# ALTAM Spring 2024 Model Solutions

## ALTAM April 2024 Model Solutions

### Question 1

The solution to this question is in the spreadsheet. It should be noted that as stated in the instructions for the exam, only work in the spreadsheet will be graded. Any work on paper is NOT graded for Excel problems.

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**Question 2**

(a) The colleague is incorrect.

The future lifetimes will be independent if and only if $\mu_{x:y}^{01} = \mu_x^{23}$ and $\mu_{x:y}^{02} = \mu_y^{13}$ .

If $\mu_{x:y}^{01} \neq \mu_x^{23}$ and/or $\mu_{x:y}^{02} \neq \mu_y^{13}$ then the force of mortality for each life depends on the status of the other life, creating dependent future lifetimes.

(b) (i) $${}_tp_{x:y}^{01} = \int_0^t {}_rp_{x:y}^{00}\, \mu_{x+r:y+r}^{01}\; {}_{t-r}p_{y+r}^{11}\, dr$$

(ii) $${}_tq_{x:y}^{\;1} = \int_0^t {}_rp_{x:y}^{00}\, \mu_{x+r:y+r}^{01}\, dr$$

(iii) The difference is the probability that ($y$) dies after ($x$), but before the end of $t$ years, i.e. $Pr[T_x < T_y \le t]$

(c) $P = 10{,}000\bar{a}_{50|60} + \frac{1}{2}P\bar{A}_{50:60}^{\;1}$

$$\bar{a}_{50|60} = (\bar{a}_{60} - \bar{a}_{50:60})$$

$$\bar{a}_{60} = \alpha(\infty)\ddot{a}_{60} - \beta(\infty) = (1.00020)(14.9041) - 0.50823 = 14.39885$$

Or $\bar{a}_{60} = \frac{1-\bar{A}_{60}}{\delta} = \frac{1-\frac{i}{\delta}\bar{A}_{60}}{\delta}\ \frac{1-(1.02480)(0.29028)}{0.04879} = 14.39887$

$$\bar{a}_{50:60} = \frac{1-\bar{A}_{50:60}}{\delta} = \frac{1-\frac{i}{\delta}A_{50:60}}{\delta} = \frac{1-(1.02480)(0.32048)}{0.04879} = 13.76454$$

$$Or\ \bar{a}_{50:60} = \alpha(\infty)\ddot{a}_{50:60} - \beta(\infty) = (1.00020)(14.2699) - 0.50823 = 13.76452$$

$\Rightarrow P = \frac{6343}{1-0.5(0.24898)} = 7245$ Maybe slightly different due to rounding

(d)

$${}_5V^{(0)} = 10{,}000\left(\bar{a}_{65} - \bar{a}_{55:65}\right) + 0.5P\bar{A}_{\overset{\ \ \ 1}{55:65}}$$

$$\bar{a}_{65} = \alpha(\infty)\ddot{a}_{65} - \beta(\infty) = (1.00020)(13.5498) - 0.50823 = 13.0442$$

$$\bar{a}_{55:65} = \frac{1 - (1.02480)(0.38891)}{0.04879} = 12.3272$$

$$\bar{A}_{\overset{\ \ \ 1}{55:65}} = \bar{A}_{65} - \bar{A}_{\overset{\ \ \ 2}{55:65}} = (1.02480)(0.35477) - 0.06091 = 0.30266$$

$$\Rightarrow\ {}_5V^{(0)} = 7171.0 + 1096.3 = 8267.3$$

(e)

Annuities are not typically commutable (i.e. convertible into cash), because the adverse selection problem is too severe.

In this case, if the survivor were in very poor health, the value of the new last survivor annuity would be very much greater than the value of the existing single life annuity.

***Examiners' Comments:***

*Part A*

*There were several common errors on this part. Many candidates assumed that they had information they didn't in terms of independence, eg that we knew something about the forces of mortality to determine independence. Other candidates incorrectly assumed that the lack of a common shock implied independence. Lastly, some candidates either failed to mention whether the hypothetical colleague in the question was correct/incorrect, or stated both sides of the argument without definitively choosing independence or dependence.*

*Part B*

*Parts i and ii were generally of the "you know it or you don't" type. Some candidates struggled with demonstrating the correct calculus, either missing the differential entirely or integrating over incorrect variables, eg x instead of time. For part iii, the most common error was simply stating what each probability meant and not what they meant together.*

*Part C*

*The most common errors here were either struggling with notation, eg calculating the wrong annuity or insurance, or incorrectly converting the reversionary annuity into single and joint annuities and from there converting from continuous to due annuities.*

*Part D*

*Generally, if candidates did well in part C they did well here. The most common error was using the wrong formula to find the value of the insurance.*

*Part E*

*Very few candidates pointed out the adverse selection situation on the part of the surviving spouse, and very few commented on the surviving spouse at all which is what we were primarily looking for. Credit was given for coherent responses regarding the new spouse.*

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**Question 3**

$(a)$

$$_{10}V^{(1)} = 12{,}000\bar{a}_{65}^{11} + 100{,}000\bar{A}_{65}^{12} - 6{,}622\bar{a}_{65}^{10}$$

$$= 12{,}000(8.8123) + 100{,}000(0.56810) - 6{,}622(0.0395) = 162{,}296.0$$

$(b)$

$(i)$ $$\frac{d}{dt}\,{}_tV^{(1)}\Big|_{t=10} = \delta\,{}_{10}V^{(1)} - 12{,}000 - \mu_{65}^{10}\left({}_{10}V^{(0)} - {}_{10}V^{(1)}\right) - \mu_{65}^{12}\left(100{,}000 - {}_{10}V^{(1)}\right)$$

$$= (0.048790)(162{,}296) - 12{,}000 - 0.001727(-118{,}046) - 0.03102(-62{,}296)$$

$$= -1945.3$$

Note: These numbers are correct based on the Standard Sickness Table parameters. However, one of the parameters given in the exam was written as $7.85 \times 10^{-5}$ when it should have been $7.58 \times 10^{-5}$.

The answer using the number given in the exam is

$$= (0.048790)(162{,}296) - 12{,}000 - 0.00229(-118{,}046) - 0.03102(-62{,}296)$$

$$= -1878.0$$

(ii) $\quad {}_{9.75}V^{(1)} \approx {}_{10}V^{(1)} - 0.25(-1945.13) = 162{,}783$

OR: $\quad {}_{9.75}V^{(1)} \approx {}_{10}V^{(1)} - 0.25(-1950) = 161{,}810$

OR: $\quad {}_{9.75}V^{(1)} \approx {}_{10}V^{(1)} - 0.25(-1878) = 162{,}765$

(c)

$$P = \frac{12{,}000(2.3057) + 100{,}000(0.39127)}{10.2478} = 6518.0$$

(d)

$$ {}_{10}V^{(1)} = 100{,}000A_{65}^{(4)12} + 12{,}000\ddot{a}_{65}^{(4)11} - 6{,}518\ddot{a}_{65}^{(4)10} $$
$$ = (100{,}000)(0.56465) + (12{,}000)(8.9373) - (6518)(0.0395) $$
$$ = 163{,}455 $$

(e)

$$ \left({}_{9.75}V^{(1)} - 3{,}000\right)(1.05)^{0.25} = {}_{0.25}p_{64.75}^{10}\left({}_{10}V^{(0)}\right) + {}_{0.25}p_{64.75}^{11}\left({}_{10}V^{(1)}\right) + {}_{0.25}p_{64.75}^{12}(100{,}000) $$

$$ = 0.00043(43{,}792) + 0.99193(163{,}455) + 0.00764(100{,}000) $$

$$ \Rightarrow {}_{9.75}V^{(1)} = 163{,}944 $$

$$ \left({}_{9.75}V^{(0)} + 0.25P\right)(1.05)^{0.25} = {}_{0.25}p_{64.75}^{00}\left({}_{10}V^{(0)}\right) + {}_{0.25}p_{64.75}^{01}\left({}_{10}V^{(1)}\right) + {}_{0.25}p_{64.75}^{02}(100{,}000) $$

$$ = 0.98779(43{,}792) + 0.00674(163{,}455) + 0.00547(100{,}000) $$

$$ \Rightarrow {}_{9.75}V^{(0)} = 42{,}732 $$

(f)

Equate the EPV of the post 65 sickness benefits with the value of the additional death benefit.

Let $D = F - 100{,}000$

$$ D\left({}_{10}p_{55}^{00}v^{10}A_{65}^{(4)02} + {}_{10}p_{55}^{01}v^{10}A_{65}^{(4)12}\right) = 12{,}000\left(10p_{55}^{00}v^{10}\ddot{a}_{65}^{(4)01} + {}_{10}p_{55}^{01}v^{10}\ddot{a}_{65}^{(4)11}\right) $$

where (using Woolhouse) $\ddot{a}_{65}^{(4)01} \approx \bar{a}_{65}^{01} = 2.8851$

$$ \Rightarrow D\left(0.7409v^{10}(0.53233) + 0.11682v^{10}(0.56465)\right) = 12{,}000\left(0.7409v^{10}(2.885) + 0.11682v^{10}(8.9373)\right) $$

$$ \Rightarrow D = 82{,}930 $$

$$ \Rightarrow F = 182{,}930 $$

(g)

Disagree

The problem is adverse selection (or anti-selection).

At issue, the EPV of future benefits is the same, but as policy approaches the end of the initial 10-years the value of the future sickness benefits is much greater if the insured is sick then than if they are healthy.

An insured who is sick would want to keep the sickness benefit, one who is not would want the larger death benefits.

***Examiners' Comments***

*3a – Most candidates were able to successfully calculate the reserve at time 10, yet some candidates failed to realize that inputs needed to solve the problem were in the Excel tables.. As a result, there was no need to do any integration to determine the continuous actuarial present values.*

*3b – Most candidates were successful at applying both Thiele's differential equation and Euler's backward method to solve each part of the problem. Given the number of terms and variables included within the formula, some candidates seemed to get tripped up by trying to take shortcuts in the calculation. Showing your work and writing out your steps in the calculation not only makes it easier for the grader, but allows partial credit to be given.*

*3c – The vast majority of candidates were able to calculate the proper annualized net premium.*

*3d – The majority of candidates were also able to calculate the appropriate reserve at time 10 for the new product, called Product Q. However, some candidates failed to recognize that they still needed to subtract out the premium payment if the policyholder returned to state 0, in which case the policyholder would be required to make the premium payment.*

*3e – While most candidates did well utilizing backwards recursion to calculate both reserves at time 9.75, some candidates were unsure how to adjust the reserve to account for the sickness benefit and premium payments that were to be paid. For example, the quarterly premium payment instead of the annual premium amount should have been added to the reserve at time 9.75 in the healthy status. Likewise, the quarterly sickness benefit versus the annual amount should have been subtracted from the reserve at time 9.75 in the sick status.*

*3f – Most candidates did not do well on this part, although several different approaches could have been taken. The most straight forward approach would have been to write the new product out in its basic elements. Essentially the new product included a 10-year annuity if the policyholder became sick and a death benefit that could have been written formulaically as a whole life policy on age 55 that paid out 100,000 and another whole life policy on age 65 that paid out (F – 100,000), given the policyholder survived either in state 0 or 1 for those first 10 years. Since the premium was set equal to the prior product's design, one could then reapply the equivalence principle to solve for F having already known the premium for the new product Q+.*

*3g – Most candidates were correct in identifying that they should disagree with the proposal. Candidates that received the most points for this part were also able to communicate that allowing policyholders to switch between products Q and Q+ would create an adverse selection issue where the policyholder could switch between products and select the product that would ultimately benefit the policyholder the most. If the company decided to move forward with this approach, they should consider the anti-selection issue within the pricing of the products.*

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**Question 4**

(a)

(i)

- Between ages 70 and 71: 0.8 year in State 0
- Between ages 71 and 72: 0.1 year in State 0, then
  0.3 year in State 1, then
  0.5 year in State 0, then
  0.1 year in State 2.

So the contributions are : 0.8 year to $T_{70}^{(0)}$, and 0 year to $T_{70}^{(1)}$ and $T_{70}^{(2)}$

0.6 year to $T_{71}^{(0)}$, 0.3 year to $T_{71}^{(1)}$, and 0.1 year to $T_{71}^{(2)}$

(ii)

There is a contribution of +1 to $D_{71}^{01}$, $D_{71}^{10}$ and $D_{71}^{02}$ (as these are the transitions observed). There are no contributions to any other $D_{71}^{ij}$, nor to any $D_{70}^{ij}$.

(b) (i)

The likelihood function for all possible transitions during an integer age $x$ is

$$L = \exp\left\{-T_x^{(0)}(\mu_x^{01} + \mu_x^{02}) - T_x^{(1)}(\mu_x^{10} + \mu_x^{12}) - T_x^{(2)}(\mu_x^{23})\right\} \times \left\{(\mu_x^{01})^{D_x^{01}}(\mu_x^{02})^{D_x^{02}}(\mu_x^{10})^{D_x^{10}}(\mu_x^{12})^{D_x^{12}}(\mu_x^{23})^{D_x^{23}}\right\}$$

$$\Rightarrow l = \log(L) = -T_x^{(0)}(\mu_x^{01} + \mu_x^{02}) - T_x^{(1)}(\mu_x^{10} + \mu_x^{12}) - T_x^{(2)}(\mu_x^{23}) + D_x^{01}\log\mu_x^{01} + D_x^{02}\log\mu_x^{02} + D_x^{10}\log\mu_x^{10} + D_x^{12}\log\mu_x^{12} + D_x^{23}\log\mu_x^{23}$$

OR

$$l = \sum_{i=0}^{3}\left(-T_x^{(i)}\mu_x^{i\bullet} + \sum_{\substack{j=0\\ j\neq i}}^{3} D_x^{ij}\log\mu_x^{ij}\right)$$

(ii)

$$\frac{\partial l}{\partial \mu_x^{01}} = -T_x^{(0)} + \frac{D_x^{01}}{\mu_x^{01}}$$

$$\frac{\partial l}{\partial \mu_x^{01}} = 0 \Rightarrow \hat{\mu}_x^{01} = \frac{D_x^{01}}{T_x^{(0)}}$$

(iii) They are independent (for different *j*) because all the cross derivatives with respect to $\mu_x^{ij}$ are 0.

(iv) Because the estimators are independent, there are no covariances. The asymptotic variance of the estimator is

$$Var(\hat{\mu}_x^{01}) \approx -\left(\mathrm{E}\left[\frac{\partial^2 l}{\partial(\mu_x^{01})^2}\right]\right)^{-1} = \left(\frac{\mathrm{E}[D_x^{01}]}{(\mu_x^{01})^2}\right)^{-1} = \frac{(\mu_x^{01})^2}{\mathrm{E}[D_x^{01}]}$$

$$\Rightarrow \mathrm{SD}\ (\hat{\mu}_x^{01}) \approx \frac{\mu_x^{01}}{\sqrt{\mathrm{E}[D_x^{01}]}}$$

OR if we substitute observed for expected $D_x^{01}$, and $\hat{\mu}_x^{01}$ for $\mu_x^{01}$, we get

$$\mathrm{SD}\ (\hat{\mu}_x^{01}) \approx \frac{\hat{\mu}_x^{01}}{\sqrt{D_x^{01}}} = \frac{D_x^{01}/T_x^{(0)}}{\sqrt{D_x^{01}}} = \frac{\sqrt{D_x^{01}}}{T_x^{(0)}}$$

Or if we substitute partially, as follows, we get

$$Var(\hat{\mu}_x^{01}) \approx \frac{(\mu_x^{01})^2}{\mathrm{E}[D_x^{01}]} \approx \frac{\mu_x^{01} \times D_x^{01}/T_x^{(0)}}{D_x^{01}} = \frac{\mu_x^{01}}{T_x^{(0)}} \Rightarrow \mathrm{SD}\ (\hat{\mu}_x^{01}) \approx \sqrt{\frac{\mu_x^{01}}{T_x^{(0)}}}$$

(c) (i)

$$\hat{\mu}_x^{0\bullet} = \hat{\mu}_x^{01} + \hat{\mu}_x^{02}$$

$$\hat{\mu}_{70}^{01} = \frac{D_{70}^{01}}{T_{70}^{(0)}} = \frac{150}{3214.0} = 0.046671 \qquad \hat{\mu}_{70}^{02} = \frac{D_{70}^{02}}{T_{70}^{(0)}} = \frac{40}{3214.0} = 0.012446$$

$$\hat{\mu}_{70}^{0\bullet} = \hat{\mu}_{70}^{01} + \hat{\mu}_{70}^{02} = 0.046671 + 0.012446 = 0.059116$$

$$\hat{\mu}_{71}^{01} = \frac{D_{71}^{01}}{T_{71}^{(0)}} = \frac{170}{3019.7} = 0.056297 \qquad \hat{\mu}_{71}^{02} = \frac{D_{71}^{02}}{T_{71}^{(0)}} = \frac{55}{3019.7} = 0.018214$$

$$\hat{\mu}_{71}^{0\bullet} = \hat{\mu}_{71}^{01} + \hat{\mu}_{71}^{02} = 0.056297 + 0.018214 = 0.074511$$

$$\hat{\mu}_{70}^{0\bullet} + \hat{\mu}_{71}^{0\bullet} = 0.059116 + 0.074511 = 0.133627 \approx 0.13$$

(ii) $${}_2\hat{p}_{70}^{\overline{00}} = \exp\{-(\hat{\mu}_{70}^{0\bullet} + \hat{\mu}_{71}^{0\bullet})\} = \exp\{-0.133627\} = 0.874916$$

(iii) An approximate 95% confidence interval for $\mu_{70}^{0\bullet} + \mu_{71}^{0\bullet}$ is given by

$$(\hat{\mu}_{70}^{0\bullet} + \hat{\mu}_{71}^{0\bullet}) \pm 1.96\text{sd}(\hat{\mu}_{70}^{0\bullet} + \hat{\mu}_{71}^{0\bullet})$$

Because of the independence of the estimators,

$$\text{Var}(\hat{\mu}_{70}^{0\bullet} + \hat{\mu}_{71}^{0\bullet}) = Var(\hat{\mu}_{70}^{01}) + Var(\hat{\mu}_{70}^{02}) + Var(\hat{\mu}_{71}^{01}) + Var(\hat{\mu}_{71}^{02})$$

$$\approx \frac{D_{70}^{01}}{\left(T_{70}^{(0)}\right)^2} + \frac{D_{70}^{02}}{\left(T_{70}^{(0)}\right)^2} + \frac{D_{71}^{01}}{\left(T_{71}^{(0)}\right)^2} + \frac{D_{71}^{02}}{\left(T_{71}^{(0)}\right)^2}$$

$$= \frac{150}{(3214.0)^2} + \frac{40}{(3214.0)^2} + \frac{170}{(3019.7)^2} + \frac{55}{(3019.7)^2}$$

$= 0.000014521+0.000003872+0.000018643+0.000006032$

$= 0.000043068$ (See Examiners' Notes below for alternatives)

Therefore, the confidence interval is given by

$$0.133627 \pm 1.96 \times \sqrt{0.000043068} \approx 0.133627 + 1.96 \times 0.006563$$

$$(0.120764, 0.146490)$$

(iv) An approximate 95% confidence interval for ${}_2p_{70}^{\overline{00}}$ is given by

$$(\exp\{-0.146490\}, \exp\{-0.120764\}) = (0.863734, 0.886243)$$

*Examiners' Comments*

*For (c) (iii), here are the alternate calculations for the variance.*

*Using the formula without substitution,*

$$\begin{aligned}\mathrm{Var}(\hat{\mu}_{70}^{0\bullet}+\hat{\mu}_{71}^{0\bullet}) &= Var(\hat{\mu}_{70}^{01})+Var(\hat{\mu}_{70}^{02})+Var(\hat{\mu}_{71}^{01})+Var(\hat{\mu}_{71}^{02})\\ &\approx \frac{(\mu_{70}^{01})^2}{\mathrm{E}[D_{70}^{01}]}+\frac{(\mu_{70}^{02})^2}{\mathrm{E}[D_{70}^{02}]}+\frac{(\mu_{71}^{01})^2}{\mathrm{E}[D_{71}^{01}]}+\frac{(\mu_{71}^{02})^2}{\mathrm{E}[D_{71}^{02}]}\\ &\approx \frac{(\hat{\mu}_{70}^{01})^2}{D_{70}^{01}}+\frac{(\hat{\mu}_{70}^{02})^2}{D_{70}^{02}}+\frac{(\hat{\mu}_{71}^{01})^2}{D_{71}^{01}}+\frac{(\hat{\mu}_{71}^{02})^2}{D_{71}^{02}}\\ &= \frac{(0.046671)^2}{150}+\frac{(0.012446)^2}{40}+\frac{(0.056297)^2}{170}+\frac{(0.018214)^2}{55}\\ &= 0.000043068\end{aligned}$$

*Using the formula with partial substitution,*

$$\begin{aligned}\mathrm{Var}(\hat{\mu}_{70}^{0\bullet}+\hat{\mu}_{71}^{0\bullet}) &= Var(\hat{\mu}_{70}^{01})+Var(\hat{\mu}_{70}^{02})+Var(\hat{\mu}_{71}^{01})+Var(\hat{\mu}_{71}^{02})\\ &\approx \frac{\hat{\mu}_{70}^{01}}{T_{70}^{(0)}}+\frac{\hat{\mu}_{70}^{02}}{T_{70}^{(0)}}+\frac{\hat{\mu}_{71}^{01}}{T_{71}^{(0)}}+\frac{\hat{\mu}_{71}^{02}}{T_{71}^{(0)}}\\ &= \frac{0.046671}{3214.0}+\frac{0.012446}{3214.0}+\frac{0.056297}{3019.7}+\frac{0.018214}{3019.7}\\ &= 0.000043068\end{aligned}$$

*In general, this question was not well done.*

*Part a) was well done, but still required careful reading. In particular, years of age had to be considered separately and the person was 70.2 at the beginning of the observation period. Also, all required values, even if zero in this particular case, had to be determined.*

*Part b) was not well done at all. Some candidates failed to appreciate the fact that all this part was for the general case, not for the specifics given in part a). In i), this meant that all possible transitions, not just those observed in a), had to be accounted for. In ii), ideally, the estimator was derived from first principles, but partial marks were given for writing down the right expression for the estimator. In iii), very few candidates came up with a valid justification. In iv), as in ii), ideally, the formula was derived from first principles, but partial marks were given for writing down the right expression for the standard deviation.*

*Part c) was done somewhat better than b) and was an application of the theoretical results from part b) to the information provided. In i), candidates had to apply the result from b) ii). In ii), the key was to know which function of the sum in i) gave the required probability. In iii), candidates had to apply the combination of the answers to b) iii) and b) iv) to generate the required confidence interval. In iv), the same transformation that led from i) to ii) also led from iii) to iv), except that the lower bound in iii) led to the upper bound in iv).*

**ALTAM April 2024 Model Solutions**

**Question 5**

(a)

$$\ddot{a}^{(12)}_{\overline{72:\overline{3}|}} = \ddot{a}^{(12)}_{\overline{3}|} + v^3\,{}_3p_{72}\ddot{a}^{(12)}_{75}$$

$$\ddot{a}^{(12)}_{\overline{3}|} = \frac{1-v^3}{0.04869} = 2.79652$$

$$\ddot{a}^{(12)}_{75} = 1.00020\ddot{a}_{75} - 0.46651 = 9.85335$$

$$\Rightarrow \ddot{a}^{(12)}_{\overline{72:\overline{3}|}} = 2.79652 + v^3\frac{85{,}203.5}{89{,}082.1}(9.85335) = 10.9376$$

(b)

$$AL_{2024} = 150\left(\ddot{a}^{(12)}_{\overline{69:\overline{6}|}}\right) + 78\left(\ddot{a}^{(12)}_{\overline{72:\overline{3}|}}\right)$$

$$\ddot{a}^{(12)}_{\overline{69:\overline{6}|}} = \frac{1-v^6}{0.04869} + v^6\frac{85{,}203.5}{91{,}936.9}(9.85335) = 12.0263$$

$$AL_{2024} = 150(12.0263) + 78(10.9376) = 2657.1$$

(c) The Normal Cost is the amount required to support the increase in accrued benefits in the year(s) following the valuation. As there are no active members, there is no increase in accrued benefits.

(d)

($i$) Let $\text{VA}_{yr}$ denote the value of the assets at yr

$$\text{VA}_{2027} = \text{VA}_{2024}(1.03)^3 - 228\ddot{s}_{\overline{3|}}^{(12)}$$
$$= 2657.1(1.03)^3 - 228\left(\frac{1.03^3 - 1}{12(1 - 1.03^{-1/12})}\right) = 2187.4$$

(ii) $\text{AL}_{2027} = 102\ddot{a}_{\overline{72:\overline{3|}}}^{(12)} + 78\ddot{a}_{75}^{(12)} + 48\ddot{a}_{\overline{3|}}^{(12)}$
$$= 102(10.9376) + 78(9.85335) + 48(2.7965) = 2018.43$$

(iii) Gain $= 2187.4 - 2018.4 = 169.0$

(iv) The expected number of deaths from the plan members is $3\,{}_3q_{69} + 2\,{}_3q_{72} = 0.18$. The actual number of deaths was 1. As excess deaths generate gain in an annuity portfolio, the plan experienced a mortality gain over the period.

OR:

The plan has made a gain, despite earning only 3% per year on investments instead of the 5% assumed in the valuation. As there are no other sources of profit or loss, the plan must have had a gain from mortality.

(e)

Advantages

(1) Protect the sponsor from investment and survivorship risks;

(2) Eliminate the need for expenses of administering plan (i.e., no more valuations required);

(3) Retirees may feel better protected by guaranteed insured benefit.

(4) Get the employer out of the business of managing an annuity portfolio (overlaps with item (2)?).

Disadvantages

(1) Annuity prices will likely exceed plan liabilities as they will include loadings for profit, expenses, and risk.

(2) Annuity prices will likely exceed plan liabilities as the interest rate the last two years has been 3% which is lower than the assumed rate of 5%.

(3) Loss of potential future gains from investment which might improve retirees' benefits or be returned to the sponsor.

***Examiners' Comments***

*Many candidates mistakenly calculated a 3-year term annuity due, perhaps stemming from a lack of experience with this notation.*

*Part B: Some candidates mentioned they used Excel to calculate a final answer, but this Excel work should be transcribed into their papers as both the work and answer are graded. For questions 2 through 6, only the information on the written paper is graded. Any work in Excel is not graded.*

*Most candidates understood part C well.*

*Part D was challenging. Many candidates used annual discounting in parts I. and II. Parts III. and IV. were generally handled well. Some calculated the gain correctly but misinterpreted it as a loss.*

*Part E was often misunderstood as being from the retiree's perspective when they were asked about the sponsor's perspective. Many candidates wrote that "high cost" of annuities are a disadvantage to the sponsor, but in order to receive credit, we needed more explanation and clarity.*

**ALTAM April 2024 Model Solutions**

**Question 6**

(a) (i) Start by assuming corridor factor does not apply (noting that $0.65(0.09) = 0.0585 > 0.025$):

$$AV_1 = \left(P - E - q^*_{70} \times ADB \times v_{i_q}\right) \times (1 + i^c_1)$$
$$= (20{,}000(0.98) - (1.50)(0.010413)(100{,}000))(1 + 0.65 \times 0.09)$$

$$\Rightarrow AV_1 = 19{,}093.3$$

Check that the death benefit exceeds 150% of the year end account value because of the corridor factor. The total death benefit is 119,093.3 while 150% of the year end AV is 28,640.

Predictably, the corridor factor does not apply.

(ii) $F_1 = ((0.98)20{,}000 - 4{,}000)(1.09) = 17{,}004$

(b) (i) Again start by assuming the corridor factor does not apply, noting that the crediting rate will be the minimum guaranteed value (2.5%):

$$AV_{10} = \left(AV_9 + P - E - q^*_{79}(FV - AV_{10})v_{i_q}\right) \times (1 + i^c_9)$$
$$= (196{,}840(0.985) + 20{,}000(0.98) - 1.50(0.029132)(100{,}000))(1.025)$$
$$= 214{,}345.5$$

The total death benefit (without the corridor factor) is 314,345 while 150% of the AV is 321,519.

$Death\ Benefit < 150\%\ of\ the\ Account\ Value$ so the corridor factor applies.

$$So\ AV_{10} = \left(AV_9 + P - E - q^*_{79}(1.50AV_{10} - AV_{10})v_{i_q}\right)(1 + i^c_9)$$
$$= \left(196{,}840(0.985) + 20{,}000(0.98) - 1.50(0.029132)(0.50AV_{10})\right)(1.025)$$
$$\Rightarrow AV_{10} = 214{,}031$$

(ii) $F_{10} = ((0.98)(212{,}000) + (0.98)(20{,}000))(1.03) = 234{,}181$

(Note that this is greater than the GMMB = 10(20,000)=200,000 )

(c) The payout from the UL policy is $1.50(214{,}031) = 321{,}047$

For the EL policy we need to check the GMDB:

The accumulated premiums are

$$20{,}000\ddot{s}_{\overline{10|}} = 20{,}000\left(\frac{(1.05)^{10} - 1}{d_{5\%}}\right) = 264{,}136$$

Since the GMDB is greater than the fund value, the death benefit is 264,136.

(d)

Advantages

1. Early withdrawal: there are guarantees on the UL, as the credited rate is at least 2.5%, but there are none for the EL.
2. The death benefit is higher in earlier years on the UL policy, and is likely higher in all years.
3. Lower "expense charges" on UL (except premium charges, which are the same), but higher cost of insurance.

Disadvantages

1. In good years, credited interest on the UL will only be 65% of the performance on the EL.
2. Because of the COI, the amount of premium "invested" in the UL is lower than in the EL each year.
3. The GMDB on the EL could be higher than the UL death benefit, if EL/UL returns are consistently low.
4. The UL premium deductions (expenses and COI) may not be guaranteed.
5. The UL typically would have higher surrender charges in the early years of the contract.

(e) (i) The Benefit Base is equal to the GMDB, i.e.

$BB = 264{,}136$
$\Rightarrow \text{Annuity benefit} = 0.112 \times 264{,}136 = 29{,}583$ per year .

(ii) The policyholder might get a higher benefit by annuitizing the final fund value at time 10 at the market rate, if the market rate is high enough.

OR

Let *c* denote the market rate, then the p/h should annuitize at market rates (without exercising the GMIB) if

$$cF_{10} > (0.112)(BB_{10})$$
$$\text{i.e. if } c > \frac{29{,}583}{234{,}181} = 12.64\%$$

***Examiners' Comments***

- *Overall, candidates found this question challenging, with very few achieving maximum points.*
- *In both part (a) and part (b), quite a few candidates did not check the corridor factor. Whenever a UL question specifies that there is a corridor factor requirement, it should be checked whenever calculating an account value or cost of insurance. Failure to check this was more critical in part (b) than part (a).*
- *Most, but not all candidates spotted that the UL policy's minimum crediting rate applies in part (b), though a few also wrongly applied it to the EL policy as well as the UL.*
- *Although many candidates correctly identified that the corridor factor applied in part (b), most did not follow through to use the corridor factor to determine the UL death benefit in part (c). The EL death benefit was done correctly by most candidates.*
- *Part (d) was done reasonably well by most candidates who attempted it.*
- *Part (e) was more challenging. Quite a few candidates correctly determined the amount of guaranteed annuity, but the understanding of why a policyholder might not annuitize through the GMIB was less common. Many candidates suggested reasons why the policyholder might not annuitize at all, but that did not answer the question asked, which was why a policyholder might annuitize without exercising the GMIB. The key here is that even though the benefit base is greater than the fund value at maturity, the market annuitization rate at maturity might be sufficiently higher than the GMIB rate as to provide a greater amount of annuity.*