

Efficient Capital Allocation through Optimization

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Abstract

In this paper, we formulate the Capital Allocation problem as an optimization problem in which we seek the mix of business that maximizes an insurance company's *Expected Net After Tax Income* subject to a constraint on the *Tail Value at Risk (TVAR)*. Using the method of Lagrange multipliers, we demonstrate that the returns on the respective TVAR contributions, so-called RORAC, are equal across all lines of business when the mix of business is optimal. We refer to this state as RORAC Equilibrium. We then investigate the impact on RORAC Equilibrium of introducing premium constraints in the optimization problem. We show that these constraints impose a cost on the company's Net After Tax Income. When the line of business returns are adjusted for the applicable costs, equilibrium is maintained. Using commercially available optimization software, we solve the optimization problem for a fictitious start-up company and we show several points on the so-called efficient frontier curve of the company. Cases with various premium constraints are also examined. Although the discussions in this paper center on the TVAR, the conclusions hold true for any conditional expected value measure.

Keywords: Optimization; Lagrange Function; Lagrange Multipliers; Capital Allocation; RORAC Equilibrium.

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