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Iowa Memorial Union, The University of Iowa, Iowa City

Abstracts

Thursday morning, first session

On Optimal Dividends: From Reflection to Refraction

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ABSTRACT: The problem goes back to a paper that Bruno de Finetti presented to the International Congress of Actuaries in New York (1957). In a stock company that is involved in risky business, what is the optimal dividend strategy, that is, what is the strategy that maximizes the expectation of the discounted dividends (until possible ruin) to the shareholders? Jeanblanc-Picqué and Shiryaev (1995) and Asmussen and Taksar (1997) solved the problem by modeling the income process of the company by a Wiener process and imposing the condition of a bounded dividend rate. Here we present some down-to-earth calculations.

Capital Allocation in Insurance: Economic Capital and the Allocation of the Default Option Value

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ABSTRACT: The determination and allocation of economic capital is important for pricing, risk management and related insurer financial decision making. This paper considers the allocation of economic capital to lines of business in insurance. We show how to derive closed form results for the complete markets, arbitrage-free allocation of the insurer default option value, also referred to as the insolvency exchange option, to lines of business for an insurer balance sheet. We assume that individual lines of business and the surplus ratio are joint log-normal although the method we adopt allows other assumptions. The allocation of the default option value is required for fair pricing in the multi-line insurer. We discuss and illustrate some other methods of capital allocation, including Myers-Read, and give numerical examples for the capital allocation of the default option value based on explicit payoffs by line.

The Economic Aspects of Life Insurance Backdating

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ABSTRACT: Backdating is a common (legal) practice in most of the U.S. to save age, and occurs when the contract bears a policy date that is prior to the application date. This works is concerned with economic aspects of backdating:

- The incentive structure, which causes this sales method to be accepted by all parties involved, and the contradictory incentives of the actuarial and marketing divisions of the insurance firm,
- Price discrimination inherent in discrete (annual) mortality pricing of life insurance, and the welfare loss resulting from it, and
- supply-side effect caused by pricing based on return-on-assets or return-on-investment, not the profit-maximization commonly assumed in microeconomics.

We also point out the apparent contradiction between the state regulatory authorities suspicious treatment of backdating, and the Patman-Robinson Act prohibition of price discrimination, when such price discrimination inhibits competition.

Thursday morning, second session

Indifference Pricing via the Probability of Ruin
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ABSTRACT: Indifference pricing in insurance via expected utility has been studied in actuarial mathematics since the work of Pratt and of Borch in the 1960s. Specifically, one finds the price of insurance that makes the insurer indifferent between insuring and not insuring the risk, in which one dictates indifference by requiring that the expected utility of the insurer's wealth be equal in both states. In the 1990s, Wang proposed using Yaari's dual theory of risk to price insurance via indifference. In fact, one could use many different risk measures to price insurance via an indifference argument. In this talk, we use the probability of ruin as the measure of risk and find the insurance premium loading that makes the insurer's probability of ruin after taking on additional risk equal to the probability of ruin before doing so. We look at a variety of models and focus our talk on a numerical example.

Optimal Asset Allocation and Ruin-Minimization Annuitization Strategies
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ABSTRACT: In this paper, we derive the optimal investment and annuitization strategies for a retiree whose objective is to minimize the probability of lifetime ruin, namely the probability that a fixed consumption strategy will lead to zero wealth while the individual is still alive. Recent papers in the insurance economics literature have examined utility-maximizing annuitization strategies. Others in the probability, finance, and risk management literature have derived shortfall-minimizing investment and hedging

strategies given a limited amount of initial capital. This paper brings the two strands of research together. Our model pre-supposes a retiree who does not currently have sufficient wealth to purchase a life annuity that will yield her exogenously desired fixed consumption level. She seeks the asset allocation and annuitization strategy that will minimize the probability of lifetime ruin. We demonstrate that because of the binary nature of the investor's goal, she will not annuitize any of her wealth until she can fully cover her desired consumption with a life annuity. We derive a variational inequality that governs the ruin probability and the optimal strategies, and we demonstrate that the problem can be recast as a related optimal stopping problem which yields a free-boundary problem that is more tractable. We numerically calculate the ruin probability and optimal strategies and examine how they change as we vary the mortality assumption and parameters of the financial model. Moreover, we solve the problem implicitly for the special case of exponential future lifetime. As a byproduct, we are able to quantify the reduction in lifetime ruin probability that comes from being able to manage the investment portfolio dynamically and purchase annuities.

Pricing Equity Linked Pure Endowments with Risky Asset Following Lévy Processes

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ABSTRACT: The pricing problem for pure endowment contracts whose life contingent payment is linked to the performance of a tradable risky asset or index is investigated. The risky asset is assumed to follow a Lévy process which introduces a second source of market incompleteness, the first being due to mortality risk. The contract is priced using the principle of equivalent utility and, under the assumption of exponential utility, the indifference price is shown to solve a non-linear partial-integro-differential equation. The affects of the jump component on the pricing equation are investigated first through a toy-model using a perturbation expansion, and then through numerical experiments with variance-gamma jump components.

Effects of Dependence among Claim Vectors on the Ruin Probability in a Multi-dimensional Risk Model

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ABSTRACT: We consider a multi-dimensional risk model. When claim vectors follow the common shock risk model or have the Marshall-Olkin distribution, we calculate the ruin probability for the multi-dimensional risk model using the expression for the convolution of multivariate phase type distributions. We concentrate on the effects of dependence among the claim vector on the ruin probability and use numerical comparisons to illustrate the effects.

Thursday afternoon, first session

Complexity and Complex Adaptive Systems: Applications for Actuarial Science, Finance, and Risk Management
Rick Gorvett --- University of Illinois at Urbana-Champaign (gorvett@uiuc.edu)

ABSTRACT: An interesting area of recent research has involved complex systems and how they adapt to emerging forces and trends. Much of this research has focused on physical and biological systems, with some attention paid to the economic and social sciences. This presentation will examine areas of potential application of complex systems theory to the insurance and enterprise risk management worlds.

On Time Value of Ruin in a Sparre Andersen Model:
Ruin Theory by Divided Differences
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Elias S. W. Shiu --- The University of Iowa (eshiu@stat.uiowa.edu)

ABSTRACT: This paper considers a Sparre Andersen collective risk model in which the distribution of the interclaim time is that of a sum of n independent exponential random variables; thus the Erlang(n) model is a special case. The analysis is focused on the function $\phi(u)$, the expected discounted penalty at ruin, with u being the initial surplus. The penalty may depend on the deficit at ruin and possibly also on the surplus immediately before ruin. It is shown that the function $\phi(u)$ satisfies a certain integro-differential equation and that this equation can be solved in terms of Laplace transforms, extending a result of X. Sheldon Lin. As a consequence, a closed form expression is obtained for the discounted joint probability density of the deficit at ruin and the surplus just before ruin, if the initial surplus is zero. For this formula and other results, the roots of Lundberg's equation in the right half of the complex plane play a central role. Also, it is shown that $\phi(u)$ satisfies Shuanming Li's Renewal Equation. Under the assumption that the penalty depends only on the deficit at ruin and that the individual claim amount density is a combination of exponential densities, a closed form expression for $\phi(u)$ is derived. In this context, known results of the Cauchy matrix are useful, which are explained in the Appendix. Surprisingly, many results are best expressed in terms of divided differences, a topic deleted from the actuarial examinations at the end of last century.

An Empirical Study on Pet Insurance
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ABSTRACT: The pet insurance in the North America has been growing very fast recently, and there is still a lot of room to grow. Traditional practice as relied on the market share of the insurance more so than on their experience. Pricing practices still continue to be performed on a haphazard basis with very little consideration for actuarial principles and

techniques. Developments of mortality and morbidity models to be use in the pricing model and new product development are essential for pet insurance.
The objectives of this article are the study of survival analysis and the search of suitable models for modeling canine and feline mortality and morbidity experience. Several survival models will be studied and applied to the pet data that I have from a Canadian insurer. The data consists of 15 years of experience on over 25,000 cats and dogs.

Thursday afternoon, second session

Fuzzy Logic in Insurance: The First 20 Years
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ABSTRACT: It has been twenty years since DeWit (1982) first applied fuzzy logic (FL) to insurance. That article sought to quantify the fuzziness in underwriting. Since then, the universe of discourse has expanded considerably and now the FL methodologies include fuzzy set theory, fuzzy numbers and fuzzy arithmetic, fuzzy inference systems, fuzzy clustering algorithms, fuzzy linear programming, fuzzy regression, and hybrids involving other soft computing technologies. Similarly, the applications have expanded to include classification, projected liabilities, future and present values, pricing, asset allocations and cash flows, and investments. This article presents an overview of these studies. The specific purposes of the article are twofold: first, to review FL applications in insurance so as to document the unique characteristics of insurance as an application area; and second, to document the extent to which FL technologies have been employed.

Fitting and Forecasting Mortality Rates for Nordic Countries Using the Lee-Carter Model
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ABSTRACT: The model proposed by Lee and Carter (LC) is being widely adopted for long-run forecasts of age specific mortality rates. That popularity is due to the model simplicity and the successful results observed in fitting the model to data from various countries (e.g. U.S., Chile, G7 countries). However, some difficulties arose when applying the model to data from the UK or Australia. In the present study, the model is applied to population data from four Nordic countries: Denmark, Finland, Norway and Sweden. The Singular Value Decomposition (SVD), the Weighted Least Square (WLS) and the Maximum Likelihood estimate (MLE) were used. These approaches give satisfactory results. However, the appropriate fitting period needed to be well chosen. A simulation study was also conducted by using bootstrapping method to verify the stability of the results.

Credibility Theory Using Copulas
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ABSTRACT: Credibility ratemaking is an actuarial technique for predicting future claims of a risk class, given past claims of that and related risk classes. Mowbray (1914) and Whitney (1918) introduced the idea of calculating a premium that was a weighted average of (1) average claims from the risk class of concern and (2) average claims over all related risk classes. Bühlmann (1967) showed that the traditional credibility premiums could be expressed as conditional expectations, where the conditioning was based on an unobserved risk type that he named as "structure variable". Modern credibility theory has its statistical foundations as the structure variable linear model.

The current research seeks to go beyond the linear longitudinal data model in two ways. First, it is common in actuarial practice to have long-tail claims data. Further, data often consist of claims frequency (count) information, or are modeled via a two-part approach (stage 1 for the number process, stage two for the amount, given the number of claims). In both situations, the normal (Gaussian) distribution is not a reasonable approximation to the marginal distribution and thus the linear model theory does not readily apply. Moreover, a linear model based credibility theory does not readily apply in situations where insurance policies have deductible and stop-loss limit provisions. In each of these situations, actuaries have well-developed methods for estimating marginal distributions; that is, estimating model parameters for each time period in isolation of the others. In this project, we use copula functions to link these period by period estimates of distributions, thus preserving all of the standard estimation machinery when developing credibility estimates. Second, we incorporate covariate (explanatory) variables into credibility estimators for these non-Gaussian situations, this feature tremendously extends the scope of potential applications. This work develops a direct link between credibility and loss distributions through the notion of a copula, a tool for understanding relationships among multivariate outcomes; it is a function that links univariate marginals to their full multivariate distribution. Copulas were introduced in 1959 in the context of probabilistic metric spaces. Recently, there has been a rapidly developing literature on the statistical properties and applications of copulas, particularly in the enterprise risk management literature. Our approach is to use all of the tools that actuaries use for parametric modeling of the marginal distributions but to connect information in the claims history using theory from copulas. Thus, we envision a highly parametric approach to claims ratemaking. We document several advantages of this new approach compared to the current paradigm in place (as well as some disadvantages). The new approach will be easy to use on a computer in that it is likelihood based. It should be applicable to a much broader set of problems, without needing special tools for each problem. We demonstrate that the copula formulation is more flexible than positing a (constant) latent variable.

We illustrate procedures that are easy to implement in today's computing environment and that will be applicable in a broad set of circumstances. We do this by positing a probabilistic model of insurance claims and developing algorithms for producing credibility forecasts based on this model. We show how the algorithms work with data of Massachusetts auto insurance claims and compare our new procedures to existing methods.

We find that copula based credibility performs well, at least for our sample data.

Managing Catastrophe Risk: An Actuarial Approach
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ABSTRACT: This paper intends to theoretically study the interrelationship between the probability that losses will exceed a given percentage of an insurer's surplus and the noncatastrophe and catastrophe risks the insurer is bearing. Since a diffusion process describes the small losses well and a jump process is suitable for modeling catastrophe losses, for our purpose, we introduce a jump–diffusion process to describe the insurer's net worth. Our results clearly show that: given similar catastrophe exposures, a geographically diversified insurer has lower probability that losses will exceed a given percentage of its surplus than a geographically concentrated insurer.

Friday morning, first session

Modeling Insurance Losses Resulting From Natural Catastrophes
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ABSTRACT: In this paper, we examine the modeling of insurance losses resulting from natural catastrophes, particularly losses to residential and commercial structures as a result of earthquakes. The model proposed can also be used for other types of natural catastrophes such as floods and hurricanes. It is based upon the three main components or characteristics of natural catastrophes, which are intensity, damage to insured properties and frequency (or occurrence). Intensity of earthquakes is defined as a function of the Modified Mercalli Index (MMI) and hence is a discrete random variable (r.v.). Extent of damage is strongly related to both earthquake's magnitude and design of the building, i.e. height, materials used in the construction, etc. In order to link damage to those characteristics, one example of solution presented is the results from the Applied Technology Council (ATC) study. Earthquake arrival during a given time period (or frequency) is represented by renewal processes, either ordinary or stationary. An ordinary renewal process is a stochastic process that counts the number of events during a given time period, when the waiting time (time between two successive events) is given by a sequence of independent and identically distributed (i.i.d.) r.v.'s. The behavior of earthquake occurrence depends upon the distribution of the time between events, particularly the failure rate function of that r.v. Many distributions such as Exponential and Weibull can be used to characterize waiting times. When the Exponential distribution is assumed, the resulting process is the well known Poisson process. Using data from earthquakes that occurred in California between 1850 and 2002, we compare the behavior of individual and aggregate losses when a Weibull renewal process (ordinary or stationary) models earthquake arrival instead of a Poisson process in an earthquake-prone area.

Ruin Probabilities in the Compound Binomial Model Defined in a Markovian Environment

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ABSTRACT: In this paper, we introduce the compound binomial model defined in a markovian environment as an extension to Gerber's compound binomial model. Rather than assuming that the increments in the total claim process are independent as in the compound binomial model, a time-dependence is introduced in the claim occurrence r.v.'s through an underlying discrete-time markov chain. We investigate the distribution of key risk measures under this extension in addition to the calculation of both finite-time and infinite-time non-ruin probabilities. Using ordering of risk concepts, we compare this model to the compound binomial model and a mixed compound binomial model. Finally, we use the proposed extension as an approximation to the continuous-time risk model based on a Markov-modulated Poisson process.

On a Class of Discrete Time Renewal Risk Processes

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ABSTRACT: We consider a class of compound renewal (Sparre Andersen) risk processes with claim waiting times that have a discrete K_n distribution (i.e., the probability generating function (p.g.f.) of the distribution function is a ratio of two polynomials of degree at most $n \in \mathbb{N}^+$). The classical compound binomial risk model is a special case when $n = 1$. Both recursive and explicit formulas are derived for the expected discounted penalty function due at ruin, for the surplus before ruin and the deficit at ruin.

Many ruin related quantities can be analyzed through the penalty function, e.g., ruin probability, the p.g.f. of the time of ruin, joint and marginal distributions of the surplus before ruin and the deficit at ruin, as well as their moments.
Detailed discussions are given in two special cases: claim sizes are rationally distributed, or the claim size distributions have a finite support.

On the Probability of Ruin in a Markov-modulated Risk Model

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ABSTRACT: In this paper, we consider a Markov-modulated risk model in which the claim inter-arrivals, claim sizes and premiums influenced by an external Markovian environment process. Models of this type have been investigated, e.g., by Reinhard (1984), Bäuerle (1996) and Snoussi (2002).
A system of Laplace transforms of non-ruin probabilities, given the initial environment state, is established from a system of integro-differential equations derived by Reinhard

(1984). In the two-state model, explicit formulas for non-ruin probabilities are obtained when the initial reserve is zero or when both claim size distributions are from K_n family, $n \in \mathbb{N}^+$. Examples with exponentially distributed claim sizes, as well as Erlang and mixture of exponentials, are given.

Decay of Ruin Probability under Uncertain Investments
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ABSTRACT: The classical result of Cramer-Lundberg states that if the rate of premium, c, exceeds the average of the claims paid per unit time, $\lambda\mu$, then the probability of ruin of the insurance company decays exponentially fast as the initial capital $\mu \to \infty$. The same asymptotic behavior of the probability of ruin as in Rolski, Schmidli, Schmidt and Teugels (2000) is rederived by means of infinitesimal generators and Laplace transforms. Using the same tools, it is shown that the probability of ruin has an algebraic decay if the insurance company invests its capital in a risky asset whose price follows a geometric Brownian motion. The latter result is shown to be valid not only for exponentially distributed claim amounts, as in Frolova, Kabanov and Pergamenshchikov (2002), but, more generally, for any claim amount distribution that has moment generating functions defined in a neighborhood of the origin.

Friday morning, second session

Another Look at Empirical Estimation of Actuarial Risk Measures
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ABSTRACT: When determining the price of an insurance risk, the central problem is to quantify the "riskiness" of the underlying distribution of losses. Various risk measures are proposed in the actuarial literature to solve this problem. However, the quality of their statistical estimators is an insufficiently explored -- yet very important in practice -- issue. In this talk, the performance of interval estimators of actuarial risk measures, represented in terms of expectation with respect to distorted probabilities, is investigated. We consider the following risk measures: proportional hazard transform, a generalized version of Wang's right-tail deviation, and the Wang transform. Confidence intervals are constructed by applying (i) the empirical approach proposed in a recent work of Jones and Zitikis (2003), (ii) the strict parametric approach based on the maximum likelihood estimators, and (iii) asymptotic theory for robust parametric procedures. Using Monte Carlo simulations, we compare the average lengths and proportions of coverage (of the true measure) of the intervals under two data-generating scenarios -- "clean" data and "contaminated" data. In the "clean" case, data are generated by one of the three (similar shape) parametric families -- Pareto, exponential, and lognormal. In the "contaminated" case, the data sets from these distributions are mixed with a small fraction of unusual observations (outliers).

VaR and CTE under Multivariate Pareto Distributions
Yongtao Laurel Hu --- University of Waterloo, Canada (y3hu@math.uwaterloo.ca)

ABSTRACT: Assume risks follow a multivariate Pareto distribution. We discuss the value at risk (VaR) and the conditional tail expectation (CTE) of the sum of the risks. We use the convolution distribution of the multivariate Pareto distribution. Numerical examples are given to compare the multivariate Pareto model with multivariate normal and elliptical
risk models.

Game Theoretic Analysis of Competitive Rate Setting
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ABSTRACT: In the process of assigning interest-crediting rates for accumulation of net premium deposits, there is competition between insurance companies. One company may set their rates based on new money rates, whereas another may set theirs based upon current portfolio rates, or any combination of the two. The different companies also use different metrics to determine their rate-setting practices. These metrics may be designed to measure competitive information, expected customer behavior, business line and/or company profitability, or other possible indicators. The determination of a competitive advantage must also account for regulatory control as well as the economic impact on the competitive and regulatory environment. The modeling of competitive rate setting must address all of these issues.

In this paper, we use game theory to model different rate-setting processes in the presence of competitors, regulators, and various states of the economy. Herein, we model both two-player and three-player games with the players consist of chance, competing insurance companies and/or regulatory agencies. Chance will depend on the level of interest rates and other possible economic factors. The two-player game will consist of two competitors where the competitors will set the credited rate based on volume vs. profitability. The three-player game will consist of the same two-players vs. the regulatory agents. We assume that the goals of the regulatory agents is to assure that both players remain solvent, pay premium tax, and maintain guaranteed rates.

Finally, we analyze these games by eliminating any sub-dominate strategies and determining any Nash, perfect, or sequential equilibriums. From these results we discuss the resulting efficient (or optimal) methodologies, which allow for a random switching of crediting strategies.

Pricing Barrier Options
Hangsuck Lee --- Soongsil University, Korea (hslee@stat.ssu.ac.kr)

ABSTRACT: This paper will derive explicit pricing formulas for eight types of inside and outside barrier options, respectively. The monitoring periods of these options start at an arbitrary date and end at another arbitrary date before maturity. The eight types of barrier

options are up-and-in, up-and-out, down-and-in and down-and-out call (or put) options. Sections 3, 4 and 5 assume that the underlying assets pay no dividends. In contrast, Section 6 will derive pricing formulas for the barrier options when their underlying assets pay dividends continuously at a rate proportional to their prices.

On the Existence of an Optimal Regression Complexity in the Least-Square Monte Carlo (LSM) Framework for Options Pricing
Yu Zhou --- University of Waterloo, Canada (y8zhou@math.uwaterloo.ca)

ABSTRACT: In this presentation, we illustrate how to value American-style options using the Least-Squares Monte Carlo (LSM) approach proposed by Longstaff and Schwartz (2001) and investigate whether there exists an optimal regression complexity in the LSM framework for options pricing. In particular, we use the smoothing spline in the regression step, which allows us to control the regression complexity on a continuous scale with just one tuning parameter. Numerical results on American put options indicate that we need to use more than a linear regression, but as the regression becomes more complex, the accuracy of the LSM method quickly deteriorates.

Friday afternoon, first session

Hedging Salary Related Pension Benefits
Kai Chen --- University of Waterloo, Canada (k22chen@math.uwaterloo.ca)

ABSTRACT: We consider the relationship between salary growth and stock returns, and use this to suggest that equities may have a role in hedging final salary benefits. Simulations from the empirical distribution for salaries, equities and interest rates are used to demonstrate the quantile hedge.

Pricing and Hedging a Hybrid Pension Plan
Hongzhen Tian --- University of Waterloo, Canada (h2tian@math.uwaterloo.ca)

ABSTRACT: A defined contribution plan with a defined benefit minimum guarantee is valued as an exchange or Margrabe option. We compare the resulting price with the actual contribution rate used by one public sector plan, and consider extensions of the exchange option theory to allow more accurately for the pension plan characteristics.

Natural Hedging of Life and Annuity Mortality Risks
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ABSTRACT: The purpose of this paper is to study natural hedging of mortality risks and to propose mortality swaps as a risk management tool. Natural hedging utilizes the interaction of life insurance and annuities to a change in mortality to stabilize aggregate cash outflows. The same mortality change has opposite impacts on life insurance and annuities. If future mortality of a cohort improves relative to current expectations, the life insurers gain because death benefit payments will be later than expected initially. However, the annuity insurers have a loss relative to current expectations because they have to pay annuity benefits longer than expected initially. If the mortality deteriorates, the situation is reversed: life insurers have losses and annuity writers have gains. Few researchers investigate the issue of natural hedging.

Most of the prior research explores the impact of mortality changes on life insurance and annuities separately, or investigates a simple combination of life and pure endowment life contracts (Frees et al., 1996; Marceau and Gaillardetz, 1999; Milevsky and Promislow, 2001). Studies on the impact of mortality changes on life insurance have mainly focused on "bad" shocks while those on the annuity mainly focused on "good" shocks.

Wang et al. (2003) analyze the impact of the changes of underlined factors guiding the process of the mortality hazard rates and propose an immunization model to calculate the optimal level of product mix between annuity and life insurance to hedge longevity risks based on the mortality experience in Taiwan. However, they do not use separate mortality tables to explore life insurance and annuity mortality experience. In practice, life insurance and annuity mortality experience can be very different, so there is "basis risk" involved in using annuities to hedge life insurance mortality risk. Their model cannot pick up basis risk.

Marceau and Gaillardetz (1999) examine the calculation of the reserves in a stochastic mortality and interest rates environment for a general portfolio of life insurance policies. In their numerical examples, they use portfolios of term life insurance contracts and pure endowment polices, like Milevsky and Promislow (2001). They focus on convergence of simulation results, but there is a hedging effect in their results. However, they are not studying natural hedging.

Froot and Stein (1998) develop a framework for analyzing the capital allocation and capital structure decisions facing financial institutions. Their model suggests that the hurdle rate of an investment opportunity consists two parts, the standard market-risk factor and the unhedgeable risk factor. Froot and O'Connel (1997) have documented the very high average hurdle rate of the catastrophe reinsurance business. On average, over the period 1980-1994, the price is on the order of four times the actuarial value. Since the risks being insured are essentially uncorrelated with the market portfolio and a classical model would imply prices roughly equal to actuarial values, this type of pattern suggests striking markup of unhedgeable catastrophe risks. Until now, no attention has been paid to the risk premium of unhedgeable mortality risks. Our hypothesis is that the insurance price is positively related to unhedgeable mortality risks after we control for each company's capital.

Although it is common for an insurer to write both life insurance and annuities, its mix of life and annuity mortality risks may not be perfectly hedged. It makes sense to create a swap with another company to acquire the missing line of business and improve the natural hedge. We propose a mortality swap between a life insurer and an annuity

insurer. It works like natural hedging within a company. Further, we consider deviations from the trend in order to investigate the hedging effect of a mortality swap. Our study investigates improving mortalities (such as medical breakthroughs) and deteriorating the mortality (such as natural disasters, war, and epidemics). Our research will investigate the overall impact of a mortality swap in a life insurer's reserve and, therefore contribute to the solution of the insurer's asset-liability management problem. If an insurer can successfully hedge its mortality risk, mortality risk premium in its products will be reduced and thus the price will be lower. It will improve its competitiveness in the market.

In summary, we will study natural hedging and propose a mortality swap between life insurer and an annuity insurer. In this way, our analysis will fill a gap in the literature on mortality risk management of life insurers.

Consistent Pricing for Equity-Linked Products

Patrice Gaillardetz --- University of Toronto, Canada (patrice@utstat.utoronto.ca)
X. Sheldon Lin --- University of Toronto, Canada (sheldon@utstat.utoronto.ca)

ABSTRACT: In this talk, we will introduce a consistent pricing method for Equity-Linked products and Equity-Indexed Annuities (EIAs) in particular. Due to their unique designs, these products involve mortality and financial risks and hence have to be valuated in an `incomplete market' framework. The no-arbitrage argument of Harrison and Pliska (1981) leads to the derivation of martingale probability measures for the valuation of these products. By assuming the separation of the insurance market and the annuity market, we derive an age-dependent, mortality risk-adjusted martingale probability measure for each market, which incorporates certain pricing information from the markets. As a result, we are able to valuate an Equity-Linked product by pricing its death benefits and survival benefits separately. We also provide an alternative approach by considering the endowment insurances market and derive an associated age-dependent, mortality risk-adjusted martingale probability measure. In this case, an Equity-Linked product is valuated in a unified manner. Numerical examples on EIAs are provided to illustrate the implementation of this method.

Friday afternoon, second session

Modeling Future Mortality Risk from Exposure to a Sudden Extreme Situation and Its Impact on Life Insurance

Yungui Hu --- Georgia Sate University (yhu3@student.gsu.edu)
Samuel H. Cox --- Georgia Sate University (samcox@gsu.edu)

ABSTRACT: This talk presents the modeling of future mortality risk from potential exposure to a sudden extreme situation such as natural disasters and terrorism, and its impact on life insurance. In the observation time window of a human's lifetime span, the duration of such situation is so short that it can be treated as an instantaneous case. Based

on this concept, the future mortality risk is modeled with an extra force of mortality consisting of two random variables—time of occurrence and severity of the sudden hazard. Both of the two parameters are independent of the variable of time to death without exposure to future mortality risk. Therefore, the total force of mortality is obtained through superposition of the extra force of mortality and the force of mortality in absence of adverse situation. With this new model, the associated survival function is derived and the impact of the mortality risk on life insurance and annuity is examined.

Variance of the Loss for Term and Pure Endowment in Actuarial Notation
Samid Viveros --- Instituto Tecnológico Autónomo de México (al81308@alumnos.itam.mx)

ABSTRACT: The variance of the loss for whole life (and endowment) has a known expression in convenient actuarial notation. The lack of such expressions for the case of term and pure endowment products has been observed in the literature, see Bowers (1997). In this talk, we present such expressions for not only term and pure endowment but also for e.g. the family income insurance. This is achieved by a natural modification of the mortality rates. As an application of the above formula, we provide an efficient computation of the percentile premium for term insurances with a common age at issue and varying insurance period. All the computing has been done using R and an Actuarial R-package.

Hattendorff Theorem - Yet Another Look
N. D. Shyamal Kumar --- The University of Iowa (shyamal-kumar@uiowa.edu)

ABSTRACT: Hattendorff Theorem is about The Risk in Life Insurance, as described by the title of the original paper by Hattendorff in 1868. Sustained interest in the result is demonstrated by the literature on it spanning 135 years. In this talk we present heuristically simple proofs of different versions of the Hattendorff Theorem. To illustrate the generality of these ideas we analyze the main example from Milbrodt (2000), where it is dealt with using martingale techniques for the multivariate counting process. All the computations have been done using the software R and an Actuarial R-package (under development). If time permits, we will briefly talk about R for Actuarial Computing.

Symbolic Computation of Moments of Loss Random Variables in Discrete Time
Bruce L. Jones --- University of Western Ontario (jones@stats.uwo.ca)

ABSTRACT: This paper presents an approach to symbolic computation of moments of loss random variables in discrete time. Such moments can be expressed as expectations of linear functions of indicator random variables. By establishing a representation for events in discrete-time stochastic processes and specifying properties of indicators of events and expected values, expressions for the desired moments are obtained. As an application of the methods, Hattendorff's theorem is verified in the Markov chain case.

Saturday morning, first session

Development and Application of the Prospective Mortality Tables in Actuarial Science

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ABSTRACT: In this paper, we present statistical techniques which consider mortality improvements in pricing and reserving life insurance contingencies. It is a well known fact that longevity risk has a large impact on the present value of the costs associated to life insurance and retirement plan annuity policies. For sake of simplicity, most actuaries use time-homogeneous lifetime distributions. This assumption is not necessarily appropriate in North America where it is now well documented that human mortality globally declined during the course of the 20th century. We consider the Lee-Carter model, the Poisson log-bilinear model and the binomial Gumbel-bilinear model to take into account the mortality improvements. These methods have been applied to the Quebec population, Canadian population and to the Quebec Pension Plan (which is the Régie des rentes du Québec or RRQ) annuitants to illustrate the impact of mortality improvements on the costs of life insurance and annuity policies. These methods can easily be used to analyze the impact of mortality evolution on the present value of costs for single and group annuities and for general and supplemental pension plans.

Smooth Monte Carlo Method for Diffusion Processes

Adam Kolkiewicz --- University of Waterloo, Canada (wakolkiewicz@math.uwaterloo.ca)

ABSTRACT: Knowledge of transition distributions is of primary importance in numerical finance as a typical pricing problem involves evaluation of one or several expectations with respect to such distributions. Since in general we are unable to represent densities of these distributions in a "closed form", several approximation methods have been considered including the Ito-Taylor expansions, numerical solutions of partial differential equations, and binomial approximations. For many high-dimensional problems, however, the only feasible methods are based on simulations. In this paper we propose a smooth Monte Carlo estimator of transition densities, which uses both simulation and numerical integration. Although non-parametric in nature, it is unbiased and exhibits a rate of convergence typical to parametric problems. When used to approximate functionals of terminal values of a diffusion process, it reduces the variance by a factor that depends on the length of time interval over which we can locally approximate the transition density. The method also provides a framework within which Monte Carlo can be combined with other numerical techniques, like quasi-Monte Carlo.

Empirical Analysis of Representative Scenario Sampling Algorithms
Yujia Zhu --- University of Waterloo, Canada (y4zhu@math.uwaterloo.ca)

ABSTRACT: In November 2003, the American Academy of Actuaries Life Capital Adequacy Subcommittee ("LCAS") issued a report entitled "C3 Phase 2 RBC for Variable Annuities: Pre-Packaged Scenarios" to address both the interest rate and equity risk associated with variable products with guarantees. In addition to providing 10,000 "pre-packaged" scenarios for the common asset classes typically needed in the stochastic cash flow projections of variable annuities, the Academy also recommends a procedure allowing companies to select a subset of the representative scenarios from the full sample of 10,000 in an attempt to reduce substantially the computational effort. The recommended representative scenario sampling algorithm is based on Chueh (1999). In this presentation, we investigate relative efficiency of the representative scenario sampling algorithms using examples such as GMMB and GMAB. We also consider the use of other sampling algorithms including based on K-means approach.

Development of Cohort Life Tables for "Other Causes" for Use in Simulation Modeling
Marjorie A. Rosenberg --- University of Wisconsin-Madison (mrosenberg@bus.wisc.edu)

ABSTRACT: In this study I apply multiple decrement theory in the development of life tables where the risk of a certain form of cancer is removed. The application of this study is for use by CISNET modelers (Cancer Intervention and Surveillance Modeling Network), a consortium of NCI-sponsored investigators whose focus is to use modeling to improve our understanding of the impact of cancer control interventions (e.g., prevention, screening treatment) on population trends in incidence and mortality. These models are also used to project future trends, and to help determine optimal cancer control strategies.
The output of this study is the development of life tables separately removing breast cancer (women only), prostate cancer (men only), and colorectal cancer (both men and women). Current work includes the development of cohort life tables removing lung cancer as a cause of death. Preliminary output will demonstrate the impact on all-cause mortality separately considering current smokers, former smokers, and never smokers for white males.

Simulated Maximum Likelihood Estimation of Stochastic Volatility Models
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ABSTRACT: The Stochastic Volatility (SV) model, introduced by Taylor (1986) is used for capturing the empirical properties of financial time series. However, most of the applications are based on the assumption that the conditional distribution of the returns given the log volatilities is normal. This paper overviews those properties and compares the SV model with the heavy-tailed error distribution (Student t-distribution) and the SV model with the normal error distribution. The Simulated Maximum Likelihood (SML)

method is applied to estimate the parameters and the latent volatility. Furthermore, an empirical analysis with several return series shows that the SV-t specification adequately accounts for the well-known properties of the financial series: a high kurtosis of the returns and low but slowly decaying autocorrelation of the squared returns.

Saturday morning, second session

Optimal Consumption Strategy in the Presence of Default Risk: Discrete-Time Case

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ABSTRACT: In this paper, we study the optimal consumption behavior in a multi-period setting. We assume that the distribution of the return of a risky asset can be different over different periods, we model this change by the Markov-Switching model. In particular, one of the regimes is an "absorption state", which represents the "default" state. If the regime switches to this default state, then an investor can only get back a certain fraction, called the recovery rate, of the amount invested. With the objective of maximizing the expected discounted utility of the consumption, we have derived the optimal consumption strategy. Analytic properties of the solution are examined. In particular, we present some properties on the effect of regime-switching and the default risk.

Risk Capital Decomposition for a Multivariate Dependent Gamma Portfolio

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ABSTRACT: Significant changes in the insurance and financial markets are giving increasing attention to the need for developing a standard framework for risk management. Today's competitive and investment oriented marketplace requires from insurance directors to use all the advantages of investing risk capitals of their enterprises. Recently, there has been growing interest among insurance and investment experts to focus on the use of a tail conditional expectation as a measure of risk, since it shares properties that are considered desirable and applicable in a variety of situations. In particular, such a method allows for a natural allocation of the total risk capital among its various constituents. This paper examines above risk measure in the case of a multivariate gamma portfolio. We demonstrate the explicit formulas for tail conditional expectation and based on it capital allocation when the proposed multivariate model consists of dependent and independent gamma marginals. Financial enterprises are always concerned of fairly allocating the total risk capital to these constituents. Consequently, this work is particularly meaningful in practice in the case of computing capital requirements for an institution who may have several lines of correlated business and whose data is distributed multivariate gamma model considered here.

Estimators for a Generalized Poisson Autoregressive Process of Order 1
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ABSTRACT: Various models have been proposed to study non-negative integer-valued time series. In this paper, we study estimators for the generalized Poisson autoregressive process of order 1, a model developed by Alzaid and Al-Osh (1993). We compare three estimation methods, the methods of moments, quasi-likelihood and conditional maximum likelihood and study their asymptotic properties. To compare the bias of the estimators in small samples, we perform a simulation study for various parameter values. Using the theory of estimating equations, we obtain expressions for the variance-covariance matrix of those three estimators, and we compare their asymptotic efficiency. Finally, we apply the methods derived in the paper to a real time series.

Price Regulation in the Automobile Insurance Market: A Discrete-time Markov Chain Model
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ABSTRACT: One of the effects of price-regulation on the automobile insurance industry is the creation a vicious circle where relatively low risk drivers, end up subsidizing individuals who are initially denied coverage by insurance companies but are later offered coverage. Suppose the insurance market is characterized by drivers who fall into two categories. First, there are the low-risk drivers who are insurable in the voluntary market (V) under the current regulatory premium structure. Second, there are the high-risk drivers who cannot obtain coverage from the voluntary market and they are sent into the residual market (R) where they are being charged a premium set by regulation, which is lower than their actuarial fair cost. Therefore losses are generated on average, in excess of the total premiums collected by R. As a result insurance companies have to raise extra funds from the insurable drivers to cover the excess R losses. As prices in V will have to increase in the following year to provide for this cross-subsidy between risk groups, some of the insured drivers in V (i.e. those with slightly higher risk characteristics) will be denied coverage and enter R. From a social welfare perspective, while these higher risk drivers will have a lower relative risk to the typical driver in R and may marginally decrease the overall risk of the group, **moral hazard** and the feeling of unfairness they will experience will probably make up for, if not exceed, the temporary decrease in the expected R losses. It is therefore possible that this circle may be perpetuated over time under certain conditions, until some insurance companies leave the market, the government subsidizes the market or the market fails completely.

In my model I will use a discrete-time Markov chain under the finite population assumption to investigate the stochastic flow of automobile insurance customers between the states of the world, V and R, as this flow is triggered by the losses they are expected to generate in the period covered by their automobile insurance contract. The population of R is expected to increase over time as people from V will be called to pay a premium higher than their actuarial fair premium something that will push high risk individuals of V into R as they be denied coverage in the voluntary market. The main purposes of this

paper are: (i) to calibrate the process by which excess losses generated by individuals in R, are covered by individuals in V as a uniform percentage added on their actuarially fair premiums, (ii) to illustrate through a discrete time Markov chain under the finite population assumption that regulated R premiums create pressure on lower risk groups under certain assumptions that make R an absorbing state in a Markov chain.

Claims Reserving When There Are Negative Values in the Runoff Triangle: Bayesian Analysis Using the Three-parameter Log-normal Distribution

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Gilberto Atondo --- Instituto Tecnológico Autónomo de México

ABSTRACT: In this paper we are concerned with the situation when there are negative values in the development triangle of the incremental claim amounts. Typically these negative values will be the result of salvage recoveries, payments from third parties, total or partial cancellation of outstanding claims, due to initial over-estimation of the loss or to possible favorable jury decision in favor of the insurer, rejection by the insurer, or just plain errors. It could be argued that the problem is more with the data than with the methods. Some of the traditional methods of claims reserving, such as the chain-ladder technique, may handle this situation. However, many can break down in the presence of a sufficient number of negative incremental claims if certain positivity constraints are not met. Although the chain-ladder is frequently used as a benchmark, due to its generalized use and ease of application, our aim is not to develop Bayesian methods that provide results close to those of the chain-ladder method. We present a full Bayesian model to consider negative incremental values, based on a three parameter log-normal distribution. The model presented here allows the actuary to provide point estimates and measures of dispersion, as well as the complete distribution for outstanding claim reserves. We apply MCMC