025
0
10
20
30
40
50
60
Time slot

Summary

- Expert opinion could (more or less) replace hard data
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Outlook

- Do it for real for mining rehabilitation costs and aviation third party liability
- R package

(Very) Selected Readings

- Roger M. Cooke, *Experts in Uncertainty*, Oxford University Press, 1991.
- A. O'Hagan, "Eliciting expert beliefs in substantial practical applications", *The Statistician*, 47 (1): 21–35, 1998.
- P. H. Garthwaite et al., "Statistical Methods for Eliciting Probability Distributions", *Journal of the American Statistical Association*, 100 (470): 680–700, 2005.