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Paid Claims Projection and Cash Flow Testing Models. Illustrative Approach.

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Abstract

Insurance companies use actuarial models to set appropriate reserves and adequately price products. This article provides illustrative examples of using a Paid Claims Projection model to estimate claim reserves for products in the runoff and a Cash Flow Testing model to determine longevity and profitability of existing products.

1 Introduction

Financial products need constant actuarial care to make sure they are performing as profitably as originally planned while they are being sold and are aging gracefully when they are in the run-off. How should an actuary treat products at different stages of products' lives and what actuarial models are appropriate to handle their claim projection and cash flows?

This paper explores the use of Paid Claims Projection model to handle claim projection and reserves estimation for products at the end of their lives and Cash Flow Testing model to determine longevity and profitability of products that are currently being sold. It offers practical guidance and advice for actuarial and financial practitioners in both insurance and consulting industry.

2 Paid Claims Projection Model

When an insurance company stops selling a product, there remains a need of a qualified claims administration service to run-off existing claims. This service includes:

- Claim litigation and mediation management
- Quality reserve establishment
- Reinsurance compliance and reporting
- Claim payment and resolution
- Salvage/Subrogation recoveries
- Ability to recognize and control Extra Contractual Obligations (ECO) and Excess of Policy Limits (EPL) exposures.

If a company is no longer equipped to handle this obligation, the claims administration can be transferred to a Third Party Administrator (TPA). If a decision is made to handle the product in the run-off within the company, an actuarially sound approach to incurred and paid claim projection as well as claims reserve establishment becomes of the utmost importance. Let us start with an example.

2.1 Paid Claims Projection Model: Problem Set Up

Suppose a product was sold during a time period between t_1 and t_m, and the current valuation date (CVD) is $t_n : t_n > t_m,\ n > m$. Our task is to:

1. Evaluate the reserves as of the CVD;
2. Project the paid claims past the CVD; and
3. Evaluate the remaining paid claims and reserves.

Let $d_1, d_2, \cdots, d_n$ be the claims durations: $d_1 < d_2 < \cdots < d_n$, and let $t_1, t_2, \cdots, t_m$ be the times when these claims incurred: $t_1 < t_2 < \cdots < t_m$. For a product in the runoff $m < n$. Note that as of CVD, claims incurred at time t_j have duration d_{n-j+1}. In particular, claims incurred at time t_1 have duration d_n.

Suppose the following completions factors were developed during the products life: $\{cf_{ij}\}_{i,j}$, where cf_{ij} is the completion factor corresponding to cumulative claims in duration d_i incurred at time t_j. Let P_j be the total claims incurred at time t_j paid for all durations $\{d_i\}_{i=1}^{n-j+1}$, and $Q_n = \sum_{j=1}^{m} P_j$ be the total paid claims at the CVD corresponding to time t_n. The following table summarizes the notation above.

Table 1: CF Table for the Current Life of the Product

	Incurred Period				
Duration	t_1	t_2	$\cdots$	t_{m-1}	t_m
d_1	cf_{11}	cf_{12}	$\cdots$	cf_{1m-1}	cf_{1m}
d_2	cf_{21}	cf_{22}	$\cdots$	cf_{2m-1}	cf_{2m}
$\vdots$					
d_m	cf_{m1}	cf_{m2}	$\cdots$	$cf_{m\ m-1}$	cf_{mm}
$\vdots$					
d_n	cf_{n1}	cf_{n2}	$\cdots$	$cf_{n\ m-1}$	cf_{nm}
Total Paid	P_1	P_2	$\cdots$	P_{m-1}	P_m

Let us calculate the claim reserves as of the CVD.

2.2 Incurred Claims and Claim Reserves as of CVD

The incurred claims and reserves as of the current date can be determined by known paid claims $\{P_j\}_{j=1}^{m}$ and completion factors $\{cf_{ij}\}_{i,j}, i = n, n-1, \cdots, n-m+1,\ j = 1, 2, \cdots, m$ and are estimated as follows:

Table 2: Incurred Claims and Reserves for the Life of the Product

Incurred Date t_j	Duration d_{n-j+1}	Paid Claims P_j	Completion Factor $cf_{n-j+1\ j}$	Incurred Claims IC_j	Reserves R_j
t_1	d_n	P_1	cf_{n1}	P_1/cf_{n1}	$IC_1 - P_1$
t_2	d_{n-1}	P_2	$cf_{n-1\ 2}$	$P_2/cf_{n-1\ 2}$	$IC_2 - P_2$
$\vdots$					
t_m	d_{n-m+1}	P_m	$cf_{n-m+1\ m}$	$P_m/cf_{n-m+1\ m}$	$IC_m - P_m$
		$Q_n = \sum_{j=1}^{m} P_j$		$IC_n = \sum_{j=1}^{m} IC_j$	$R_n = \sum_{j=1}^{m} R_j$

Note that the incurred claims in this case are estimated in a classical way of estimating incurred claims for creditable months. This is possible due to the time t_n being removed from the time t_m, the last incurred claims date.

2.3 Paid Claims and Reserves Projections

To project paid claims and reserves past the CVD, an actuary will need to perform a completion factor study for the entire life of the product:

Table 3: CF Table for the Entire Life of the Product

	Incurred Period				
Duration	t_1	t_2	$\cdots$	t_{m-1}	t_m
d_1	cf_{11}	cf_{12}	$\cdots$	cf_{1m-1}	cf_{1m}
d_2	cf_{21}	cf_{22}	$\cdots$	cf_{2m-1}	cf_{2m}
$\vdots$					
d_m	cf_{m1}	cf_{m2}	$\cdots$	$cf_{m\ m-1}$	cf_{mm}
$\vdots$					
d_n	cf_{n1}	cf_{n2}	$\cdots$	$cf_{n\ m-1}$	cf_{nm}
d_{n+1}	$cf_{n+1\ 1}$	$cf_{n+1\ 2}$	$\cdots$	$cf_{n+1\ \ m-1}$	$cf_{n+1\ m}$
$\vdots$					
d_N	cf_{N1}	cf_{N2}	$\cdots$	$cf_{N\ m-1}$	cf_{Nm}

Here N is the last projected period of product's life.

Then the paid claims can be projected into the future periods $t_{n+1}, t_{n+2}, \cdots, t_N$ as follows:

$$Q_{n+1} = \left[\frac{P_1}{cf_{n\,1}} - \frac{P_1}{cf_{n+1\,1}}\right] + \left[\frac{P_2}{cf_{n-1\,2}} - \frac{P_2}{cf_{n\,2}}\right] + \cdots +$$
$$+ \left[\frac{P_m}{cf_{n-m+1\,m}} - \frac{P_m}{cf_{n-m\,m}}\right] = \sum_{j=1}^{m} P_j \left[\frac{1}{cf_{n-j+1\,j}} - \frac{1}{cf_{n-j\,j}}\right]$$
$$Q_{n+2} = \left[\frac{P_1}{cf_{n+1\,1}} - \frac{P_1}{cf_{n+2\,1}}\right] + \left[\frac{P_2}{cf_{n\,2}} - \frac{P_2}{cf_{n+1\,2}}\right] + \cdots +$$
$$+ \left[\frac{P_m}{cf_{n-m+2\,m}} - \frac{P_m}{cf_{n-m+1\,m}}\right] = \sum_{j=1}^{m} P_j \left[\frac{1}{cf_{n-j+2\,j}} - \frac{1}{cf_{n-j+1\,j}}\right]$$
$$\vdots$$
$$Q_N = \left[\frac{P_1}{cf_{N-1\,1}} - \frac{P_1}{cf_{N\,1}}\right] + \left[\frac{P_2}{cf_{N-2\,2}} - \frac{P_2}{cf_{N-1\,2}}\right] + \cdots +$$
$$+ \left[\frac{P_m}{cf_{N-m\,m}} - \frac{P_m}{cf_{N-m-1\,m}}\right] = \sum_{j=1}^{m} P_j \left[\frac{1}{cf_{N-j\,j}} - \frac{1}{cf_{N-j-1\,j}}\right]$$

Summarizing for $k = 1, 2, \cdots, N-n$:

$$Q_{n+k} = \left[\frac{P_1}{cf_{n+k-1\,1}} - \frac{P_1}{cf_{n+k\,1}}\right] + \left[\frac{P_2}{cf_{n+k-2\,2}} - \frac{P_2}{cf_{n+k-1\,2}}\right] + \cdots +$$
$$+ \left[\frac{P_m}{cf_{n+k-m\,m}} - \frac{P_m}{cf_{n+k-m-1\,m}}\right] = \sum_{j=1}^{m} P_j \left[\frac{1}{cf_{n+k-j\,j}} - \frac{1}{cf_{n+k-j-1\,j}}\right]$$

The remaining reserve is:

$$R_{n+1} = R_n - Q_{n+1}$$
$$R_{n+2} = R_n - Q_{n+1} - Q_{n+2} = R_n - \sum_{j=1}^{2} Q_{n+j}$$
$$\vdots$$
$$R_N = R_n - Q_{n+1} - Q_{n+2} - \cdots Q_N = R_n - \sum_{j=1}^{N-n} Q_{n+j}$$

Summarizing for $k = 1, 2, \cdots, N - n$:

$$R_{n+k} = R_n - Q_{n+1} - Q_{n+2} - \cdots Q_{n+k} = R_n - \sum_{j=1}^{k} Q_{n+j}, \quad \text{or, recursively:}$$

$$R_{n+k} = R_{n+k-1} - Q_{n+k}$$

The following numerical example will illustrate the general Paid Claims Projection problem just discussed.

2.4 Paid Claims Projection Model: Example

Suppose a product was sold during a time period between January and June of 2008 and the current valuation date (CVD) is March 31, 2010. Note that as of the CVD the oldest product's claim is in its 27th duration and the youngest one is in its 22nd duration.

Let us first evaluate the reserves as of the CVD.

Suppose the completion factor chart and the paid claims for each incurred date up to the CVD are:

Table 4: CF Table for the Current Life of the Product

	Incurred Period					
Duration	200801	200802	200803	200804	200805	200806
22	0.9919	0.9928	0.9911	0.9883	0.9876	0.9861
23	0.9943	0.9929	0.9915	0.9888	0.9881	0.9881
24	0.9946	0.9935	0.9918	0.990	0.9900	0.9900
25	0.9954	0.9937	0.992	0.9920	0.9920	0.9920
26	0.9958	0.994	0.9940	0.9940	0.9940	0.9940
27	0.996	0.9960	0.9960	0.9960	0.9960	0.9960
Total Paid	1000	2000	3000	4000	5000	6000

The incurred claims and reserves as of the CVD can be determined by known paid claims $\{P_j\}_{j=1}^{6}$ and completion factors $\{cf_{ij}\}_{i,j}, i = 22, 23, \cdots, 27, \; j = 1, 2, \cdots, 6$ and are estimated as:

Table 5: Incurred Claims and Reserves for the Life of the Product

Incurred Date t_j	Duration d_{n-j+1}	Paid Claims P_j	Completion Factor $cf_{n-j+1\ j}$	Incurred Claims IC_j	Reserves R_j
200801	27	1000	0.996	1004	4
200802	26	2000	0.994	2012	12
200803	25	3000	0.992	3024	24
200804	24	4000	0.990	4040	40
200805	23	5000	0.9881	5060	60
200806	22	6000	0.9861	6085	85
Total		21,000		21,225	225

Here, for example, $IC_1 = 1004 = 1000/0.996$ and $R_1 = 4 = 1004 - 1000$.

To calculate paid claim and reserve projections, suppose we developed completion factors for the future life of the product:

Table 6: CF Table for the Entire Life of the Product

	Incurred Period					
Duration	200801	200802	200803	200804	200805	200806
22	0.9919	0.9928	0.9911	0.9883	0.9876	0.9861
23	0.9943	0.9929	0.9915	0.9888	0.9881	0.9881
24	0.9946	0.9935	0.9918	0.990	0.990	0.990
25	0.9954	0.9937	0.992	0.992	0.992	0.992
26	0.9958	0.994	0.994	0.994	0.994	0.9940
27	0.996	0.996	0.996	0.996	0.9960	0.9960
28	0.998	0.998	0.998	0.9980	0.9980	0.9980
29	0.999	0.999	0.9990	0.9990	0.9990	0.9990
30	1.00	1.0000	1.0000	1.0000	1.0000	1.0000
Total Paid	1000	2000	3000	4000	5000	6000

Then, projecting paid claims and estimating remaining reserve, we obtain:

Table 7: Estimated Future Paid Claims and Reserves

	27	26	25	24	23	22	Total Est Paid	Remaining Reserve
201004	2.01	4.02	6.05	8.08	10.12	12.17	42	183
201005	1.00	4.02	6.04	8.06	10.10	12.15	41	142
201006	1.00	2.00	6.02	8.05	10.08	12.12	39	102
201007	0.00	2.00	3.01	8.03	10.06	12.10	35	67
201008	0.00	0.00	3.00	4.01	10.04	12.07	29	38
201009	0.00	0.00	0.00	4.00	5.01	12.05	21	17
201010	0.00	0.00	0.00	0.00	5.01	6.01	11	6
201011	0.00	0.00	0.00	0.00	0.00	6.01	6	0
201012	0.00	0.00	0.00	0.00	0.00	0.00	0	0
Total							225	555

Here, for example, the future paid claims in duration 27 for April 30, 2010 valuation date are projected to be $2.01 = 1000/0.996 - 1000/0.998$, and the total projected paid claims as of April 30, 2010 are calculated following the gold set of completion factors in Table 6

$$1000\left(\frac{1}{0.996}-\frac{1}{0.998}\right)+2000\left(\frac{1}{0.994}-\frac{1}{0.996}\right)+3000\left(\frac{1}{0.992}-\frac{1}{0.994}\right)+$$
$$+4000\left(\frac{1}{0.99}-\frac{1}{0.992}\right)+5000\left(\frac{1}{0.9881}-\frac{1}{0.99}\right)+6000\left(\frac{1}{0.9861}-\frac{1}{0.9881}\right)=42$$

The remaining reserve as of April 30, 2010 is then $225 - 42 = 183$.

Similarly, the future paid claims in duration 26 for June 30, 2010 valuation date are projected to be $2.00 = 2000/0.998 - 2000/0.999$, and the total projected paid claims as of June 30, 2010 are calculated following the violet set of completion factors in Table 6

$$1000\left(\frac{1}{0.999}-1\right)+2000\left(\frac{1}{0.998}-\frac{1}{0.999}\right)+3000\left(\frac{1}{0.996}-\frac{1}{0.998}\right)+$$
$$+4000\left(\frac{1}{0.994}-\frac{1}{0.996}\right)+5000\left(\frac{1}{0.992}-\frac{1}{0.994}\right)+6000\left(\frac{1}{0.990}-\frac{1}{0.992}\right)=39$$

The remaining reserve as of June 30, 2010 is then $142 - 39 = 103$. The difference due to rounding.

In this example, by the end of the year we can expect to pay another $225 in claims and hold a total of $555 in claim reserve.

In the next section we will show how to apply a different model to test longevity and profitability of a product that is currently being sold.

3 Cash Flow Testing Model

A typical life, health, or property/casualty insurance company periodically performs cash flow testing of its products. This is done for purposes of:

- Reserve and Capital Adequacy
- Product Development
- Investment Strategy Evaluation
- Financial Projections
- Actuarial Appraisals
- Testing of Nonguaranteed Elements

The cash flows of some assets can be projected on the basis of asset structure alone (e.g., high quality noncallable bonds) or on the basis of a combination of their structure and external factors (e.g., callable bonds or mortgage-backed securities). When performing cash flow analysis, an actuary should consider:

- Reinvestment strategy
- Asset segmentation
- Use of derivative contracts
- Insolvency or other nonperformance of reinsurers
- Expenses associated with maintaining, collecting, or paying out policy cash flows
- External factors such as interest rates
- Policyholder options
- Claim settlement and benefit payment practices
- Expense-control strategies

The following example illustrates a cash flow testing scenario.

3.1 Cash Flow Testing Model: Assumptions

The goal of the cash flow testing analysis is to estimate the present value of its cash flows. The cash flows could be determined as the difference between the break-even and projected claims that will be determined below. Before we get to the discussion of the break-even claims, let us first make assumptions about the product's lapse rates, persistency rates and loss ratios.

Let $d_1, d_2, \cdots, d_n$ be the claims durations: $d_1 < d_2 < \cdots < d_n$. Let P_i be the premiums paid, l_i, p_i, and LR_i be the lapse, persistency and loss ratio rates at durations d_i, $i = 1, 2, \ldots, n$ expressed as follows:

Table 8: Premium by Duration

Duration	Premium
d_n	P_n
d_{n-1}	P_{n-1}
$\vdots$	$\vdots$
d_2	P_2
d_1	P_1
Total	$P = \sum_{i=1}^{n} P_i$

Table 9: Lapse by Duration

Duration	Lapse
d_{n+}	l_{n+}
d_{n-1}	l_{n-1}
$\vdots$	$\vdots$
d_2	l_2
d_1	l_1

Table 10: Persistency by Duration

Duration	Persistency
d_{n+}	p_{n+}
d_{n-1}	p_{n-1}
$\vdots$	$\vdots$
d_2	p_2
d_1	p_1

Table 11: Loss Ratio by Duration

Duration	Loss Ratio
d_{n+}	LR_{n+}
d_{n-1}	LR_{n-1}
$\vdots$	$\vdots$
d_2	LR_2
d_1	LR_1

3.2 Cash Flow Testing Model: Projections

Given the lapse, persistency and loss ratio assumptions above, the lapse, persistency and loss ratio *projections* can be expressed as follows:

Lapse: $\{l_{ij}\}$, where

$$l_{ij} = \begin{cases} l_{i+j-1} & i+j \leq n \\ l_{n+} & i+j > n \end{cases}$$

Or

Table 12: Lapse Projection

	Projection Year				
Duration	1	2	$\cdots$	$n-1$	$n+$
d_{n+}	l_{n+}	l_{n+}	$\cdots$	l_{n+}	l_{n+}
d_{n-1}	l_{n-1}	l_{n+}	$\cdots$	l_{n+}	l_{n+}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
d_2	l_2	l_3	$\cdots$	l_{n+}	l_{n+}
d_1	l_1	l_2	$\cdots$	l_{n-1}	l_{n+}

Persistency: $\{p_{ij}\}$, where

$$p_{ij} = \begin{cases} p_{i+j-1} & i+j \leq n \\ p_{n+} & i+j > n \end{cases}$$

Or

Table 13: Persistency Projection

	Projection Year				
Duration	1	2	$\cdots$	$n-1$	$n+$
d_{n+}	p_{n+}	p_{n+}	$\cdots$	p_{n+}	p_{n+}
d_{n-1}	p_{n-1}	p_{n+}	$\cdots$	p_{n+}	p_{n+}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
d_2	p_2	p_3	$\cdots$	p_{n+}	p_{n+}
d_1	p_1	p_2	$\cdots$	p_{n-1}	p_{n+}

Note that $p_{ij} = 1 - l_{ij}$.

Loss Ratio: $\{LR_{ij}\}$, where

$$LR_{ij} = \begin{cases} LR_{i+j-1} & i+j \leq n \\ LR_{n+} & i+j > n \end{cases}$$

Or

Table 14: Loss Ratio Projection

	Projection Year				
Duration	1	2	$\cdots$	$n-1$	$n+$
d_{n+}	LR_{n+}	LR_{n+}	$\cdots$	LR_{n+}	LR_{n+}
d_{n-1}	LR_{n-1}	LR_{n+}	$\cdots$	LR_{n+}	LR_{n+}
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
d_2	LR_2	LR_3	$\cdots$	LR_{n+}	LR_{n+}
d_1	LR_1	LR_2	$\cdots$	LR_{n-1}	LR_{n+}

Then the premium $\{P_{ij}\}$ for projection year j and duration i can be found using the current year premium and persistency above:

$$P_{ij} = P_i \prod_{k=0}^{j-1} p_{i+k},\ Q_j = \sum_{i=1}^{n} P_{ij}$$

Recursively:

$$P_{ij} = P_{i\ j-1} * p_{ij} = P_{i\ j-1} * p_{i+j-1}$$

Or

Table 15: Premium Projection

	Projection Year				
Duration	1	2	$\cdots$	$n-1$	$n+$
d_{n+}	$P_n * p_{n+}$	$P_n * p_{n+}^2$	$\cdots$	$P_n * p_{n+}^{n-1}$	$P_n * p_{n+}^n$
d_{n-1}	$P_{n-1} * p_{n-1}$	$P_{n-1} * p_{n-1} * p_{n+}$	$\cdots$	$P_{n-1} * p_{n-1} * p_{n+}^{n-2}$	$P_{n-1} * p_{n-1} * p_{n+}^{n-1}$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
d_2	$P_2 * p_2$	$P_2 * p_2 * p_3$	$\cdots$	$P_1 * \prod_{j=2}^{n-1} p_j$	$P_2 * \prod_{j=2}^{n} p_j$
d_1	$P_1 * p_1$	$P_1 * p_1 * p_2$	$\cdots$	$P_1 * \prod_{j=1}^{n-1} p_j$	$P_1 * \prod_{j=1}^{n} p_j$
Total	$Q_1 = \sum_{i=1}^{n} P_{i1}$	$Q_2 = \sum_{i=1}^{n} P_{i2}$	$\cdots$	$Q_{n-1} = \sum_{i=1}^{n} P_{i\ n-1}$	$Q_n = \sum_{i=1}^{n} P_{in}$

3.3 Cash Flow Testing Model: Break-Even and Anticipated Claims

To find the cash flows, let us first define the break-even loss ratio for each projection year.

Let NII_j be percent of investment income, C_j – commissions, PT_j – premium taxes, LAE_j – loss adjustment expense, and PA_j – premium administration expense for a projection year j. Then the break-even loss ratio in year j for the next n years is:

$$LR_{BE_j} = 100\% + NII_j - C_j - PT_j - LAE_j - PA_j$$

For a discount rate of $r\%$, let us now calculate the cash flow CF_j for the product for a projection year j.

Let MP_j be the projected annualized earned premium:

$$MP_j = \frac{Q_{j-1} + Q_j}{2}$$

Then for a projection year j, the anticipated loss ratio ALR_j:

$$ALR_j = \frac{\sum_{i=1}^{n} P_{ij} \cdot LR_{ij}}{\sum_{i=1}^{n} P_{ij}}$$

Projected claims:

$$CL_j = MP_j \cdot ALR_j$$

Break-even claims:

$$CL_{BE_j} = MP_j \cdot LR_{BE_j}$$

and the cash flow:

$$CF_j = MP_j \cdot (LR_{BE_j} - ALR_j)$$

Finally, the total anticipated loss ratio and the break-even loss ratio are calculated by taking present values of these amounts:

$$ALR = \frac{\sum_j PV_j\,(CL_j)}{\sum_j PV_j\,(MP_j)}$$

$$LR_{BE} = \frac{\sum_j PV_j\left(CL_{BE_j}\right)}{\sum_j PV_j\,(MP_j)}$$

The following numerical example will illustrate the general Cash Flow Testing problem just discussed.

3.4 Cash Flow Testing Model: Example

Suppose we are given the following premium, lapse rates, persistency rate and loss ratio assumptions.

Table 16: Premium by Duration

Duration	Premium
5	1000
4	1200
3	1100
2	1000
1	900
Total	$P = 5200$

Table 17: Lapse by Duration

Duration	Lapse
5	25%
4	25%
3	25%
2	41%
1	55%

Table 18: Persistency by Duration

Duration	Persistency
5	75%
4	75%
3	75%
2	59%
1	45%

Table 19: Loss Ratio by Duration

Duration	Loss Ratio
5	62%
4	65%
3	68%
2	67%
1	55%

Given the lapse, persistency and loss ratio assumptions above, the lapse, persistency and loss ratio projections can be expressed as follows:

Lapse:

Table 20: Lapse Projection

	Projection Year				
Duration	1	2	3	4	5
5	25%	25%	25%	25%	25%
4	25%	25%	25%	25%	25%
3	25%	25%	25%	25%	25%
2	41%	25%	25%	25%	25%
1	55%	41%	25%	25%	25%

Persistency:

Table 21: Persistency Projection

	Projection Year				
Duration	1	2	3	4	5
5	75%	75%	75%	75%	75%
4	75%	75%	75%	75%	75%
3	75%	75%	75%	75%	75%
2	59%	75%	75%	75%	75%
1	45%	59%	75%	75%	75%

Note that $p_{ij} = 1 - l_{ij}$.

Loss Ratio:

Table 22: Loss Ratio Projection

	Projection Year				
Duration	1	2	3	4	5
5	62%	62%	62%	62%	62%
4	65%	62%	62%	62%	62%
3	68%	65%	62%	62%	62%
2	67%	68%	65%	62%	62%
1	55%	67%	68%	65%	62%

Therefore, the premium projection using the current year premium and persistency above is:

Table 23: Premium Projection

		Projection Year				
Duration	0	1	2	3	4	5
5	1000	750	563	422	316	237
4	1200	900	675	506	380	285
3	1100	825	619	464	348	261
2	1000	590	443	332	249	187
1	900	405	239	179	134	101
Total	5200	3470	2538	1903	1427	1071

Here, for the first two projection years:

Table 24: Premium Projection:Detail

	Projection Year	
Duration	1	2
5	$750 = 1000 \cdot 0.75$	$563 = 1000 \cdot 0.75^2 = 750 \cdot 0.75$
4	$900 = 1200 \cdot 0.75$	$675 = 1200 \cdot 0.75^2 = 900 \cdot 0.75$
3	$825 = 1100 \cdot 0.75$	$619 = 1100 \cdot 0.75^2 = 825 \cdot 0.75$
2	$590 = 1000 \cdot 0.59$	$443 = 1000 \cdot 0.59 \cdot 0.75 = 590 \cdot 0.59$
1	$405 = 900 \cdot 0.45$	$239 = 900 \cdot 0.45 \cdot 0.59 = 405 \cdot 0.45$
Total	3470	2538

For the break-even point calculation, we assume no investment income, 22% commission in the 1st year, 10% in the 2nd year and 8.4% in the subsequent years, 2.4% premium tax, 4% loss adjustment expense (LAE), and 8.4% administrative expense (as percentage of premium). Then

Table 25: Break-even Loss Ratio

		Projection Year				
	0	1	2	3	4	5
Total	100%	100%	100%	100%	100%	100%
Investment Income	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Commissions	22%	10%	8.4%	8.4%	8.4%	8.4%
Premium Taxes	2.4%	2.4%	2.4%	2.4%	2.4%	2.4%
LAE	4.0%	4.0%	4.0%	4.0%	4.0%	4.0%
Administrative Expense	8.4%	8.4%	8.4%	8.4%	8.4%	8.4%
Net (Break-even point)	63.2%	75.2%	76.8%	76.8%	76.8%	76.8%

Finally, using discount rate of $r = 4.0\%$,

Table 26: Cash Flow Analysis

		Projection Year					
	0	1	2	3	4	5	PV
Projected Annualized Earned Premium	5200	4335	3004	2220	1665	1249	15,932
Anticipated Loss Ratio (ALR)	55.0%	64.2%	64.3%	63.1%	62.3%	62.0%	60.9%
Break-even Loss Ratio	63.2%	75.2%	76.8%	76.8%	76.8%	76.8%	72.1%
Projected Claims	2860	2785	1930	1401	1037	774	9702
Break-even Claims	3286	3260	2307	1705	1279	959	11,492
Cash Flow	426	475	377	304	242	185	1789

Here the Projected Annualized Earned Premium is interpolated between two years and the Anticipated Loss Ratio in each projection year is calculated as the projected premium in each duration multiplied by the corresponding projected loss ratio divided by the total projected premium for the year.

For example, in year two the Anticipated Loss Ratio calculation is:

Table 27: ALR Calculation: Year 2

Dur	LR_{i2}	P_{i2}
5	62%	563
4	62%	675
3	65%	619
2	68%	443
1	67%	239
Total		2538

$$ALR_2 = \frac{\sum_{i=1}^{5} P_{i2} \cdot LR_{i2}}{\sum_{i=1}^{5} P_{i2}} =$$
$$= \frac{563 \cdot 0.62 + 675 \cdot 0.62 + 619 \cdot 0.65 + 443 \cdot 0.68 + 239 \cdot 0.67}{2538} =$$
$$= \frac{1631.28}{2538} = 0.6427$$

The Cash Flow is calculated as a difference between Break-even Loss Ratio and Anticipated Loss Ratios multiplied by the Projected Annualized Earned Premium.

For example, in year three the cash flow will be $(0.768 - 0.631) \cdot 2220 = 304.14$:

Table 28: Cash Flow Calculation: Year 3

Dur	3
Proj EP	2220
ALR	63.1%
BE LR	76.8%
Proj Clms	1401
BE Clms	1705
CF	304

The calculation in Table 26 shows that the product is going to be profitable for at least 5 years.

Here we have chosen a scenario where the Anticipated Loss Ratios were based on lapse rate and calendar year loss ratio assumptions. Other scenarios are possible. For example, we could have Anticipated Loss Ratios following scenario above for the first two years, deteriorating to a break-even point in year three and beyond.

4 Conclusion

This paper showed an application of different actuarial models to answer questions for a product at different stages of life.

At the end of a product's life, when the product is no longer sold, or is in a run-off, an actuary needs to make competent paid claims projections as well as current and remaining reserves estimates. Paid Claims Projection model was appropriate to handle such questions.

At the beginning or mid-life of a product, when a product is being sold, it is important to make periodic estimates of the present value of net cash flows that the product will bring. Cash Flow Testing model was appropriate to accomplish this.

Both models are necessary to insure a product's and, ultimately, company's, financial health and vitality.

Finally, it is important to note that a more comprehensive approach toward solving these and similar problems would be application of the Monte Carlo simulation techniques which provide a number of advantages over deterministic, or single-point estimate analysis, presented in this paper.

References

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