

Capital Allocation in the Property-Liability Insurance Industry

Stephen P. D'Arcy, FCAS, MAAA, Ph.D.

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Abstract

Capital allocation is a theoretical exercise because all of a firm's capital could be depleted to cover a significant loss arising from any one segment. However, firms do need to allocate capital for pricing, risk management and performance evaluation. One versatile allocation method, the Ruhm-Mango-Kreps algorithm, has several key advantages: additivity, simplicity and flexibility. However, the approach is so flexible it can be used to produce many different values instead of having a single answer. In this paper, the cost of capital in financial markets is incorporated into the Ruhm-Mango-Kreps algorithm to yield one allocation that reflects the true cost of capital an insurer would face.

1. Introduction

Financial firms have developed economic capital models to determine the level of financial resources they need to have to remain solvent over the next year at a particular level of probability. Capital adequacy and capital allocation are two related applications of economic capital models, but with very significant differences. Capital adequacy (or economic capital) is the total amount of capital a firm is required to hold to meet specific regulatory, rating agency or company-imposed benchmarks. The capital requirement is determined based on a financial model, one that is either externally imposed or internally generated. The resulting capital requirement is calculated very precisely, although it must be recognized that model, parameter and process risk all contribute to potential errors in this measurement, relative to the actual solvency probability and associated capital.

Capital allocation also depends on a financial model to determine how the capital of a firm is allocated to particular subdivisions within the firm. However, any capital allocation is only a theoretical division of the firm's resources, as any business segment would have a claim on the entire capital of the firm if extremely adverse results were to occur. A variety of capital allocation techniques have been proposed and there is no single commonly accepted capital allocation method. Capital needs to be allocated for a variety of reasons, including pricing, risk management and performance evaluation. Since different capital allocation methods can often be used by the same firm for these different applications, it is vitally important the distinctions among these approaches be well understood.

2. Background

Capital adequacy provisions for property-liability insurers have been in place since insurers were first subject to regulation. One early capital adequacy tenet was the Kenney Rule, which stated that fire insurers should maintain a level of surplus equal to the annual premium written. As liability insurance grew in importance and insurers began to operate as multi-line carriers, this rule was relaxed to allow a 3:1 premium to statutory surplus ratio. The National Association of Insurance Commissioners (NAIC) Early Warning Tests incorporated this 3:1 ratio in its calculations in the 1970s. Gradually, some of the inherent weaknesses of this measure were recognized. No provision was made for loss reserves, no distinction was made for the investment policy of the firm, and no recognition was given to factors that could reduce risk, such as diversification by line or firm size.

In the 1990s, a risk-based capital approach was adopted by regulators in the United States that established a minimum level of statutory surplus based on a formula that was applied to each firm. This formula did reflect diversification, firm size and different investment categories, but the formula produced a rather rough measure of the required capital. Different regulatory actions were triggered when a firm's statutory capital fell to certain multiples (200 percent, 140 percent, 100 percent) of the risk-based capital figure. The objective of this approach was not to dictate the amount of capital a firm should hold, but to initiate regulatory oversight early enough to reduce the likelihood of an insolvency. A similar external formula was applied by regulators in Europe under Solvency I.

The current movement in capital adequacy is toward the use of internal models, developed and used by the insurer. Rating agencies currently allow this option and Solvency II includes this provision as well. The benefits of internal models are that they can better reflect the

specific risks a firm faces and are likely to be more widely used as a management tool than any externally imposed model. Extensive use of the model could lead to a more effective integration of risk management into company operations.

Capital allocation within an insurance company was not a major consideration for the property-liability insurance industry until the 1960s when investment income began to be recognized in the ratemaking process. Once investment income had to be reflected in rate filings, capital had to be allocated by line of business by state both to calculate the expected amount of investment income and to determine the return on capital. The early methods were fairly straightforward, based on arbitrary premium-to-surplus and liabilities-to-surplus ratios. In many cases, the capital allocation was done independently for each application (e.g., Massachusetts auto rate filings, New York workers' compensation rate filings) and no effort was made to allocate all of the insurer's capital, or to see if the total allocation would equal the firm's total capital if one method were applied consistently to all lines and all states.

In the 1990s, insurers and consultants developed dynamic financial analysis (DFA) models for the property-liability insurance industry that incorporated both the underwriting and investment sides of insurance operations. DFA models applied advanced actuarial approaches to model underwriting operations, sophisticated financial tools to model interest rates, inflation and equity returns, and credit default models for the asset side of operations. DFA models can serve a variety of functions, including strategic planning, reinsurance analysis, pricing and capital adequacy determination. The importance of capital adequacy eventually led the DFA type models to focus on economic capital issues for regulatory, rating agency and internal company needs.

As insurers were working on DFA models for insurers, the field of risk management grew in scope and importance for all financial organizations. Traditional risk management formally developed as a recognized business function in the 1960s, growing out of the insurance purchasing area. Based on this heritage, and the fact that at the time few other risks were as important to firms or as subject to effective management techniques, risk management focused on pure risks, those subject to a loss or no-loss outcome. Many of these risks were insurable, but risk management also considered other approaches to handling these risks. In the 1970s, financial instability generated by inflation, interest rate volatility and foreign exchange rate fluctuations created a new class of risk for firms, financial risk. New instruments to hedge these emerging risks were created and the field of financial risk management developed, unfortunately independently from traditional risk management. The infamous failures of financial risk management that occurred in the 1990s, including Long-Term Capital Management, Barings Bank and Orange County, led regulators and boards to take a more comprehensive approach to risks, one that considered all the risks a firm faced in aggregate, and to move away from the traditional silo approach of hazard risks, financial risks, operational risks and other risks on a stand-alone basis. This comprehensive approach, commonly termed enterprise risk management, represents the extension of common risk management principles to all the risks facing an organization on a consistent application. The key metric that has emerged to manage these diverse risks is economic capital.

Insurers now allocate capital within the company for three key reasons—pricing, risk management and performance measurement. Economic capital models are increasingly used to determine the capital allocation within companies. A variety of approaches have been proposed as the basis for capital allocation, although no single approach has been accepted as a standard.

3. Capital Allocation Methods

One method of capital allocation is based on value at risk (VaR). This approach has been adopted from financial risk management, which developed and used VaR as a rough measure of the risk of a portfolio of derivatives. The VaR is the level of losses that is not expected to be exceeded with a specified confidence (or probability) level over a stated time horizon. For example, if the 99 percent daily VaR were \$10 million, then the financial institution would expect to lose more than \$10 million only once every 100 days. VaR does not provide any information about how large the loss on that one in 100 days would be, or about the average gain or loss of the portfolio. In the insurance industry, the focus is on longer intervals than a single day, so the common time period used is one year. When capital is allocated based on VaR, each line of business or profit center is assigned the capital equal to a particular percentile for an annual VaR.

One problem with VaR is that it only considers one point on the probability distribution and ignores the rest of the distribution. Tail value at risk (TVaR), which is also termed tail conditional expectation (TCE), takes the average of all the values in the tail beyond a particular percentile for a specific time period. This approach still ignores the majority of the values in the distribution (all loss values less than the cutoff percentile) and does not indicate the extent of the worst cases. TVaR only provides the average loss if a loss in excess of the TVaR threshold were to occur. There are two approaches to using TVaR for capital allocation. The first is to allocate capital solely based on the TVaR levels for each line of business or profit center individually. In this case, the TVaR would be the average of the worst x percent losses for a particular line of business. The second approach is to rank all losses for the firm as a whole, in the manner that VaR for the firm is determined. Then for each line of business or profit center, the losses in the x

percent worst cases for the firm as a whole are averaged. Capital is allocated in proportion to these TVaR levels.

Marginal capital allocation is an alternative approach based on option pricing theory. In this case, the value of the insolvency put option is calculated for the firm. This is the estimated cost of hedging, using option valuation approaches, the value of the loss that policyholders would incur if the firm became insolvent. Capital is then allocated to individual lines of business based on the changes in the value of the insolvency put option when shifts in the line of business mix are made. Myers and Read (2001) propose an allocation approach based on the marginal effect on the insolvency put option of the firm by adding an incremental amount to a particular line of business. Merton and Perold (1993) propose an allocation based on adding or removing an entire line of business. One drawback of these approaches is the need to accurately measure the covariance terms for different lines of business. Another is that option pricing models that lead to closed-form solutions, the ones commonly used in these approaches, assume that losses are either normally or lognormally distributed; insurance losses tend to be much more skewed than these distributions allow.

Both of the marginal capital allocation methods described above calculate the capital required for an individual line as if that line were the last line to be added to the firm, in essence assigning all the diversification effect to that one line of business. Another technique, derived from the game theory approach of Shapley (1953), considers all possible orders for combining lines of business and allocates capital on that basis. This approach requires a large number of steps to complete the process, especially for situations involving many lines of business. The end result, though, can produce a more consistent capital allocation. An alternative approach, based on the work of Aumann and Shapley (1974), is a much simpler procedure that examines

marginal changes rather than total line changes. This approach can be shown to be equivalent to several other methods, including the Ruhm-Mango-Kreps (RMK) approach discussed below, for a broad class of risk measures.

A more robust capital allocation method has been developed that focuses on aggregate portfolio risk, including both underwriting and investment operations. Any decline in the value of an investment generates the same type of call on capital that an underwriting loss would for a line of business, and should be treated similarly in the capital allocation process. Other capital allocation methods tend to focus on allocating capital to lines of business or regional profit centers. In these methods, investment risk is either considered to be a component of a line of business or eliminated by assuming risk-free investment returns. While this approach may be acceptable for pricing, it is not appropriate for risk management or performance evaluation applications of capital allocation. An insurer needs to maintain capital to support risky investments in the same way that capital needs to be allocated to a line of business that generates risk. If an investment portfolio takes on additional risk, then additional capital needs to be allocated to support this new position.

A weighting factor (termed risk leverage) is determined as a function of the aggregate portfolio outcome; the worse the aggregate outcome, the higher the risk leverage factor. The risk leverage can be 0 for favorable outcomes when there is no need to draw on capital at the aggregate level, in which case the focus is only on downside risk. The risk leverage factor is then applied to the amount of capital consumed (or supplied) by each component of the portfolio in scenarios when the total portfolio consumes capital. Kreps (2005), Ruhm and Mango (2003) and Ruhm, Mango and Kreps (2010) describe different aspects of this approach. Bear (2005), Clark (2005) and Ruhm (2003) demonstrate applications of this approach.

Using stochastic simulation, the Ruhm-Mango-Kreps algorithm is applied by generating a large number of iterations and sorting these based on the aggregate outcomes. The risk leverage factor is then determined for each iteration. The worse the outcome, the higher the risk leverage factor. This risk leverage factor is then multiplied by the outcome for each component of the portfolio. Weighted averages are calculated for each portfolio component to determine the capital allocation. Depending on the set of risk leverage factors selected (i.e., the “kernel” of the risk measurement), the Ruhm-Mango-Kreps approach can duplicate the Myers-Read marginal capital approach, the value-at-risk and tail value-at-risk methods, Black-Scholes pricing, the capital asset pricing model (CAPM) pricing, other basic approaches or new allocations. The advantage of the Ruhm-Mango-Kreps approach is its relative simplicity; it is easy to explain to less mathematical audiences and to calculate using a firm’s own DFA or economic capital model.

Another view of capital allocation reflects the fact that capital is not only needed for the extreme losses but is also needed to support writing a line of business in the first place (Mango 2006). This view proposes a two-step approach to allocating capital. The first is to recognize that an individual line of business requires a temporary, nonconsumptive use of capital to support writing business. This is equivalent to renting real estate or equipment. Regulators do not allow insurers to write business without underlying capital, even if the subsequent experience is favorable and does not draw down that capital. The second step is to consider the consumptive use of capital, the chance that a particular line of business will consume capital. The cost of capital is then dependent on the level of capital consumed and the length of time for it to be replaced.

4. Using Financial Markets to Determine the Riskiness Leverage Factors

A variety of different capital allocation methods have been proposed to determine how an insurer's capital can be assigned to segments of the firm in order to establish prices, manage risk or evaluate performance. No single method has yet been adopted as an industry standard or best practice. The Ruhm-Mango-Kreps algorithm has several significant advantages: it allocates a firm's capital down to any desired level, it is relatively straightforward to apply and it can utilize any mathematical relationship to determine the relative riskiness of particular outcomes. The flexibility of this approach allows this method to be used to produce a wide variety of results depending on the weighting system selected.

The key concept of the Ruhm-Mango-Kreps approach is that capital is allocated to individual components based on how much of the need for capital is generated by that component in each scenario for which capital is drawn down at the aggregate level. The key concept of the Mango shared capital approach is that there are two different needs for capital, a nonconsumptive need that simply requires capital be available to support accepting a particular risk (writing a line of business or investing in a risky asset class) and a consumptive need when capital is actually required to support adverse situations. As discussed in Bear (2005), both approaches provide useful insights and they can be combined to produce a more robust capital allocation approach.

The riskiness leverage ratio is the weight applied to capital consumed by adverse scenarios. As described in Kreps, there are an infinite number of riskiness leverage factors that could be applied in the Ruhm-Mango-Kreps algorithm. The trick is to find the one management believes is most appropriate. The flexibility of the Ruhm-Mango-Kreps algorithm is its greatest flaw. This flexibility provides management with too much choice, and no solid grounds for

selecting a particular factor. It also risks the choice of a measure that is not appropriate. Kreps (2005) suggests that management can select the riskiness leverage ratio that seems most reasonable. However, what seems reasonable to the manager of one division may seem quite unreasonable to the manager of a different division. The Ruhm-Mango-Kreps algorithm has been illustrated with many different risk leverage models, including variance, value at risk, tail value at risk, excess tail value at risk, semi-variance, mean downside deviation and proportional excess (Kreps 2005; Bear 2005; Ruhm, Mango and Kreps 2010). All of these approaches, however, depend solely on the performance of the company and ignore external capital markets. Without any guidance on what the correct riskiness leverage factors should be, there can be no confidence that the results are valid.

An approach that incorporates the capital markets to determine the riskiness leverage factor would provide two advantages. First, it would provide a single approach to determining the riskiness leverage factor, and end any internal debates about which method should be used. Second, it would reflect the actual cost of recapitalizing the firm, instead of a method that relies on functions that have desirable mathematical properties. This approach fits in well with Kreps' view, stated when discussing coherent risk measure properties, that, "While axiomatic treatments may prefer one form or another, *it would seem plausible that the risk measure should emerge from the fundamental economics of the business* and the mathematical properties should emerge from the risk measure, rather than vice versa" (Kreps 2005, 43). If capital is depleted, then calculate the cost of replenishing that capital by issuing new equity. The additional equity would restore the capital to the desired level. This new equity will dilute the current owners' share of future earnings, so it bears a cost. This cost is the cost of capital.

5. The Cost of Capital

The cost of capital is a widely used term in finance, and is one that has different meanings for different usages. One use of the cost of capital is as a hurdle rate, or the minimum rate of return a company uses to make investment decisions. If a potential investment is not expected to generate the hurdle rate of return, then the company should forego that investment. The hurdle rate is what the company considers its own cost of capital to be, even though the firm may not need to raise capital for a particular investment, so any investment that does not generate returns in excess of this level would not enhance the value of the firm. The hurdle rate is frequently determined by judgment rather than a formula- or market-based approach.

A second usage of the cost of capital is in regulatory hearings, particularly for utilities. The cost of capital, often termed the weighted average cost of capital (WACC) in these situations, is the target return for the utility, and the prices approved are based on a projection that would yield this return. WACC is determined by taking a weighted average of the cost of equity and the cost of debt. The cost of equity is often estimated based on the CAPM, which is the sum of the risk free rate (the current rate on U.S. Treasury bills) plus β (the covariance of the utility's stock returns with the total stock market return) times the market risk premium. As β for most utilities is less than one, the cost of equity would be less than the general stock market return. The cost of debt in the WACC is the current interest rate for outstanding long-term debt issued by that or similar firms. This rate will depend on the utility's bond ratings (Moody's, Standard & Poor's, etc.) as well as the current economic environment. The weights applied to each portion of the WACC are based on the capital structure of the firm: how much of the firm's value is equity and how much is debt. Expert testimony is generally called upon to opine on each

of these variables during the course of the rate hearing. Once values for each of the variables are set, the WACC is easily calculated.

The third usage of the cost of capital, and the one that applies in this situation, is what a firm would have to pay to raise capital at a particular time. The cost of capital would depend on how much the firm needs to raise, why it is raising capital and what source of funds are being used. Firms generally have three major sources of capital—equity, debt or internally generated funds. Each major source has subdivisions. Equity could be common or preferred, voting or nonvoting. Debt comes in even more varieties—bank loans, commercial paper, fixed or floating rate, senior or subordinated, secured or unsecured, straight or convertible, and maturity. Internally generated funds can come from normal operations or from reducing adjustable payments such as dividends.

The pecking-order theory of financial choices states that firms prefer to raise capital by internal funds first, then use debt and, finally, will use equity (Brealey, Myers and Allen 2008, 517-22). Substantial evidence based on the capital structure of firms from many countries supports this theory. However, trying to model the cost of capital based on the wide variety of financing methods presents a significant challenge. Fortunately, a simplifying assumption is available to simplify this process.

The Modigliani-Miller irrelevance of capital structure theory starts with the premise that the value of a firm is independent of its capital structure (Modigliani and Miller 1958). A firm is worth the same whether it is 100 percent equity or 100 percent debt financed. Therefore, the cost of capital can be determined either by the cost of equity or the cost of debt. In practice, capital structure does seem to matter due to differential tax treatment of debt and equity, the greater risk of insolvency for a firm with a high ratio of debt, the lower cost of raising capital by issuing debt

and signaling issues. For example, if a firm is issuing equity, that implies management thinks the stock is overvalued (Myers and Majluf 1994). However, equating the cost of equity and the cost of debt is a reasonable approximation. This approach will be utilized in determining the cost of capital for insurers by measuring the cost of equity. One significant advantage to using equity to replace lost capital is its permanence. A key question in issuing debt to generate needed capital is how long that capital is needed. That question does not have to be addressed when equity is used to replace capital.

6. The Market-based Cost of Capital Approach

In the market-based cost of capital approach, firms face the need to raise capital under certain outcomes of their economic capital models. The model will indicate how much the firm would need to raise to return to the capital level considered acceptable by management. This economic capital level would be affected by regulatory and rating agency considerations. How much capital the firm needs to raise would affect the cost of capital. The firm would also face different costs depending on whether it were the only firm needing to raise capital (such as after a unique event applicable only to that company), many insurance companies were also attempting to raise capital (such as after a natural disaster) or if a broad spectrum of firms were in need of replenishing capital (such as due to a recession or credit crisis).

Therefore, the riskiness leverage factor should depend both on the size of the aggregate loss and the cause of the loss. If the loss is idiosyncratic for the firm, then a low recapitalization charge could apply. If the loss occurred when a major insured catastrophe occurred, then a higher recapitalization charge could apply. If the loss occurred when a credit crunch or recession occurred, then an even higher recapitalization charge would likely apply. Economic capital models would have this information for each simulation.

The riskiness leverage factors need to be based on the true cost of capital for each scenario. This cost will vary depending on the financial condition of the company and of the industry and on general economic conditions. The more capital the firm needs to raise, the greater the cost of capital will be. In addition, how many other firms are attempting to raise capital at the same time will also affect the cost.

All the simulations used to illustrate the RMK approach in tables 1 and 2 are based on fixed distributions. The model is assumed to reflect all possible situations with the correct

probabilities. Thus, when an outlier occurs, it is the result of an unfortunate draw (process risk) and does not affect the distributions or any future outcome. In reality, the distributions and parameters are not known, so any adverse occurrence would change the perception of the expected future results (a Bayesian approach). After a major catastrophe, for example, individuals are more likely to purchase the affected coverage for a while, which increases demand and can increase profitability. Investors may be more willing to supply capital to the industry after an industrywide loss expecting markets to harden and profits to increase. However, if the model is properly parameterized in the first place, these changes in the perception of the industry's risk do not reflect any real change in circumstances.

Assume three states of the world that the company will face when issuing equity to replace lost capital. The first would be idiosyncratic in which the event affected only that firm. An example would be an industrial disaster at a policyholder's plant that generates a catastrophic level of workers compensation and property damage losses. As no other insurer would be impacted, the need to raise capital is idiosyncratic to the firm. Another example would be when new management determines they want the company to have a higher level of capitalization to reduce the risk of financial difficulties. This need for capital would be just for this company and not for the industry in general.

The second state would be industrywide. In this case, a catastrophe that affected a large part of the insurance industry would be the cause of the loss. A hurricane, earthquake or other natural disaster would exemplify this event. In this case, many insurers would be seeking to replenish lost capital at the same time, creating competition among insurers. In this event, the cost of capital could be higher than in the idiosyncratic case, since there is a greater demand for capital and potential investors have a choice of insurance firms in which to invest. On the other

hand, the cost of capital could be lower than for the idiosyncratic case if investors thought the industrywide catastrophe would lead to a greater demand for insurance, hard markets and greater profitability for insurers in the future. Also, the idiosyncratic loss may be viewed as an indicator of poor management (inadequate reinsurance, excessive exposures to large losses, poor underwriting, etc.). Studies of the cost of capital for insurers under a variety of idiosyncratic and industrywide circumstances would provide an indication of the size and sign of this differential.

The third state of the world would be systemic, in which general economic conditions adversely affect a large number of firms across a wide range of industries. A severe recession or credit crunch would be an example of this type of event. In this case, the cost of capital could be much higher than when capital needs are confined to the insurance industry. One recent example of this type of financing would be Warren Buffet's company Berkshire Hathaway providing \$5 billion of capital to Goldman Sachs Group Inc. and \$3 billion to General Electric Co. during the height of the financial crisis in the fall of 2008. In both cases, Berkshire Hathaway received preferred stock with a 10 percent dividend plus five-year warrants to purchase \$5 billion of Goldman Sachs at \$115/share and \$3 billion of GE common stock at \$22.25/share. This costly financing was necessary due to the precarious financial position of each company (Craig, Karnitschnig and Lucchetti 2008; Lohr 2008; Crippen 2009). Other companies, including AIG Inc. and General Motors Co., were forced to turn to the U.S. government for financing in circumstances that were quite costly and diluted shareholders' equity.

Based on this approach, the cost of capital is not simply company specific, as prior applications of the Ruhm-Mango-Kreps method assume. Therefore, the firm's potential cost of capital depends on two factors, how large the loss is and what precipitated the loss. Fortunately, both can be determined from a typical DFA model.

Bear (2005) proposes combining Mango's shared capital approach with the Ruhm-Mango-Kreps algorithm. The cost of the nonconsumptive capital is the firm's cost of capital. The cost of consumptive uses is based on the RMK algorithm. This approach will be followed here, but the riskiness leverage factor for the RMK algorithm will be based on both the size of the loss and the cause of the loss.

A very simple example similar to the one used in Ruhm (2003) and Bear (2005) will be used to illustrate this approach. An insurer writes equal amounts (\$100 million) of two lines of business, workers compensation and homeowners, and has three types of investments, stocks (\$150 million), bonds (\$400 million) and credit derivatives (\$50 million). For WC, the base case has an expected underwriting profit of 5 percent, but there is a 1 percent chance the firm experiences a catastrophic WC loss equal to 50 percent of the line's earned premium. For homeowners, the base case is an expected underwriting profit of 7.5 percent, but there is a 5 percent chance the firm experiences a catastrophic homeowners loss (Hurricane Andrew or Katrina level) that would be equal to 50 percent of premiums. In normal circumstances, stock returns average 8 percent, bond returns average 5 percent and credit derivative returns average 15 percent, but there is a 10 percent chance of a credit crunch that affects stocks and credit derivatives but not bonds since they are valued on an amortized basis. In the event of a credit crunch, the expected value of stocks is reduced by 2,000 basis points (from 8 percent to -12 percent) and the expected return on credit derivatives is reduced by 4,000 basis points (from 15 percent to -25 percent).

Results of 100 iterations are displayed in Table 1. This number is small enough to be able to show all the iterations but large enough to illustrate the effect of each type of catastrophic loss included in the model. In practice, hundreds of thousands of iterations would be run in order to

produce results that display a wide range of possible outcomes and to reflect the relationships among different subdivisions of the company in the distribution's tail. The iterations are sorted based on the size of the aggregate loss. The cost of capital is equal to the assumed market cost of capital (15 percent) plus the ratio of aggregate losses to the firm's actual capital (\$150 million) plus another 10 percent if the market has experienced a credit crunch or 5 percent if the insurance industry has experience a major catastrophe but there is no general credit crunch. A more realistic model would allow the market cost of capital and the adjustments to the cost of capital for a credit crunch or catastrophe to be stochastic variables, with the risk premiums dependent on the severity of these events. However, the fixed values used here are adequate to illustrate this approach. Although these parameters were selected arbitrarily, reasonable parameters could be determined based on a study of the cost of raising equity capital based on the size of the offering relative to the value of the firm and the cost of credit default swaps during a credit crunch. During 2009, the credit default swap spreads peaked in March, with the Standard & Poor's U.S. Investment Grade Index hitting 377 basis points on March 9, then declining to approximately 100 points by the end of the year (Kondas 2010). The simulated cost of capital could be set to track this level of movement. The riskiness leverage factor is the ratio of the cost of capital divided by the normal cost of capital (15 percent). In this example, the firm needs to raise capital in only nine of the 100 iterations, and it is the component results for those nine iterations that determine the capital allocation for the risk capital.

Table 2 displays the overall results in a format similar to Ruhm (2003). The average values for each column of the 100 iterations are shown on the first line. The second line, risk weighted expected value, is the sum of the product of the respective column times the risk leverage factors, divided by the sum of the risk leverage factors. In essence, this is a weighted

average (weighted by the risk leverage factors) of the results when a loss occurs. The third line, risk measurement, is the risk weighted expected value minus the average, or how much below average the risk weighted values were. These values are calculated for all the risks the firm faces, both underwriting and investments. For workers compensation, the average underwriting return was \$5.35 million, the risk weighted expected value \$4.03 million, for a risk measurement of negative \$1.32 million. For stocks, the average investment return was \$7.91 million and the risk weighted expected value was negative \$30.19 million, so the risk measurement is negative \$38.10 million. The capital allocation, shown on line 4, is the risk measurement for each component divided by the total risk capital. Based on this example, workers compensation would get 2 percent of the total capital, homeowners 17 percent, stocks 48 percent, bonds 7 percent and credit derivatives 27 percent. The Ruhm-Mango-Kreps approach would stop there and allocate all of the firm's capital (\$150 million) based on those percentages.

Mango (2006) and Bear (2005) incorporate the nonconsumptive aspect of capital allocation as well. This approach is recognized in the next section of Table 2. Line 5 is the Ruhm-Mango-Kreps risk capital amount, which is the negative of the risk measurement. Regulatory or rating agency capital is determined in the next three lines. A simplified approach is illustrated here. The firm must maintain a 3-to-1 premium to capital level (line 6), and a similar ratio of reserves to capital must be maintained until all losses are paid. The average duration that capital is needed (similar to a funds generating coefficient for capital asset pricing model applications to insurance) is shown on line 7. Line 8, regulatory capital, is premiums divided by premium to surplus ratio times the duration of capital need. In this example, regulatory capital is required only for lines of business. Risk-based capital also requires capital for investments based on perceived riskiness and investment concentration, but these are not included in this example.

The total calculated capital, line 9, is the sum of the RMK capital allocation and the nonconsumptive capital. Since this total, \$213.08 million, exceeds the actual capital of the firm, \$150 million, line 10 scales the calculated capital down to the actual capital level. This approach would achieve the objectives of allocating all of a firm's capital, reflecting the relative riskiness of each component of the firm, and incorporating any regulatory or rating agency capital requirements.

One application of capital allocation is for risk management. The last line calculates the expected total return on this capital by, for lines of business, adding the underwriting income to a risk-free investment times the allocated capital and dividing the total by the allocated capital. For investments, the risk-free return is subtracted from the average return, since any risky investment needs to justify the risk it generates by examining returns over the risk free rate. On this basis, workers compensation produces a rate of return of 13 percent, which is below the cost of capital. Homeowners generates a return of 21 percent, which exceeds the cost of capital. Based on these results, the firm should reduce its exposure in workers compensation, while increasing its homeowners exposure. For investments, stocks produce a return of 2 percent over the risk-free rate, bonds produce a return of 23 percent over the risk-free rate and credit derivatives produce a return of 8 percent over the risk-free rate. This would be compared with the market cost of capital, which at 15 percent is 10 percent over the risk-free rate. Thus, it would appear that bonds are the optimal investment, no doubt driven by amortized value accounting. However, the entire bond return is based on the fact that the average returns were slightly above, at \$20.88 million versus \$20 million (5 percent of \$400 million), the expected value for bonds. Since bonds generated so little capital allocation, this slight, random, fluctuation generated this extremely

high excess return. This is a good example of why the results of models should not be relied upon if the user does not understand their limitations.

7. Conclusion

This paper expands on the approach of Ruhm, Mango and Kreps to consider different impacts of calls on capital depending on the circumstances facing the insurer and to incorporate investment risk directly into the capital allocation process. In some instances, when other lines are profitable, a need for capital by one line would not require any additional financing and would not generate a capital charge. At the other extreme, one line's need for capital may occur when the company faces financial distress and incurs an extremely high cost of capital, one that may even require the sale of the company to a more adequately capitalized organization. In between, the cost of capital would vary according to the financial condition of the insurer and the economic conditions of the time.

In addition, the need for capital associated with particular investments, such as equities, mortgage-backed securities or other credit derivatives, is illustrated. For insurers investing in financial derivatives, such as writing interest rate floors or credit default swaps, a capital allocation method that assigns more capital to the investment area would be a clear indication of the additional risk the insurer is facing. This unified approach to capital allocation illustrated in this paper should facilitate more effective risk management across the firm and help insurers avoid some of the investment problems that occurred recently.

By incorporating event-specific risk charges into the Ruhm-Mango-Kreps approach, the uncertainty over the appropriate risk leverage factors is eliminated, replaced with market-based values. The result is a capital allocation method that considers the individual risk elements an insurer holds with respect to the overall portfolio of risks. This method is dependent on accurate models but, if the model is accurate, leads to a reliable allocation of capital within the organization. In adopting this approach, a firm would eliminate fruitless arguments over which

theoretical approach to capital allocation is appropriate. Any disagreement over allocated capital would instead focus on perceived flaws in the model. This focus could improve the firm's model, which would provide a valuable, tangible benefit for the organization.

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Contact information for author:

Stephen P. D'Arcy, FCAS, MAAA, Ph.D.
 Professor Emeritus of Finance at the University of Illinois
 Robitaille Endowed Chair in Risk and Insurance
 Center for Insurance Studies, California State University-Fullerton
 Mihaylo College of Business and Economics
 800 N. State College
 Fullerton, CA 92834
 Telephone: 217.621-1618
 Email: s-darcy@illinois.edu