

Next Steps for ERM: Valuation and Risk Pricing

Gary G. Venter, FCAS, ASA, CERA, MAAA

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Abstract

ERM has methodology for quantifying the risk to capital, but to determine optimal capital the impact of capital level on firm value is needed. This gets into the realm of valuation. Also capital allocation provides risk quantification for business units, but comparing return among units is fundamentally a risk-pricing exercise. When considering risk pricing, optimal returns on allocated capital are not necessarily constant across business units. The current ERM methodology, as well as actuarial literature on valuation and pricing, are reviewed, and possible directions for application of pricing and valuation methods to risk management problems are outlined.

Keywords:

ERM; risk pricing; firm value; capital allocation.

1. Introduction

Enterprise risk modeling has developed tools for risk quantification and attribution of risk to business sources. Quantifying the risk to capital, for instance being able to say that the probability of losing half of capital in a single year is less than 1 percent, does not determine how much capital to hold, however. Quantifying capital risk is useful for regulatory review of capital levels, but setting optimal capital includes other issues, like customer risk preferences, competition, etc. The insurance finance literature suggests that financially stronger insurance companies are more profitable (see, for example: Epermanis and Harrington (2006); Grace, Klein and Kleindorfer (2004); Phillips, Cummins and Allen (1998); and Sommer (1996)). Thus increasing capital strength can enhance insurer firm value to some degree. Franchise value, defined as firm value less capital, is a reasonable target to optimize. Models of firm value are discussed below.

Attributing risk to business units is used to allocate capital or determine the capital costs of the business units. Units can be compared based on return to allocated capital or value added, i.e., profits less cost of capital. This is an attempt to determine which business units are earning more profit compared to the risks they entail. Fundamentally this is a risk-pricing exercise. However its typical application ignores much of the risk-pricing literature. The allocation of risk measures used to quantify capital risks does not necessarily capture the value of the risk for risk transfer purposes. For instance, capital is sometimes allocated based on tail risk only, but a company would want to charge for the risks it bears even when not in the tail. Capital allocation and capital cost quantification methods are reviewed below in the context of their risk-pricing implications.

Section 2 discusses criteria for risk quantification and attribution in the context of risk pricing. Section 3 reviews related ERM methodology with those criteria. Section 4 addresses risk pricing in general and suggests possible refinements. Section 5 surveys directions for firm value modeling. Section 6 concludes.

2. Criteria for Risk Quantification and Attribution

If risk attribution is used compare profits of insurer business units, the risk attributed to the business unit should in some way reflect the value of the risk the unit is taking. This is not a precise requirement, but will be used as a broad evaluation standard. Value here could mean market value in general or company-specific market value, such as the price the company has to charge to make the business add value. Some more precise criteria follow.

- Risk grows more than linearly with loss potential. This is a fundamental risk attitude reflected in utility theory, for example. Simply put, a loss that is twice as big is more than twice as bad.
- Risk-taking is not cost free. Any degree of risk-taking needs some charge, even if small. At least scenarios generating an economic loss should have a charge, and probably anything worse than plan should.
- The charge for adding a new obligation should be commensurate with the increase it produces in the overall risk of the firm. This is a formulation of the principle of marginal pricing.

Risk measures and attribution methodologies can be examined with reference to these criteria.

3. Review of ERM Risk Quantification and Attribution

In this section, concepts and methods of risk measurement and risk attribution are reviewed and compared to the risk-pricing criteria. The notation is: Y is a random variable with cumulative distribution $F(y)$ for a company, with $Y = \sum X_i$, the sum of business units (which could even be individual policies). $\rho(Y)$ is a risk measure on Y (i.e., a functional mapping F to a real number) and r is the allocation, i.e., $\rho(Y) = \sum r(X_i)$.

Marginal allocation: allocating in proportion to the impact of the business unit on the company risk measure. This can be: **last-in marginal allocation** where the impact of the business unit is measured by $\rho(Y) - \rho(Y - X_i)$, the company risk measure with and without the unit; **Aumann allocation**, where the impact is averaged over every coalition of business units that unit can be in; or **incremental marginal allocation** where the impact is $[\rho(Y) - \rho(Y - \varepsilon X_i)] / \varepsilon$, the change in the company risk measure from ceding away a small proportional part of the unit, grossed up to the size of the whole unit. In the limit $\varepsilon \rightarrow 0$ this is the derivative of the company risk measure with respect to the volume of the business unit.

Marginal allocation is required to meet the criteria of marginal pricing, but the allocations should add up to the entire risk of the company. This is possible with incremental marginal allocation.

Marginal decomposition: when the incremental marginal impacts add up to the whole risk measure, the allocation is called a marginal decomposition of the company risk measure. By Euler's Theorem, this happens when the risk measure is homogenous degree 1, i.e., for a positive constant k , $\rho(kY) = k\rho(Y)$. Marginal decomposition is also called Euler allocation. It was introduced to the actuarial literature in Patrik, Bernegger and Rüegg (1999). Several examples are derived in Venter, Major and Kreps (2006), who also tie down the application of Euler's Theorem to random variables.

As an example, the standard deviation has a marginal decomposition equal to the covariance of the unit with the company, divided by the standard deviation of the company. This can be seen by taking the derivative of the company standard deviation wrt the volume of the business unit, using l'Hôpital's rule for the limit. More examples are below. Marginal decomposition would seem to be a fundamental requirement needed for capital allocation to meet the marginal pricing criterion.

Proportional allocation: allocating a risk measure by calculating the risk measure on the company and each business unit, and allocating by the ratio of the unit risk to the company risk: $r(X_j) = \rho(Y)\rho(X_j)/\Sigma\rho(X_j)$. This will not usually provide a marginal decomposition, but will if the risk measure is the mean under a transformed probability distribution. Then the risk measure on each unit is the transformed mean for the unit, and these add up to the transformed mean for the firm.

Suitable allocation: defined by Tasche (2000). Under a suitable allocation, if you allocate capital by the allocation of a risk measure, and compute the return on allocated capital, then proportionally increasing the size of a business unit that has a higher-than-average return on capital will increase the return on capital for the firm. Venter, Major and Kreps (2006) show that marginal decomposition always produces a suitable allocation, and it appears that it is the only method that guarantees suitability. This seems to be a fundamental property that an allocation should provide, and adds further support to requiring that allocation be a marginal decomposition.

To see what can happen otherwise, suppose that there is a marginal decomposition for a risk measure but for some reason a different allocation is selected. Thus some units will be allocated more capital and some less capital than under the marginal decomposition. Some units will have lower than average return on allocated capital and some higher than average under each allocation. It is thus quite possible that a unit that gets lower allocation under the selected method has a higher-than-average return under that method and a lower-than-average return under marginal decomposition. Growing that unit proportionally, perhaps by buying less quota share reinsurance, and increasing capital by the marginal impact on the overall company risk measure, will reduce the company return on capital due to the properties of marginal decomposition. Thus the selected method can result in wrong decision-making.

Co-measure: defined if $\rho(Y)$ can be expressed as:

$$\rho(Y) = E\{\sum_i[h_i(Y)L_i(Y) | i^{\text{th}} \text{ condition on } Y]\},$$

where the h_i 's are additive, i.e., $h(V+W) = h(V)+h(W)$, and the only restriction on the L_i 's is that the conditional expected value exists. Then the co-measure is defined by:

$$r(X_j) = E\{\sum_i[h_i(X_j)L_i(Y) | i^{\text{th}} \text{ condition on } Y]\}$$

By the additivity of the h 's, the co-measures add up over the business units to the whole risk measure. For instance if there is only one h and L , with the condition $Y > F^{-1}(0.99)$, and $L(Z) = 1$

and h is $h(Z) = Z$, then $\rho(Y) = E[Y | Y > F^{-1}(0.99)]$ is TVaR at 99% and $r(X_i) = E[X_i | Y > F^{-1}(0.99)]$ is its co-TVaR, which is a marginal decomposition. The ability to take a sum of several functions allows risk measures like the sum of TVaR at different probability levels to be used. This formulation of co-measures is from Venter, Major and Kreps (2006).

A less trivial example is risk-adjusted TVaR, or RTVaR, first defined by Furman and Landsman (2006) (although they called it something else). This has two sets of h 's and L 's. The condition on Y is $Y > F^{-1}(\alpha)$ for both sets of functions, with $h_1(Z) = h_2(Z) = Z$, $L_1(Y) = 1$, and $L_2(Y) = c[Y - E(Y)] / \text{Stdev}(Y | F(Y) > \alpha)$ for some constant c , which will usually be between zero and one. Then:

$$\begin{aligned} \rho(Y) &= E[Y | Y > F^{-1}(\alpha)] + c \text{Cov}(Y, Y | F(Y) > \alpha) / \text{Stdev}[Y | F(Y) > \alpha] \\ &= \text{TVaR}_\alpha + c \text{Stdev}[Y | F(Y) > \alpha]. \end{aligned}$$

The co-measure, co-RTVaR, which is a marginal decomposition, is:

$$r(X_i) = \text{co-TVaR}_\alpha(X_i) + c \text{Cov}(X_i, Y | F(Y) > \alpha) / \text{Stdev}[Y | F(Y) > \alpha].$$

The function $L(y)$ was used by Kreps (2005) to express the risk-adjusted value of a random variable Y as $\rho(Y) = E[(Y - aEY)L(Y)]$. Then the co-measure is $E[(X_i - aEX_i)L(Y)]$. For instance if $L(Y)$ is the indicator function for $F(Y) > 0.99$, and $a=0$, this is TVaR at 99 percent. The special case $a=0$, which includes TVaR, was addressed in Mango and Ruhm (2003), so the effort to find an appropriate riskiness leverage function for a company and use it to define a risk measure and co-measure is called the RMK algorithm. The Kreps and Mango-Ruhm papers were originally in the risk-pricing context.

When a risk measure can be expressed in the form applicable for co-measures, there are many equivalent expressions, leading to different co-measures. At most one of these can be a marginal decomposition. As an example, taking $h(X) = X$ and $L(Y) = \text{Stdev}(Y)/E(Y)$ gives $\rho(Y) = \text{Stdev}(Y)$ and $r(X_i) = \text{Stdev}(Y)E(X_i)/E(Y)$. This is not a marginal decomposition. Taking $h(X) = X - E(X)$ and $L(Y) = [Y - E(Y)]/\text{Stdev}(Y)$ gives the marginal decomposition of standard deviation $r(X_i) = \text{Cov}(X_i, Y)/\text{Stdev}(Y)$. A similar example with VaR is given below.

Showing this is the marginal decomposition involves taking the derivative of the risk measure of the firm wrt X_i , that is, $\lim_{\varepsilon \rightarrow 0} \frac{\rho(Y) - \rho(Y - \varepsilon X_i)}{\varepsilon}$. Often the limit is most easily accomplished by l'Hôpital's rule, which gives $-\rho'(Y - \varepsilon X_i)|_0$, where the prime denotes the derivative wrt ε . For standard deviation, the derivative of $-\text{Std}(Y - \varepsilon X_i) = -[\text{Var}Y - 2\varepsilon \text{Cov}(X_i, Y) + \varepsilon^2 \text{Var}(X_i)]^{1/2}$ gives at $\varepsilon = 0$ $r(X_i) = \text{Cov}(X_i, Y) / \text{Std}(Y)$. Venter, Major and Kreps (2006) add εX_i to define the derivative, but that gives the problem of how to imagine a proportional increase in business volume. Using the derivative to the left takes a proportional decrease, which is easy to imagine as a small quota share ceded. The same approach works for Myers-Read, below.

Even if a co-measure allocation is marginal, it might not meet the other pricing criteria of not ignoring risk and risk increasing more than linearly. For instance, tail risk measures like VaR and TVaR ignore some smaller losses. If the tail probability level is chosen low enough to include all adverse scenarios, TVaR does not ignore the small risks. However TVaR is linear in losses above the threshold, violating the non-linearity property. Insurers have found as that allocating by TVaR over a low threshold attributes less risk than seems reasonable to units exposed to large losses.

A reasonable alternative is to use RTVaR over a low threshold. RTVaR increases more than linearly, and the low threshold would include the smaller losses. This would also meet the more general standard of being plausible as a risk-pricing metric, as standard-deviation loading is a traditional risk-pricing method.

A similar effect can be produced by taking the risk measure to be a weighted sum of TVaRs at different probabilities, from quite low to quite high. For instance, 60 percent, 90 percent, 98 percent and 99.6 percent could be used. This would include the impact of smaller losses, but emphasize the large losses. There is a good deal of flexibility here, which could be used to calibrate risk loads to market prices.

Aumann-Shapley method: an accounting method of cost allocation for the cost of production facilities that are used to produce a variety of products—see Billera, Heath and Verrecchia (1981). For risk measures or cost functions that are homogeneous of degree 1, this comes out the same as the Euler method, or marginal decomposition. For other functions it is a similar average, but over all production levels from zero to full capacity, which is not really relevant to insurance

lines of business. Thus basically this is useful only for homogeneous risk measures and is then the Euler method.

Myers-Read allocation: an additive marginal allocation method that requires that the value of the default put option as a fraction of expected loss be the same for each business unit. The risk measure is required capital itself, and the incremental marginal change in required capital from a small proportional reduction to a business unit is the amount by which capital can be reduced and still keep the same company-wide ratio of default put value to expected losses. This incremental marginal change in capital is grossed up to the volume of the whole business unit to give the capital allocated to the unit. It turns out that these allocations add up to the overall capital. Thus the Myers-Read method is a marginal decomposition. Calculating the default put value requires the whole probability distribution and it is more impacted by large losses, so the other criteria are met as well.

Even with these properties, it does not appear that the Myers-Read allocation was ever intended to be the denominator of return on capital. It was proposed for allocating the frictional costs of holding capital, such as taxation on investment income, but not as a measurement of the price for bearing risk.

Allocation by layer: an allocation method introduced by Bodoff (2008). It is easiest to describe in the context of a simulation model. Let $X_{i,k}$ be the loss to unit i in the k^{th} simulation, which has total company losses Y_k . The capital C is to be allocated to unit.

Layers of losses up to C are needed for the allocation. To take a fairly extreme case, assume that layer z is the layer from $z - 1$ US¢ to z ¢, and that all simulations have been rounded to whole cents. (Yen would be approximately the same.) Define n_z as the number of simulations Y_k that are z

or greater. The allocation of layer z to unit i is $\frac{1}{n_z} \sum_{k \text{ with } Y_k \geq z} \frac{X_{i,k}}{Y_k}$. The allocation of C to unit i is

$$\sum_{z=1}^C \frac{1}{n_z} \sum_{k \text{ with } Y_k \geq z} \frac{X_{i,k}}{Y_k}. \text{ As a check, summing the allocation over the units (i's) gives } \sum_{z=1}^C \frac{n_z}{n_z} = C.$$

Bodoff considers the case $C = \text{VaR}_\alpha$, but the method works more generally for any risk measure. For instance, if you want to allocate TVaR, just use the layers up to TVaR, and similarly for standard deviation, etc. The allocation has some good properties. All layers contribute to the allocation, so it does not ignore smaller but potentially painful losses. Also the larger simulations get into the allocation for all lower layers, so they get a greater weight overall. Thus the units that generate

large losses get a bigger allocation. As this is an allocation of a specific capital level, it is not meaningful to say it is marginal or not. If you reduce business, C is still C , so the marginal impact is zero. However if C is set equal to a risk measure and allowed to change with the volume of the writings, the resulting allocation is not marginal in any known cases.

Merton-Perold: a method of attribution of the cost of capital to business unit that does not require capital allocation, based on Merton and Perold (1993). The firm is viewed as providing each business unit with the option to use the firm capital if its losses exceed its premiums. The value of that option is the implicit capital cost to the firm of carrying that business. The value added of the unit is the excess of its profits over the capital cost. Merton and Perold assume a standard options-pricing framework so they can use the Black-Scholes formula for the value of the option. The distributions involved and the one-period time frame make such assumptions inappropriate for insurance risk, but the basic concept can be applied with more realistic distributional assumptions.

A more fundamental criticism is that the profits are also an option, in this case the option the firm has to take all the profits of the business unit if there are any. However, the combination of the two options is not a contingent claim at all: the firm takes all the profits and pays all the losses. Thus the value added of the business unit would traditionally be calculated as the net present value of its cash flows under a risk-adjusted rate, having nothing to do with capital or options. Or other pricing concepts, like arbitrage theory or capital asset pricing model (CAPM), could be used to value the unit. Still, the division of the value into the cost and profit components may provide insights. This approach meets all the criteria in Section 2.

Capital consumption: an application of Merton-Perold to insurance risk that uses distributions and risk pricing more realistic to the non-life insurance business, from Mango (2005).

Coherent risk measure: Artzner et al. (1999) define the concept of a coherent risk measure, which is a risk measure meeting a few mathematical requirements, the most controversial and most often failing being subadditivity: the risk measure of a sum of independent random variables should not be greater than the sum of their risk measures. This is a useful criterion if the question being addressed is measuring the diversification benefit of combining business units, and the analyst wants to guarantee in advance that the answer will not be negative. Otherwise it is not really a necessary requirement. Since marginal allocation does not look at the risk of individual units, but rather their contribution to the risk of the whole, subadditivity is not relevant in this context.

Distortion measure: from Wang (1996), defined by a distribution function $G(x)$ on the unit interval (that is, $G(0)=0$, $G(1) = 1$ and G is non-decreasing) so that the risk measure $\rho(Y) =$

$\int_0^\infty G[S(y)]dy$, where $S(y) = 1 - F(y)$ is the survival function of Y . Actually $G[S(y)]$ is itself a survival function. Thus the role of G is to transform the probabilities of Y , and since the mean of a distribution is the integral of its survival function, a distortion risk measure is a transformed mean. The marginal decomposition of a transformed mean is the transformed mean of the business unit, where the transform uses the transformed probabilities of the aggregate firm variable. Applying G directly to the survival functions of the business unit will not necessarily give a marginal decomposition. However if $\rho(Y)$ is defined as $\Sigma r(X_i)$ this will work.

Famous examples of such distortion measures are $G(p) = p^a$ (proportional hazards transform) and the Wang transform $G(p) = 1 - T_a[\Phi^{-1}(1-p) - b]$, where T_a is the t-distribution function with a degrees of freedom, and Φ is the standard normal distribution. However $\text{VaR}_{0.99}$ and $\text{TVaR}_{0.99}$ are also distortion measures. They both have $G(p) = 1$ if $p > 0.01$. Note that $G[S(y)]$ then is 1 when $F(y) < 0.99$, so the portion of the integral from 0 to $F^{-1}(0.99)$ is $F^{-1}(0.99)$. VaR has $G(p) = 0$ otherwise, whereas TVaR has $G(p) = p/0.01$ otherwise.

Distortion measures are in fact **spectral measures**. These are of the form $\rho(Y) = E[Y \cdot \eta(F(Y))]$ for some nonnegative scalar function η . A bit of calculus can show that taking $\eta(p) = G'(1-p)$ will put any distortion measure into the spectral form.

Transformed means are arbitrage-free if the transformation is made on the probabilities of events. However this may not be the case if the transform is on the probabilities of outcomes of deals—see Venter (2004). Distortion measures can be applied either way, so may or may not be arbitrage-free. In a simulation model, the events represented are the simulated scenarios. Thus to apply a distortion measure in an arbitrage-free manner, the probabilities of the scenarios would be modified, and those probabilities would be applied to the results of every business unit in the scenario. This could end up with bumpy implied distributions for the business units themselves, but would give a marginal allocation of the firm transformed mean to the units.

This would be fairly simple in practice. Say the Wang transform is selected. Wang's (2004) results suggest that taking a around 5 or 6 is consistent with some market prices, so the main issue is the choice of b . The scenarios would be sorted by firm loss to get the observed survival function.

Then the transformed mean can be calculated as a function of b , and the b that gives the target return for the firm can be found. This would then give the transformed probability for each scenario, and these then give the transformed unit means.

Distortion measures have marginal decompositions and since they are transformed means, they are plausible as pricing tools. However some distortion measures satisfy the criteria of using the entire distribution of losses and being non-linear in large losses, and some fail to. These criteria give rise to types of distortion measures, although these concepts can be applied to other measures too.

Complete risk measure: as defined by Balbás, Garrido and Mayoral (2009), the risk measure uses the entire probability distribution of Y in a non-trivial way. This can be formally defined for distortion risk measures by requiring that $G(p)$ is not constant on any interval, and so is an increasing function on the unit interval. No tail measures satisfy this definition. The motivation is that if the risk measure is to be used to express preferences among random variables, this cannot be done using the tail alone. Complete distortion measures like the Wang transform meet all the pricing criteria.

Adapted risk measure: one meeting two more requirements, also from Balbás, Garrido and Mayoral (2009). If a risk measure is going to be used in pricing, typically you would not want it to be less than the mean of Y . For a distortion measure this requires that $G(p) \geq p$. However another practical requirement is that in the tail the relative risk load is unbounded. This would be needed to get behavior like a minimum rate on line, where the ratio of risk load to expected losses can be very large. For distortion measures this can be expressed as $G'' < 0$ and G' goes to infinity at $p = 0$. The Wang transform is an example. If the unlimited variance of Y is infinite, then the standard deviation loading of higher layers would also increase without bound, and so would meet this criterion. Usually minimum rates on line are used only with heavy-tailed distributions, so this is a realistic example.

Transformed distributions: not every transformed distribution is a distortion measure. Consider, for instance, the Esscher transform $f^*(y) = f(y)e^{y/c}/Ee^{Y/c}$. This does not exist for many heavy-tailed distributions, but in practice losses will be capped by policy limits which will make the transform finite. It has a free parameter c that determines the change in level. The change in probability depends on the value y of the loss. This does not happen with distortion measures since they are spectral measures, that is the transform is a function of the probability but not the value of the loss. The Esscher transform is thus not a spectral measure.

If Y is a compound frequency-severity process, a combined transform can be defined that simultaneously applies to the frequency and severity probabilities. A popular transform in the finance literature is the minimum entropy martingale measure, which is the martingale that minimizes a particular information distance measure from the original process. Møller (2004) shows how to apply this to an insurance profit process consisting of a fixed premium flow minus compound Poisson losses. If Y is the severity with density f , this transform uses a constant c which determines the profit load, with the frequency and (Esscher) severity transforms:

$$\lambda^* = \lambda Ee^{Y/c}, f^*(y) = f(y)e^{y/c}/Ee^{Y/c}.$$

Venter, Barnett and Owen (2004) find this provides a reasonable representation of catastrophe reinsurance prices, including high risk loads that resemble minimum rates on line for high layers. Note that the constant c does not have any specific meaning—it can be quite different for different distributions with the same profit loading, and is not likely to be the same across lines. A starting point for c might be somewhere around the 75th to 90th percentile of Y . If the policy limit is a significant multiple of that, the upper layer probabilities can increase by a factor of 100 or more.

This approach might work well if distributions used in pricing were transformed for each line to match the target profit loads, giving a c for each line, and all of these were used in the simulation of aggregate results. This would be a different way to organize enterprise risk management, starting from line profit goals instead of providing them. Some method to compare the profit goals to see which lines were more aggressive might be needed. Perhaps comparing c to percentiles of each line's severity might be helpful.

An alternative would be to use the Esscher transform on the aggregate losses from simulated scenarios. This could be applied to simulations as described above for the distortion measures to get transformed probabilities by scenario, which would then be used for each line.

Transformed distributions can be applied to other statistics than the mean. TVaR under transformed probabilities is called weighted TVaR, or WTVaR. This is no longer linear in large losses, so if it is used over a low probability threshold, it also meets the pricing criteria.

Summary of ERM risk and attribution: either capital consumption or marginal decomposition can meet the criteria set out in Section 2. Risk measures such as a weighted sum of TVaRs, RTVaR, WTVaR, or some of the probability transforms meet all the criteria set out as long as they are allocated by marginal decomposition. Myers-Read would be in this category as well. As far as

being plausible as a pricing method, the Wang transform and the minimum entropy martingale transform would be the most commensurate with pricing theory. However all of them would have a problem with pricing that considers the viewpoint of a diversified investor. Some other probability transforms would work in that context, however. This is part of the topic of the next section.

4. Whither Risk Pricing?

ERM professionals in the insurance industry are looking to capital allocation to price risk because in many insurance companies virtually no one, including actuaries, accountants, risk managers, and senior management, accepts standard risk-pricing approaches such as CAPM and arbitrage pricing theory. Some reasons for this reluctance are discussed below. This is in stark contrast to the academic finance profession, who routinely use these pricing methods, and some of whom have expressed skepticism about the role of capital allocation in pricing.

For instance, Gründl and Schmeiser (2007) say “However, we could not find reasons for allocating equity capital back to lines of business for the purpose of pricing. This holds true for cases that do not integrate frictional costs and also when such costs are considered.” And later, “We explained the main difficulties an insurance firm runs into when using capital allocation models for capital budgeting decisions such as expanding or contracting certain business segments. (This) ... typically leads to wrong decisions by an insurance company.”

Sherris (2006) marginally allocates a risk measure—the default put option—to lines of business in a pricing context, but does so in order to subtract it from the price otherwise developed. He discusses capital allocation, but finds that it does not affect prices. Both these papers assume that pricing is done with transformed expected values within the arbitrage pricing framework. Thus they are essentially finding that capital allocation is not necessary if there is an available pricing mechanism.

The non-life insurance industry has been reluctant to embrace financial risk pricing, such as CAPM and arbitrage theory. One problem with CAPM is that the distributional assumptions are inconsistent with the heavy-tailed distributions in non-life insurance. Another may be that the betas are regarded to be similar across lines and so can be ignored. There is also a strong emphasis in practice on pricing specific risk with no reference to the market risk. However ERM is breaking down the resistance to pricing in a diversified context, at least within a single insurance enterprise. Thus it is not as much as a conceptual leap to pricing from the investor point of view as it once was.

The distributional argument is perhaps the strongest conceptually. However this is not unique to the insurance industry. Rubinstein (1973) already pointed out that investor preferences related to higher moments should and do influence security prices. Kozik and Larson (2001) provide a more detailed history of the consideration of higher moments in asset pricing, and how this can be

applied in insurance pricing. Basically investors prefer negative odd moments and low even moments for portfolio returns.

Fama and French (1992) report another problem with CAPM: other factors besides covariance are priced. In particular they find a higher return for larger companies, and for companies with low market-to-book ratios. There are different ways to interpret their result. One strong contender is the efficient market view: portfolios of these securities get higher returns because they have higher risk, after controlling for covariance. Efficient market theory is perhaps a faith-based matter these days, but there is some support for it here, mainly because these effects persist even though they are well-known. If they were just due to pricing mistakes, the market would have corrected for them already. However the implication is that the Fama-French factors (FFF) are surrogates for risk measures. There has therefore been some attempt at replacing them with more direct risk quantification.

Hung (2007) shows that including the 3rd and 4th co-moments gives better fits than does CAPM plus FFF, even though those factors still improve the fit further. Chung, Johnson and Schill (2006) find that the FFF become insignificant if enough higher co-moments are included. However this might require as many as 10 higher moments, which seems complicated. That gives some impetus to looking for other measures of risk besides moments.

One such possibility was suggested by Wang (2002). He proposed a sort of co-moment generating function risk measure $E[Xe^{Y/c}]/Ee^{Y/c}$, which he calls the exponential tilting of X with respect to Y . If X and Y are bivariate normal and Y is the market portfolio, then c can be found that makes this risk measure the CAPM price—see Landsman and Sherris (2007). Thus exponential tilting can be regarded as an extension of CAPM to non-normal risks, including with a non-normal market, without explicit reference to moments. Empirical work is needed to see how well this works, however.

Pricing by probability transforms has not fared much better than CAPM in insurance practice. Even though this appears to be risk-specific pricing, there are other areas of resistance. Often the comment is heard that arbitrage pricing, being additive, does not reflect the benefit of diversification. Also there is not much secondary trading in the insurance market, so continuous hedging strategies are not available. However there is enough competition in the insurance business to force non-diversified companies to match the prices of their diversified competitors. Also the theory of pricing in incomplete markets requires pricing by transformed expected values even if hedging is not

possible. Thus the objections to using this approach are a bit weak.

The interaction with CAPM is another issue. In a complete market the arbitrage price is unique and has an implied hedging scheme, so beta does not have to be addressed, e.g., as in the Black-Scholes formula. In an incomplete market, pricing with a transformed mean is not unique, there is no complete hedging, and just being arbitrage-free does not guarantee that risk is properly priced. Thus the relationship with market risk is an operative issue. This would appear to require recognition of dependency with the market in the transformed-mean pricing.

One thing that helps in that context is that CAPM can be expressed as a transformed mean and so is a type of arbitrage-free price. To see this, let r be the risk-free rate, and suppose that Y has density $f(y)$ with $EY = (1+r)Y_0$. The transformed distribution will be $g(y) = f(y)h(y)$, and since this must integrate to unity, $Eh(Y) = 1$. Take $h(y) = 1 + b(E[M|y] - EM)$, where M is the market portfolio and b is a constant small enough that h is not negative. It is easy to see that $Eh(Y) = 1$. The transformed mean is $E^*(Y) = E[Yh(Y)] = EY + b\{E[YE(M|Y)] - [EM][EY]\} = EY + b\{E[YM] - [EM][EY]\} = EY + bCov[Y,M]$. Let the returns for Y and M be $R = Y/Y_0 - 1$ and $R_M = M/M_0 - 1$. Then $E^*(Y) = (1+r)Y_0 + bCov[RY_0, R_M M_0] = (1+r)Y_0 + bY_0 M_0 Cov[R, R_M]$ and so $E^*(R) = r + bM_0 Cov[R, R_M]$. To get CAPM, bM_0 should be replaced by setting $b = (R_M - r)/(M_0 \sigma_M^2)$ to get the expected return under transformed pricing of $E^*(R) = r + Cov[R, R_M](R_M - r)/\sigma_M^2$. Also $E^*(Y) = EY + Cov[Y, M][M - (1+r)M_0]/(M_0 \sigma_M^2)$.

Comparable transforms can be made for higher co-moments as well. These are defined similarly to other co-measures. The n^{th} co-moment $s_n(Y, M) = E\{[Y - EY][M - EM]^{n-1}\}$. Note that this is not commutative, that is it is not usually the same as $s_n(M, Y)$ for $n \neq 2$. Higher co-moments can also be represented as probability transforms. For instance, expanding the co-3rd moment shows that it is $E[YM^2] - EM^2EY - 2E[YM]EM + 2EY(EM)^2$. Then similar arguments as for covariance show that setting $h(y) = 1 + b\{E[M^2|y] - EM^2 + 2[EM]^2 - 2E[M|y]EM\}$ gives the loaded co-3rd moment as a transformed mean. Perhaps other transforms can retain the connection with the market risk as well as distort the probabilities in other ways.

Thus no-arbitrage pricing has the potential to represent the risk needs of a diversified investor. However to do this requires certain types of probability transforms. Hopefully there is a way to do this without using 10 co-moments, perhaps with exponential tilting. Further research is needed to flesh out these ideas.

Recognizing jump risk may also be necessary in order to get a suitable pricing formula for insurance risk. Arbitrage-free pricing formulas for processes including jumps are fairly widespread. See, for example, Kou and Wang (2004), Jang (2007) or Jang and Krvavych (2004). Ramezani and Zeng (2007) find that U.S. stock indices can be best modeled with about 10 jumps per year in addition to a diffusion process. Dunham and Friesen (2007) find about 50 jumps per year for equity futures, and that jumps make up a large part of their price risk. Also a market with jump risk makes perfect hedging impossible. For all these reasons, diversified shareholders probably care about jump risk, and pricing models should price for it. Perhaps an approach would be to use co-jumps, i.e., the jumps in a process when the market has a jump. This is another area for development of pricing tools.

Defining co-jump risk is most readily done specific to a particular model of pricing movements. One possibility would be a compound-Poisson model, or other such compound frequency-severity model. The minimum-entropy martingale transform discussed above for a compound-Poisson process applies to the jump risk component by simply increasing the expected number of jumps. Another possibility would be a Levy process. In general these processes allow infinitely many small jumps in a finite period, but only finitely many of these are greater than ε no matter how small ε is. Thus it should be reasonable to ignore the small jumps and consider a model that is a continuous process plus a compound-Poisson process. The jumps would be at the Poisson events, and the co-jump risk could simply be a co-measure of the risk jumps with the market jumps. For something like earthquake insurance, the insurance losses could have a strong link with the market losses, so a reasonable first approximation to the co-jump would be the risk's own jump. Thus just pricing jump risk as idiosyncratic risk may be reasonable as a starting point.

There are only finitely many trades in a day, so observed prices are never continuous. But small jumps from trade to trade are consistent with an underlying continuous process. Adding a jump process is a way to reconcile many small price movements, which define a specific volatility, with some occasional larger movements, which have a different volatility. There might be an ambiguity between the two volatilities that could produce competing models. While traded prices are discrete, bid and ask prices exist over continuous ranges. These can be thought of as a sort of cloud of probability around where the price is between trades. One potential model is a quantum-type approach, where the price process does not have actual values between trades—only the cloud of probability exists. This is perhaps easier to accept conceptually for security prices than it is for parti-

cles. Whether or not such a model would have any practical implications is not clear.

Many insurers feel that they do not need models for market prices. They know the market prices already. What they want to know is whether or not the market prices for the various business segments give them adequate returns for the risks they are taking. That is why they look towards capital allocation, etc., to evaluate returns. But so doing is just substituting a naïve, undeveloped and poorly understood pricing model for more well-researched though still incomplete pricing models. The standard pricing models, perhaps with some more development as outlined above, can be regarded in a comparative framework as giving the relative risk-price adequacy among business units, which is what the strategic planners are looking for. However since these models usually have a free parameter or two, it can be a problem to create a consistent comparison. This may be easier with spectral transforms like the Wang transform that transform probabilities only than with transforms like Esscher, which need the parameter to relate to the random variable being transformed. Perhaps making the Esscher parameter a fixed percentile of the distribution could provide a basis for comparisons.

5. Firm Value, Capital and Risk

Modeling firm value involves capturing the impacts of both capital and risk. Broadly speaking, more capital makes the firm more valuable, as does less risk. These effects are reflected very differently in different models, however. Traditional actuarial analysis featured the probability of ruin, with more capital increasing the expected time to ruin. This was put into a firm-value context beginning with de Finetti (1957), who expressed firm value as the expected present value of future dividends to shareholders, which is very much in the tradition of analysis of life insurance policies. This firm value is not the same as market cap, as it is supposed to reflect a somewhat stable underlying value. Market cap is based on the price those interested in buying or selling shares immediately would pay or take for those shares, which are typically a small fraction of outstanding shares. When a company is bought as a whole it is usually at a significant multiple of market cap.

In de Finetti's approach, more capital increases the likelihood of the firm being around in the future to pay more dividends. However a dividend paid earlier has a higher present value than one paid later, which creates a tension between keeping capital and paying it out. This suggests that there is an optimal capital level. Taking more risk can reduce the probability of survival as well, but if it comes with the possibility of more return, it might increase firm value. In some cases, reducing risk, even by costly risk transfer, can increase firm value by increasing the likelihood of extended survival.

De Finetti was assuming the company starts with a fixed capital and survives unless it is used up. In the model of Modigliani and Miller (1958), the company is assumed to be able to raise capital at any time at a fixed borrowing-lending rate. Risk and value are measured by CAPM. It is not too surprising that they find that costly risk transfer is never worthwhile. Thus in 1957–1958, the actuarial and financial worlds were off on quite different paradigms.

The direction of the actuarial approach is well illustrated by Gerber and Shiu (2006). They use a compound-Poisson process with a sum-of-exponentials severity distribution, which has been shown to be a good approximation to any severity if you take enough exponentials, and optimize firm value using the approach of Bellman (1954). They find that the optimal dividend strategy is a barrier strategy: keep all earnings until capital reaches its optimal level, and pay out anything beyond that in dividends. (They do not consider share repurchasing, but for these purposes, that can be viewed as equivalent to paying dividends.) Reinsurance is also considered in the actuarial approach.

Bather (1969) includes proportional reinsurance, and Asmussen, Højgaard and Taksar (2000) look at excess. The pricing assumed for excess makes a difference in these models, however.

In the financial literature, Mayers and Smith (1982 and 1990) provide a list of reasons why Modigliani and Miller does not apply. Probably the most accepted single reason, advocated by Froot, Sharfstein and Stein (1993), is that raising new capital is more expensive than retaining earnings. This makes risk transfer, even if costly, value accretive, especially for firms with expanding capital needs, such as growing firms and capital-intensive firms, like those that profit from research. Also firms with potential exposure to capital losses, such as insurers, would be in this category.

Insurance finance papers have explored these issues empirically. Staking and Babbel (1995) report an increase in market value from using risk management to avoid financial distress. Sommer (1996) finds that the profit load insureds are willing to pay decreases as the ratio of insurer capital to assets declines, and also decreases as the volatility of that ratio increases. Phillips, Cummins and Allen (1998) estimate the price discount that insureds demand for accepting a higher expected cost of insurer default. They find the discount is about 10 times the economic value of the default put value for long-tailed lines and 20 times for short-tailed lines. Grace, Klein and Kleindorfer (2004) find that insurer security issues impact insureds' buying decisions for homeowners' insurance. Epermanis and Harrington (2006), who look at financial strength as measured by rating agencies, find that growth rates are higher for higher rated insurers, and that the growth rate of a company moves up and down with rating changes.

One conclusion from these studies is that the value of an insurer increases non-linearly with capital. For a weakly capitalized insurer, value can increase steeply as capital is added, while with a lot of capital, these effects are minimal but the frictional costs of holding capital makes the value increase less than the capital increase. The opposite effects occur from losing capital: the drop in firm value can be much greater than the capital lost. There are many examples of this in the financial histories of insurers. Froot (2003) uses the term "M-curve" to describe the response of market value to capital. He shows that besides addressing capital targets, the M-curve is a general tool for risk-management. If losing \$X reduces firm value by more than gaining \$X increases it, then reducing risk by costly reinsurance can be valuable. On the other hand, increasing investment risk, even with higher expected returns, can sometimes reduce value.

It is difficult to construct the M-curve directly from empirical studies. Major (2007) reports on efforts to combine the actuarial and finance approaches into a model, using the Bellman equa-

tion, to find the capital and reinsurance options that optimize firm value while taking into account the possibility of refinancing, albeit expensively, and insurance buyer risk preferences. He uses the insurance finance literature results to model profitability as a function of capital. Besides the optimal capital and reinsurance strategies, the M-curve is an output from which other risk management work can then be undertaken. Further understanding of the relationship between profitability and financial strength could improve this aspect of the model. Also the cost of distress financing has to be input. The insights of Myers and Majluf (1984) provide a starting point for that aspect.

6. Conclusions

Capital allocation is an attempt to do risk pricing while avoiding the rigors of the pricing project. But to work well it has to face up to the problems of risk pricing and incorporate them. This can be done to some degree within the typical ERM setting if specific risk measures and risk attribution methods are employed. An alternative is direct risk pricing. However, risk pricing needs further development for this application, such as addressing higher moments and jump risk. Also its emphasis on market price for risk needs to be further tailored to get to the adequacy of market prices given a firm's unique risk profile. The impact on firm value is the bottom line for strategic and pricing decisions, and this is also the case for optimal capital. Better understanding of the relationship between financial strength and profitability is the key to advancing firm-value models.

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Gary G. Venter, FCAS, ASA, CERA, MAAA, is actuary in residence at Columbia University. He can be reached at gary.venter@gmail.com.