A Flowchart Representation of a Modern Tontine

Arnold F. Shapiro[1]

Penn State University, Smeal College of Business, University Park, PA 16802, USA

Abstract

During my presentations at the 26th International Congress on Insurance: Mathematics and Economics (I:ME) and the 58th Actuarial Research Conference (ARC), which dealt with tontines on the blockchain, there was considerable interest in the flowchart portions of my presentations. In view of this, I decided to focus this ARC Proceedings note on a flowchart representation of a modern tontine.

The note begins with a short overview of key characteristics of a modern tontine. This is followed by a representation of a tontine flowchart that explicitly displays the components of a tontine and their relationships. The remainder of the note discusses the specifics of those components, including: the valuation and buy-in module, the active life test, settlement options, and periodic payments.

The note concludes with a commentary.

Keywords: tontine, flowchart, modern, validation, active life test, periodic payment, termination

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1 Introduction

During my presentations at the 26th International Congress on Insurance: Mathematics and Economics (I:ME) and the 58th Actuarial Research Conference (ARC), which dealt with tontines on the blockchain, there was considerable interest in the flowchart portions of my presentations. In view of this, I decided to focus this ARC Proceedings note on a flowchart representation of a modern tontine.

The note begins with a short overview of key characteristics of a modern tontine. This is followed by a representation of a tontine flowchart that explicitly displays the components of a tontine and their relationships.

Next we discuss the specifics of those components, including: the validation and buy-in module, which involves nominee authentication, where the nominee is the designated individual upon whose life the tontine payouts depend, subscriptions and asset allocation; the active life test that assigns a status to the nominee, which includes active, non-active, deceased and redeemed; settlement options, based on no remaining nominees, less than a minimum number of nominees, and more than or equal to a maximum term; and periodic payments.

The note concludes with a commentary.

2 Overview of key features of a tontine

A classical tontine is a financial arrangement in which investors form an asset pool and then, mutually and irrevocably, agree to receive payouts from it while an individual they designate, called their nominee, is living and to forfeit their accounts upon the death of their nominee, with the forfeiture apportioned among the investors whose nominees are still living.[2] As a consequence, in addition to investment returns from the asset pool, the returns include "mortality credits," due to the mortality experience of the nominees.

In contrast, a modern tontine, which is the type discussed in this note, may include provisions for settlements in the event of redemption of the tontine by an investor or death of a nominee.[3]

The tontine players include:[4]

The sponsor, who maintains the tontine and determines the payout policy,

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[2] Adapted from Fullmer (2019: 4).

[3] Throughout this note, when the term tontine is used, it should be interpreted to mean "modern tontine."

[4] Adapted from Milevsky (2022: 27).

The fund manager, who selects investments and manages the asset allocation of the tontine fund,

The nominee, who is the designated individual upon whose life the tontine payouts depend, and

The investor (subscriber), who purchases and owns shares in the tontine, and upon whose behalf tontine payouts are paid.[5]

As far as payouts, there are two ways for the investors to receive payouts from the tontine fund: periodic payments in the form of dividends (see §11.1) and terminal payments on account of the dissolution of the tontine. In this regard, the tontine might be terminated, and its funds distributed, if the number of surviving nominees drops below a critical number or the maximum duration for the tontine is attained (see §9).

Finally, with respect to limitations of this note, to simplify the discussion, we will only consider closed-end tontines, in that there are no provisions for new entrants to replace nominees who have died or whose sponsor has redeemed her or his contract and it is assumed that the nominees are homogeneous in age and gender. [6]

3 A Tontine flowchart for a cohort of n nominees

A representation of a tontine flowchart for a cohort of n nominees is depicted in Figure 1.

The flowchart starts at the upper left of the figure.

Key features are the tontine specifications, the validation and buy-in module, the active life test, the stopping rules, and the periodic payments. Each of these features is discussed in the pages that follow.

4 The tontine specifications

Step ① of Figure 1, which lays the foundation for the tontine process, displays an abbreviated list of tontine specifications. The topics mentioned include:

The target cohort size, n,
The subscriber's buy-in price/unit,
The specified minimum number of active nominees, n*, which triggers a settlement,
The specified maximum term for the tontine, t*, which triggers a settlement, and

[5] Of course, the subscriber may be the nominee.

[6] In general, this need not be the case. See, for example, Milevsky and Salisbury (2016).

The tontine payout function, d(t), per dollar initially invested.

This list is not exhaustive, of course, there are other topics that need to be specified, like age and subscription limits. These topics will be discussed in context, as we come to them.

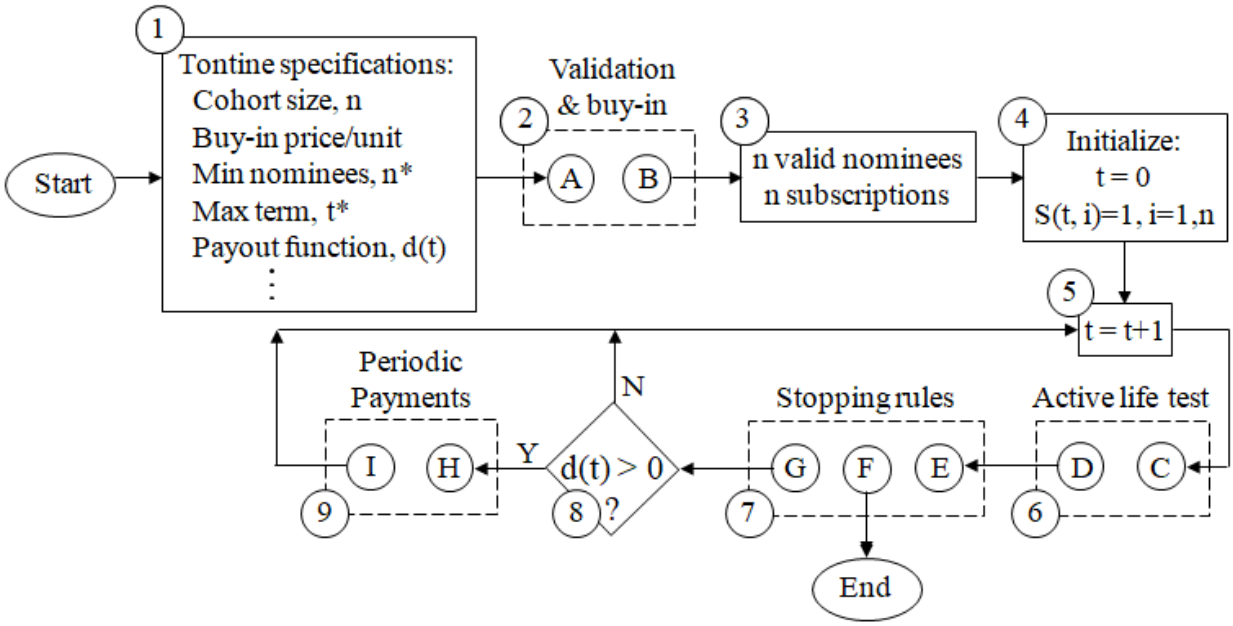

Figure 1: Flowchart for a tontine cohort

5 Validation of nominees and buy-in prices

The implementation of the tontine begins with the validation of potential nominees and the payment of related subscriptions. In terms of the flowchart of Figure 1, we pass to the validation & buy-in module through porthole A of step ②.

The validation and buy-in process begins by initializing the indexes i and j in step ① of Figure 2, where the latter, j, will be used to track the valid nominees and subscriptions.

The goal is to develop a cohort of n nominees[7].

[7] For the sake of this discussion, we assume that the goal of n nominees is attainable, which may or may not be the case in a particular situation.

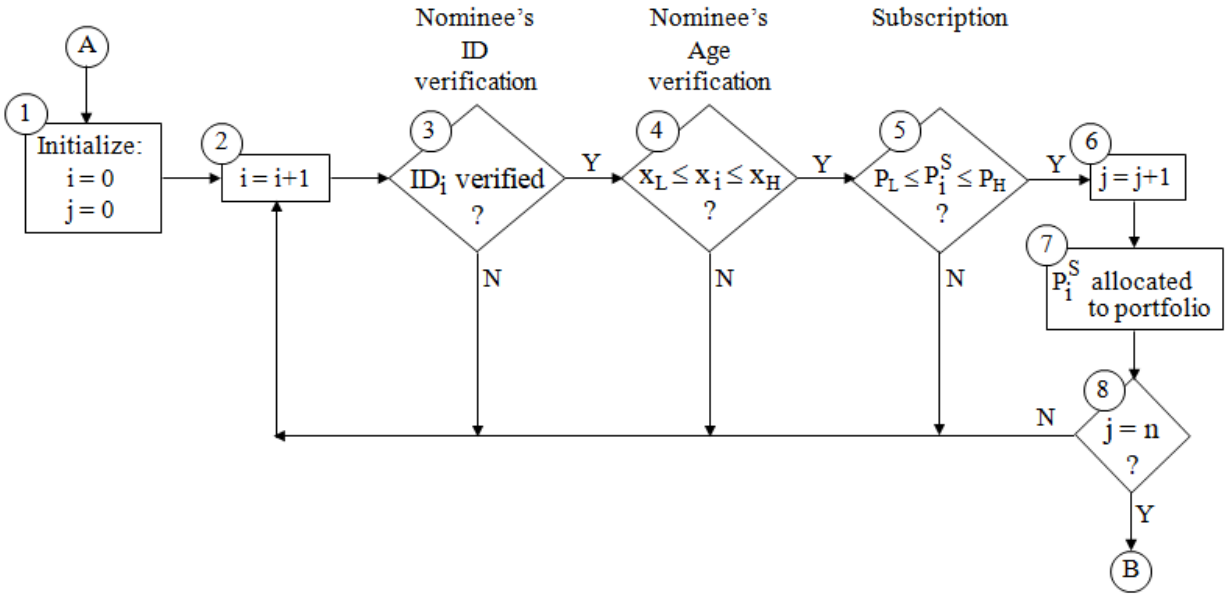

Figure 2: Validation / Buy-in

Here

$ID_i \equiv$ ID of i-th nominee

$x_i \equiv$ age of i-th nominee

$x_L \equiv$ lower bound on the age of the nominee

$x_H \equiv$ upper bound on the age of the nominee

and

$P_i^S \equiv$ buy-in price of i-th subscriber

$P_L \equiv$ lower bound on the buy-in price of the subscriber

$P_H \equiv$ upper bound on the buy-in price of the subscriber

$n \equiv$ initial number of nominees for this cohort

5.1 Validation of nominee's ID and age

Steps ③ and ④ of Figure 2 highlights common nominee participation requirements of a tontine:

First the ID must be verified,
Then the age requirements must be satisfied; in this case, the age at issue must fall between X_L and X_H.

In a typical situation, a clear and legible copy (either physical or electronic) of documents, such as a valid passport (photo page) or driver's license is acceptable as proof of age and/or identity.

In addition, the company issuing the tontine may, as part of its due-diligence, independently verify this information with that gathered during its "Know Your Client" (KYC) inquiries.

5.2 Subscription buy-in price and payment

Given that the ID is verified and the age requirement is satisfied, we move to step ⑤ of Figure 2 to validate the proposed subscription buy-in price and payment. In this example, the buy-in price of the subscriptions, P_i^S, must fall within the parameters specified by the contract, here $P_L \leq P_i^S \leq P_H$.

Just as there is an age restriction to promote equity among the nominees, there might need to be economic restrictions on the minimum and maximum investment to ensure against what Milevsky (2022: 125) calls "a lopsided gamble."[8]

5.3 Asset Allocation

Guardian Capital, a Canadian financial services firm that provides tontines, and TontineTrust, a blockchain-based tontine provider in Ireland, illustrate two examples of the asset allocation policy of step ⑦ of Figure 2.

With Guardian Capital, the investment objective for the Fund is set by the firm, and is to "provide long term capital appreciation by investing the tontine trust's assets in equities and fixed income securities. ... The fund will seek to achieve the investment objective by following a "glidepath" [target] approach to asset allocation." [Guardian Capital (2023)]

With TontineTrust "The capital in the ... tontine is deployed into a diversified portfolio of leading ETF index funds balanced across global markets and asset classes following classical portfolio design theory." [TontineTrust (2021)]

6 A cohort of n valid nominees and their subscribers

At this stage, a cohort of n valid nominees and their subscribers[9] has been compiled, as displayed in step ③ of Figure 1.

Now, as depicted in step ④, the time index, t, is set to 0 and all the nominees are assigned an active status, that is, S(t, i) = 1, t=0, i=1, ..., n.

[8] From the tontine sponsor's perspective, no attempt need be made to ascertain the appropriateness of the tontine for the subscriber, although the sponsor might put the subscribers on notice that there may not be a good fit.

[9] Of course, the nominee and subscriber may be one and the same person.

7 The iteration step

Step ⑤ of Figure 1 is the iteration step where the index, t, is incremented.

8 The Active Life Test

After incrementing the time, in step ⑤ of Figure 1, we proceed to the Active life test (ALT) by passing through porthole C of step ⑥.

The primary role of the ALT is to identify the current active nominees, and to store their number, N(t), for future use (see §9). An additional role is to accommodate settlements with respect to redemption and death.

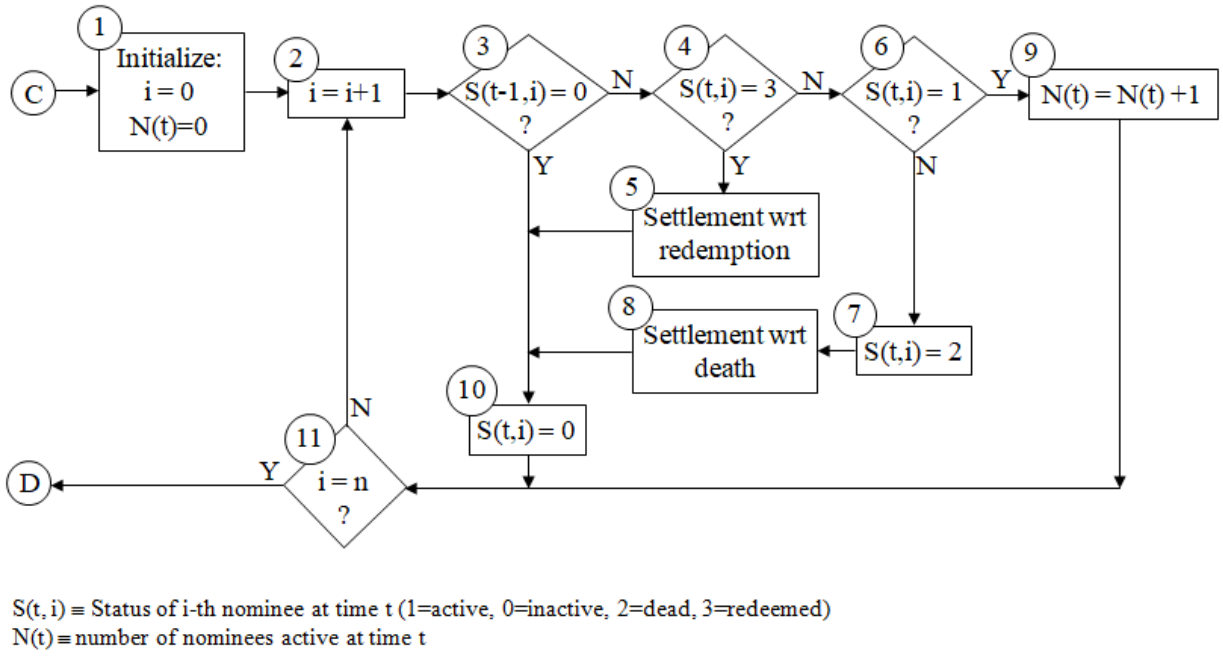

Figure 3: Active life test

We begin the ALT by setting the counter, i, and the number of active nominees, N(t), to 0, in step ① of Figure 3.

Next, we increment the counter, i, by 1, in step ②.

8.1The Gatekeeper

In step ③ of Figure 3, the gatekeeper, we check if the i-th nominee is already in an inactive status, in which case, S(t-1, i) = 0.

If so, step ③ returns a "Y", and, in step ⑩, a 0 status is reassigned to the nominee at time t, that is, S(t, i) = 0.

If that was the last nominee of the cohort, i = n in step ⑪, and the ALT is terminated. Otherwise, we proceed to the next nominee, step ②.

If, on the other hand, the nominee's status at time t-1 was active, step ③ returns an "N", and we proceed to the core of the active life test, steps ④ and ⑥ of Figure 3, to determine the nominee's current status.

8.2 Redemptions

Figure 3, step ④, queries whether the tontine associated with the i-th nominee was redeemed in the current year. That is, whether S(t, i) = 3. If so, step ④ returns a "Y" and, if provided for by the tontine,[10] there is a settlement with respect to the redemption, in step ⑤.

Next, the status of the i-th nominee is changed to inactive, S(t, i) = 0, in step ⑩.

If that was the last nominee, i = n in step ⑪, and the ALT is terminated. Otherwise, we proceed to the next nominee.

If step ④ returns a "N", the tontine associated with the i-th nominee was not redeemed during the current year, and we proceed to step ⑥,where the query is whether the i-th nominee is still an active life, S(t, i)=1.

8.3 Deaths

If the i-th nominee died during the current year, step ⑥ returns a "N", and the status of that nominee changes to S(t, i) = 2, in step ⑦.

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[10] Although classical tontines did not provide for a payment in the event of redemption or death, modern tontines need not have this restriction. An example of a settlement option with respect to redemption or death is provided by Guardian Capital's tontine, GuardPath Modern Tontine 2042 Trust, which only provides for a tontine payout of the fund after 20 years, for those still living. However, in the event of death or redemption during the first four years of the tontine, the recovery is 95% of the net asset value. Thereafter, the recovery decreases by 5% per year, until a 50% recovery rate is reached, and the recovery rate remains at 50% thereafter.

If provided for by the tontine, a death benefit settlement occurs in step ⑧.

Next, the nominee's status is changed to inactive, S(t, i) = 0, in step ⑩.

If that was the last nominee, i = n in step ⑪, and the ALT is terminated. Otherwise, we proceed to the next nominee.

If step ⑥ returns a "Y", the i-th nominee is still active, S(t, i) = 1, and we proceed to step ⑨.

8.4 Active lives

If the i-th nominee's status at time t-1 was active, S(t-1, i) = 1, and it also is currently active, S(t, i) = 1.[11] In this case, Step ⑥, of Figure 3, returns a "Y", and we proceed to step ⑨, where the total number of active lives at time t is increased by 1, that is, N(t) = N(t) + 1.

If that was the last nominee, i = n in step ⑪, and we terminate the ALT. Otherwise, we proceed to the next nominee.

8.5 Exiting the ALT

Here, we just make the point that, when the index counter gets to n, in Figure 3 step ⑪:

The active life test terminates, and

The numbers of active lives is fixed at N(t) for the remainder of the t-th iteration.

We return to Figure 1 through porthole D of step ⑥.

9 Conditions for terminating the tontine

We turn now to conditions for terminating the tontine, which are addressed in Figure 1, step ⑦, the stopping rules.

Three basic conditions under which the tontine might be terminated are:

1. N(t) = 0, no remaining nominees, due to deaths and/or redemption

2. N(t) ≤ n*, the remaining nominees are less than or equal to a target number, and

[11] As far as the evaluation of step 6 of Figure 3, we might follow TontineTrust (2018), for example, and use facial recognition to verify that a nominee is alive.

3. t = t*, the specified maximum term for the tontine has been reached.

Each of these are depicted in Figure 4 and discussed in the following subsections.

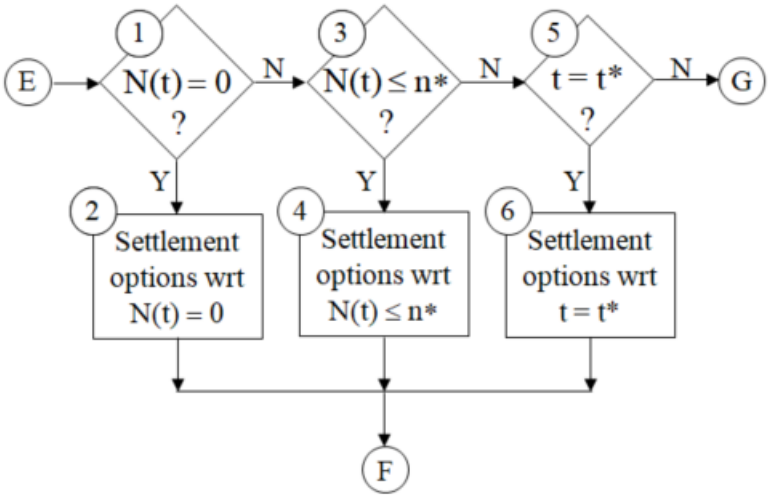

Figure 4: Stopping rules & settlement options

9.1 N(t) = 0

We begin this phase by checking whether there are any remaining nominees, Figure 4, step ①. If there are no remaining nominees, N(t) = 0, step ① returns a "Y", the settlement option(s) with respect to N(t) = 0 are invoked, step ②, after which we proceed through porthole F and the tontine terminates.

One interesting suggestion, as far as a settlement option in this case, came from Daya and Bernard (2023: 19), who suggested distributing the funds to a designated charity that would be named ex ante at the inception of the contract.

If step ① returns an "N", we proceed to step ③.

9.2 N(t) ≤ n*

Having arrived at step ③ of Figure 4, we know that N(t) > 0.

Now, however, the issue is how the number of nominees with an active status at time t compares with the minimum number of nominees specified in the contract, n*.

If the number of active lives at time t is less than or equal to n*, N(t) ≤ n*, step ③ returns a "Y" and the settlement option(s) with respect to N(t) ≤ n* are invoked[12], after which we proceed to porthole F and the tontine terminates. The settlements would be on a pro rata basis.

If N(t) > n*, step ③ returns an "N", and we proceed to step ⑤ of Figure 4.

9.3 t = t*

In step ⑤ of Figure 4, we check if the maximum term, t*, has been reached.[13] If it has, the plan assets are distributed according to the settlement option with respect to t*, in step ⑥, after which we proceed to porthole F and the tontine terminates. Once again, the settlements would be on a pro rata basis.

If the maximum term, t*, has not been reached, step ⑤ returns an "N", and we exit the stopping rules through porthole G, and proceed to the periodic payment stage, Figure 1, steps ⑧ and ⑨.

10 Periodic Payments

As mentioned previously, there are two ways for the investors to receive payouts from a tontine fund: periodic payments in the form of dividends and terminal payments on account of the dissolution of the tontine. The previous section addressed the latter. In this section, we turn our attention to the periodic payments and their payout function.

The periodic tontine payments are based on two sources: the investment returns from the asset pool, the dividends; and the mortality credits by virtue of deceased nominees, which is the core of tontines.

Following Milevsky and Salisbury (2016: 577), define the tontine payout function, d(t), to be the rate at which funds are paid out, at time t, per initial dollar invested.

10.1 d(t) = 0

To begin, we check to see if there is nothing to allocate, d(t) = 0, in step ⑧ of Figure 1.

This would be the case, for t < 20, for example, where the tontine only provided a single payout to subscribers whose nominees survived for 20 years.

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[12] Notice that we use ≤ to make the comparison, since > 1 may have become inactive during the year.

[13] Guardian Capital, for example, uses t* = 20 in their tontine, the duration to which half of the original nominees are expected to survive.

If d(t) = 0, step ⑧ returns an "N" and we bypass the periodic payment stage and proceed to time t+1, in step ⑤ of Figure 1.

10.2 d(t) > 0

If d(t) > 0, there is something to allocate and we enter the periodic payment stage, step ⑨ of Figure 1, through porthole H.

11 Modeling the periodic payments

Figure 5 is a representation of a provision for a periodic payment if the i-th nominee is in active status at time t, that is, if S(t, i) = 1.

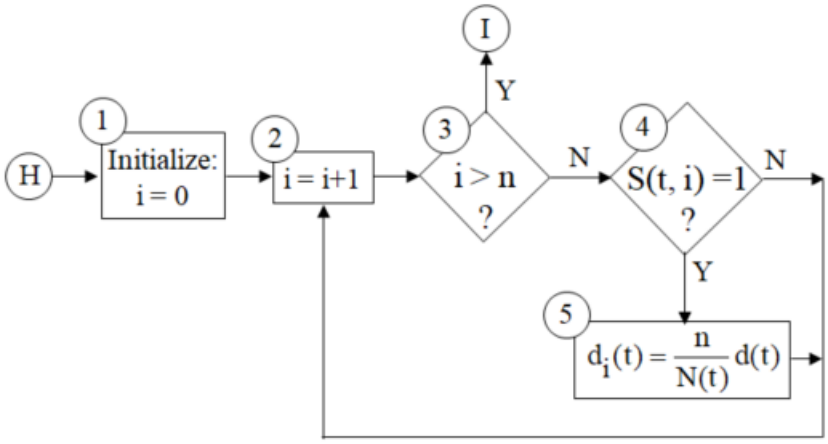

Figure 5: Periodic payment with respect to an active nominee

11.1 Payments with respect to active nominees

We start by initiating the counter to 0, in step ① of Figure 5.

Then, the counter is increased by 1, in step ②, and if that number is less than or equal the original number of nominees, i ≤ n, in step ③, the active status of the i-th nominee is checked.

In particular, if S(t, i) = 1 in step ④, which returns a "Y", an allocation of a pro-rata share, $d_i(t)$, of the payout, per unit of dollar invested, is received on behalf of the i-th investor, in step ⑤, where

$$d_i(t) = \frac{n}{N(t)} d(t).$$

We continue through this periodic payment stage until the active status of all n nominees has been queried, and a payment allocated when appropriate.

11.2 Nonactive nominees

If the i-th nominee is not active, there is no periodic payment.

In this case, the query at step④, in Figure 5, returns an "N", and we proceed directly to the counter, step②.

11.3 Exiting the periodic payment stage

When the counter, i, exceeds the cohort size, n, step③ of Figure 5 returns a "Y", and we exit the periodic payment stage though porthole I.

11.4 Proceeding to the next iteration

The iteration at time t is now complete, and we proceed from porthole I in Figure 1, step ⑨, to the next iteration at time t+1, in step⑤.

This completes our flowchart representation of a tontine.

12 Commentary

The purpose of this note was to present a flowchart representation of a modern tontine. To this end we first presented a tontine flowchart for a cohort of nominees that explicitly displayed the components of the tontine and their relationships. This was followed by a discussion of the specifics of those components, including: the buy-in module, the active life test, settlement options, and periodic payments.

While the note covered the basics of a modern tontine, there were topics that were not addressed, including a non-homogeneous cohort of nominees and/or subscription prices and open-ended models, which provide for new entrants to replace nominees who have died or who were associated with contacts that were redeemed.

Such exclusions notwithstanding, to the extent that this note helps the reader conceptualize the basic components of a modern tontine and their relationships, it will have served its purpose.

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