Allocation of Required Capital by Line of Business

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ABSTRACT

Insurance companies have various methods of determining required capital appropriate to their mix of businesses and risks. Most utilize a formula such as a multiple of the NAIC Risk-Based Capital (RBC) formula or that of a rating agency. To make rational decisions about allocating actual capital resources, management must know how to allocate their total company required capital by line of business. This is complicated because most RBC formulae recognize the benefits of size and diversification and are nonlinear.

This paper will show how the NAIC formula and others can be allocated by line of business by the use of partial derivatives of RBC. The basis of the method is that the formulae are homogeneous functions of their major parameters. This is the theoretical approach presented in a paper by Fraser for allocating taxes by line of business under the 1959 Tax Act for life companies. The particular form of the nonlinearity in the NAIC RBC formula makes the allocation simple and elegant. This method has the property of distributing the benefits of diversification. It will be contrasted with alternative methods commonly used by companies and shown how it can lead to different business strategies that could lead to higher returns on equity.

The paper starts with a brief description and analysis of the current NAIC Life RBC formula. A description of homogeneous functions will show how marginal analysis can be used to give a simple, easily implemented method of allocating the total required capital by line of business. This method is intuitively appealing, additive to the total company RBC, and in some ways optimal.

INTRODUCTION

All managers must decide how to ration their limited equity capital among lines of business or business units in order to achieve their company's goals. If capital is allocated to those businesses with the highest returns on equity (ROE), the total company's ROE can be improved. The first step in this process is to determine the capital needs of the business unit or line of business (I will use LOB to denote either of these.) For life insurance companies this often starts with the Regulatory Capital requirements, which in the U.S. is defined by the NAIC Life Risk-Based Capital (RBC) formula. Many companies use a multiple of this formula for their target level of statutory surplus. For GAAP capital management and ROE computations, GAAP adjustments are added to this statutory target surplus to determine the GAAP capital requirement for each LOB, which serves as the denominator in the ROE. In addition, investment income on the target surplus by LOB is usually added to the line's results when computing the returns. Thus the allocation of RBC by LOB is crucial to this process. Different methods of allocation, which require more or less capital in a line, will change the computed ROE and possibly lead to drastically different decisions regarding which businesses appear attractive.

A characteristic of the NAIC Life RBC formula is that the benefits of diversification and size are reflected, resulting in a formula that is nonlinear. Some companies have an internally developed formula or process, which will share some of the characteristics of the NAIC Life RBC formula and allow a development and analysis similar to the one presented here. The same is true of the NAIC P&C RBC formula. When the formula is applied to each LOB and the result added, the total is higher than the total company's RBC. I will refer to the application of the RBC formula for each line of business as the "separate company" approach. Regulatory requirements and the publicly disseminated information concern the total company's capital in relation to the total company's RBC, making this the important measure, not the "separate company" values or their total. Actuaries use several methods to determine the RBC by LOB. These implicitly incorporate a method of distributing the benefits of diversification and size. The situation is somewhat analogous to the distribution of fixed costs or overhead in expense allocations. It is very important to know what the method being used implies in order to interpret the results intelligently.

Commonly used methods of allocating RBC start with the separate company approach. Many companies have a dominant line of business which becomes the "plug," that is, the smaller lines are given their calculated RBC and the dominant line is given the balance which adds up to the correct total. This gives the entire benefit of diversification and size to this dominant line and puts a higher burden on the smaller lines. They would have to achieve higher returns to achieve ROE targets.

Another approach takes the separate company RBC for each line and their sum. The actual total company RBC is then multiplied by the LOB's separate company RBC divided by the sum of all the separate company RBC values. This method distributes the benefit of diversification and size in proportion to the lines' separate company RBC values. This places a somewhat lower burden on the smaller lines, but may still underestimate the diversification benefit that these lines provide.

An alternative algorithm will be described in this paper, which is a simple modification of the separate company approaches and is easily implemented. It is based upon a marginal analysis of the RBC formula, which provides other useful insights into the company's risk exposure and RBC results. Some life insurers used the same procedures in the 1960's and 1970's to allocate federal income tax by line of business. The federal tax law at that time involved nonlinearities, which caused the separately computed tax for each LOB to not add to the total company's computed tax. Fraser (1962) noted that a marginal analysis could be applied to the tax formula to develop each line's tax so that the sum of the lines' taxes was additive to the total company's tax. In the case of the RBC formula, the nonlinearity is much simpler so the algorithm is simple and elegant.

This paper will start with a brief overview of the NAIC formula, without going into too much detail. I will then define and present some properties of homogeneous functions. The NAIC RBC formula will be shown to be an example of a homogeneous function. The algorithm for computing the RBC allocation will be presented as well as the marginal analysis of the formula upon which it is based. An example will both illustrate the procedure and its differences with the other methods.

THE NAIC RBC FORMULA

Actuaries have divided the risks of insurers into 4 broad categories. They are asset risk (C-1), insurance risk (C-2), interest rate risk (C-3) and general business risk (C-4). The NAIC started with these and over the years sub-divided them. The formula has become considerably more complex in an attempt to better assess the risks of insurers. Each of the risks C-j is a formula which is a function of the elementary risk parameters such as amount of life insurance inforce, the amount of NAIC Class 1-6 bonds, group health premium, reserves for life and annuities, etc. Most of these enter linearly in the C-j but some are piecewise linear (such as insurance inforce) and some are even discrete variables (such as the number of distinct bond issuers in the bond portfolio.) In fact, some calculations may require cashflow testing in order to evaluate the C-j.

The current formula is

$$RBC = C\text{-}0 + C\text{-}4a + \sqrt{(C\text{-}1_o + C\text{-}3a)^2 + (C\text{-}1_{cs})^2 + (C\text{-}2)^2 + (C\text{-}3b)^2 + (C\text{-}4b)^2} \quad , \quad \text{where}$$

C-0 = subsidiary insurance & investment companies' RBC
C-4a= business risk
$C\text{-}1_o$= asset risk, for other than common equity-like assets
C-3a= interest rate risk
$C\text{-}1_{cs}$= risk from common equity-like assets (common stocks, affiliated preferred stocks, and Schedule BA assets classified as common equity)
C-2 = insurance risk
C-3b= health credit risk
C-4b=health administrative expense business risk.

The formula actually has a cross-term involving the product of ($C\text{-}1_o$ +C-3a) and $C\text{-}1_{cs}$, which allows for the possibility of a nonzero correlation between the two factors.

This correlation is currently set to zero. The analysis below could be easily extended to the nonzero correlation case. In the formula, C-0 is computed by a look-through approach by applying the formula to the appropriate subsidiaries. Some of those subsidiaries could be P&C insurers and their variant of the formula would have to be applied. While this article will not investigate the P&C formula, it is of similar form to the life companies' version and is completely amenable to the same technique presented here. C-4a is a simple linear function of premiums and separate account liabilities. Thus C-0 and C-4a are allocable either directly or through the procedures that we will develop here. We need to concentrate mainly on $C\text{-}1_o$, C-3a, $C\text{-}1_{cs}$, C-2, C-3b, C-4b, which occur in the square root.

Letting $SQRT = \sqrt{(C\text{-}1_o + C\text{-}3a)^2 + (C\text{-}1_{cs})^2 + (C\text{-}2)^2 + (C\text{-}3b)^2 + (C\text{-}4b)^2}$ and taking

the partial derivatives we find that

$$\frac{?RBC}{?C\text{-}1_0} = \frac{(C\text{-}1_o + C\text{-}3a)}{SQRT}$$, which I will call $w\text{-}1_0$ and this also equals $\frac{?RBC}{?C\text{-}3a}$.

Similarly, the other partial derivatives of RBC with respect to the various C's are

$$w\text{-}1_{cs} = \frac{?RBC}{?C\text{-}1_{cs}} = \frac{C\text{-}1_{cs}}{SQRT}$$

$$w\text{-}2 = \frac{?RBC}{?C\text{-}2} = \frac{C\text{-}2}{SQRT}$$

$$w\text{-}3b = \frac{?RBC}{?C\text{-}3b} = \frac{C\text{-}3b}{SQRT}$$

$$w\text{-}4b = \frac{?RBC}{?C\text{-}4b} = \frac{C\text{-}4b}{SQRT}\ .$$

The w's are weights reflecting the ratio of each risk component to the total square root of the sum of the squares of all the risk components in the square root. They summarize the company's risk profile, at least according to the NAIC formula. A company with a high $w\text{-}1_0$ near 1 has most of its risks in asset and/or interest rate risk. A health insurer, a term insurance company, or certain reinsurers would have a higher w-2 and possibly higher w-3b and w-4b weights. Each company could be represented by a profile vector $(w\text{-}1_0, w\text{-}1_{cs}, w\text{-}2, w\text{-}3b, w\text{-}4b)$ in five dimensions, indicating which risks dominate its profile. Note that $(w\text{-}1_0)^2+(w\text{-}1_{cs})^2+(w\text{-}2)^2+(w\text{-}3b)^2+(w\text{-}4b)^2 = 1$, which implies that all companies' profile vectors are on the unit sphere.

The weights w-j moderate the effect of the elementary risk parameters on the value of the RBC. Since if x is any one of these independent variables,

$$\frac{?RBC}{?x} = \frac{?C\text{-}0}{?x} + \frac{?C\text{-}4a}{?x} + w\text{-}1_0\,\frac{?C\text{-}1_0}{?x} + w\text{-}1_0\,\frac{?C\text{-}3a}{?x} + w\text{-}1_{cs}\,\frac{?C\text{-}1_{cs}}{?x} + w\text{-}2\,\frac{?C\text{-}2}{?x} + w\text{-}3b\,\frac{?C\text{-}3b}{?x} + w\text{-}4b\,\frac{?C\text{-}4b}{?x}\ .$$

The RBC factors often associated with the variable x are the derivatives of the C-j with respect to x. In fact, the C-j is mostly linear functions of their independent variables. For example, the RBC factor for commercial mortgages in good standing is .026 multiplied by the Mortgage Experience Adjustment (MEA) Factor and further multiplied by (1-tax factor), that is $\partial C\text{-}1_0/\partial x = .026\times MEA\times(1\text{-tax factor})$. The formula above indicates that this has to be multiplied by $w\text{-}1_0$, the weight for asset risk, to get a better estimate of the effect of additional mortgages on the company's RBC. Similarly, the effect of adding group health premium into the mix would be modified by the weight for C-2 risk, w-2. If the company is heavily asset risk dominant, then w-2 will be close to zero and the additional health premium may have negligible effect. This should be considered in the methodology used for the allocation of RBC by line.

Before proceeding, it should be pointed out that marginal analysis might aid in solving the problem of the circularity of determining product mix and capital requirements. The capital needs for new or expanded product lines can be determined by the marginal analysis of RBC and the effect on return on equity analyzed more accurately. Also, investment decisions are sometimes made with RBC requirements in mind. Often, the factors that appear in the RBC formula are used for this purpose. But that may overstate the real RBC requirement because the weights $w\text{-}1_0$ and $w\text{-}1_{cs}$ have to be reflected. For more on the marginal analysis of RBC, see Zeppetella (1993 and 2002).

HOMOGENEOUS FUNCTIONS

A function f is homogeneous of degree n if for any $\lambda>0$,

$$f(\lambda x_1,\lambda x_2,\ldots,\lambda x_k) = \lambda^n f(x_1,x_2,\ldots,x_k).$$

Thus a homogenous function evaluated at a scalar multiple of its vector argument is a simple multiple of the original function. This property leads to a useful result: If n>0 and this formula is differentiated with respect to λ and λ is set equal to 1, we get

$$x_1\frac{\partial f}{\partial x_1} + x_2\frac{\partial f}{\partial x_2} + \ldots + x_k\frac{\partial f}{\partial x_k} = nf(x_1,x_2,\ldots,x_k).$$

In the case of a function homogeneous of degree n=1, the function is equal to the sum of all its partial derivatives with respect to each independent variable multiplied by the respective independent variable. It was Fraser (1962) who noted that the federal income tax formula based upon the 1959 Tax Act was homogeneous of degree one and that each variable, when multiplied by the marginal change in the tax with respect to that variable and summed over all independent variables would give the total company tax exactly. That fact led to a methodology for allocating tax by LOB that was used in the 1960s and 1970s. A similar technique can be applied to the RBC allocation.

The RBC function

$$RBC= C\text{-}0 + C\text{-}4a + \sqrt{(C\text{-}1_o+C\text{-}3a)^2+(C\text{-}1_{cs})^2+(C\text{-}2)^2+(C\text{-}3b)^2+(C\text{-}4b)^2}$$

is homogeneous of degree one in the variables C-j. Therefore, since the derivatives of RBC with respect to C-0 and C-4a are equal to one and the others are the weights w-j,

$$RBC = C\text{-}0 + C\text{-}4a + (C\text{-}1_o + C\text{-}3a) \times w\text{-}1_0 + (C\text{-}1_{cs}) \times w\text{-}1_{cs} + (C\text{-}2) \times w\text{-}2 + (C\text{-}3b) \times w\text{-}3b + (C\text{-}4b) \times w\text{-}4b.$$

Therefore the procedure for allocating RBC by use of this formula is as follows:

1. Calculate the total company's C-j and the resulting RBC for the total company.
2. Calculate from the total company C-j, the weights w-j that are the ratios of the various C-j (or $C\text{-}1_o$+C-3a, in the case of $w\text{-}1_0$) to the square root of the sum of the squares.
3. For each LOB, calculate its C-j as if it were a separate company.
4. Use the C-j for each LOB together with the weights w-j for the total company in the above formula, to evaluate the RBC for each LOB.

In step 3, some shortcuts may, and in fact, should be taken. For instance, one might not want to be so precise as to count the number of distinct bond issuers for each LOB, but simply use the total company's number. There would likely be questions about issuers shared by several lines among other problems anyway. In any event, for the total of the lines' RBC to add to the total company RBC exactly, it would be necessary for each of the lines' C-j to add up to the total company's C-j. So the separate company C-j's should be calculated by trying to divide the total company C-j up into the parameters attributable to the various lines. Another example of this would be the asset concentration factors. One shouldn't look for the highest exposures in each line separately, but instead allocate only those of the company's ten highest exposures to the lines. Dividing the C-j in this way may require some expedient decisions where the C-j are piecewise linear functions e.g., life net amount at risk which may have to be divided among more than one life LOB.

AN EXAMPLE OF RBC ALLOCATION BY LOB

As an example to illustrate this procedure and how it differs from some other allocation methods, consider a company with three lines of business: Life, Annuity and Group Health. Without giving all the details, the most important characteristics are summarized in the following table (amounts in millions):

	Life	Annuity	GH	Total Company
Bonds				
NAIC 1	3,700	500		4,200
NAIC 2	1,450	750	45	2,245
NAIC 3	200	100		300
NAIC 4	200			200
NAIC 5	50			50
NAIC 6	50			50
Commercial Mortgages	750	750		1,500
Unaffiliated				

Common Stock	500		5	505
Real Estate	200			200
Policy Loans	500			500
Short-term Investments	150	150		300
Total Invested Assets	7,750	2,250	50	10,050
Reserves	6,750	2,250	50	9,050
Net Amount at Risk	14,000			14,000
Premiums	600		300	900

Under a few more assumptions, a computation gives the following values:

	Life	Annuity	GH	Total Company
$C\text{-}1_o$	140.7	28.3	0.5	169.5
C-3a	31.3	37.5	0	68.8
$C\text{-}1_{cs}$	156.3	0	1.5	157.8
C-2	12.2	0	26.2	38.4
C-4a	12.0	0	1.5	13.5
Separate Company RBC	244.7	65.9	27.8	301.9

Notice that the separate company RBC amounts total 338.4, so the diversification benefit in the formula is 338.4-301.9 =36.5. Also note that GH seems to have a heavy RBC requirement which is driven mainly by the 9% of premium in the formula, times the incurred claims ratio (which I have assumed is .833). It is natural to think that elimination of the GH business would reduce RBC commensurately. However, this is not the case.

For the marginal allocation method, the weights of the various risks are calculated for the total company:

$w\text{-}1_0 = (C\text{-}1_o + C\text{-}3a)/SQRT = .826357$

$w\text{-}1_{cs} = C\text{-}1_{cs}/SQRT = .547182$

$w\text{-}2 = C\text{-}2/SQRT = .133136$

Then the RBC allocated to Life is from the C-j for Life and the total company w-j:
12.0 + (140.7+31.3) ×.826357 + 156.3×.547182 + 12.2×.133136 = 241.3
For the Annuity LOB we get (28.3+37.5) ×.826357 = 54.4. For Group Health,
1.5 + 0.5×.826357 + 1.5×.547182 + 26.2×.133136 = 6.2. These add up to the correct 301.9, which is the total company's RBC.

Notice that most of the diversification benefit has been allocated to Group Health and to a lesser extent, the Annuity line. Clearly in the total company, the Life business dominates the other businesses and asset risk dominates insurance risk. The weight w-2 shows that Group Health's C-2 risk only gets about a 13.3% weight. This is in marked contrast to the other separate company allocation methods, which would give most or all of the diversification benefit to the Life line.

Which is the "correct" way to allocate RBC? Like the similar question of allocating fixed costs or overhead, companies use different approaches. However, if the management of this hypothetical company used the higher 27.8 million separate company value for Group Health and decided that its ROE was too low, it might make the wrong decision. Eliminating the Group Health business would only reduce the RBC by 5.0 million, because the RBC with all the Group Health related items eliminated is 296.9. So eliminating the Group Health business may not have the expected effect on ROE.

SUMMARY

This paper has presented an analysis of the NAIC Risk-Based Capital Formula for life insurance companies. The formula is a homogeneous function of degree one in the several types of risk C-j. Marginal analysis of RBC shows how a company's risk profile leads to certain weights that measure the effect of changes in the C-j on the total RBC. The marginal analysis and homogeneity of the formula lead to a method of allocating RBC by line of business, which spreads the diversification benefit among the lines. This method appears to give results which aid management in making better capital allocation decisions.

REFERENCES

Fraser, J. 1962. "Mathematical Analysis of Phase 1 and Phase 2 of 'The Life Insurance Company Income Tax Act of 1959'." *TSA*, XIV, p51.

Zeppetella, T. 1993. "Marginal Analysis of Risk Based Capital." *The Financial Reporter*, June, 1993, p19

Zeppetella, T. 2002. "An Update on the Marginal Analysis of Risk-Based Capital." *The Financial Reporter*, April, 2002, p18