QFI IRM Model Solutions
Fall 2024

1. Learning Objectives:

1. The candidate will understand and be able to identify and describe types of risk present in investment management.

Learning Outcomes:

(1c) Identify behavioral risks and explain how they factor into investment management.

Sources:

Quantitative Enterprise Risk Management, Mary Hardy and David Saunders, 2022, Ch. 19: Behavioural Risk Management

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a) Explain whether your client is risk-neutral, risk-seeking, or risk-averse.

Commentary on Question:

Overall, candidates performed above average on this part of the question. Most candidates were able to use an appropriate approach to determine that ln(w) indicated a risk-averse client. Credit was given for candidates that described concavity without taking the second derivative.

The client is risk-averse.

This determination can be made by determining if u''(w) < 0 (risk-averse), > 0 (risk-seeking), or = 0 (risk-neutral)

$u'(w) = \frac{1}{w}$

$u''(w) = \frac{-1}{w^2}$

As u''(w) < 0, the client is risk-averse.

1. Continued

(b)

(i) Calculate the expected value and expected incremental utility of each investment option.

(ii) Identify the most appropriate investment option based on results above.

Commentary on Question:
Candidates generally performed above average on this part of the question. Most candidates were able to perform the calculation correctly and make an appropriate recommendation based on their calculated results. A common mistake candidates did was adding the initial capital of $5000 (instead of using the payoffs) when calculating the expected incremental utility, even when the question provided the additional direction to use it as a reference point for the calculation.

(i)

Expected Value = $\sum_{j=0}^{n} w_j * p(w_j)$

Expected Value for Option A = 4500 * 0.5 + 5500 * 0.95 = 5450
Expected Value for Option B = 500 * 0.01 + 1000 * 0.05 + 5000 * 0.1 + 6000 * 0.84 = 5595

Expected Incremental Utility = $\sum_{j=1}^{n} (u(W_0 + x_j) - u(W_0)) * p(x_j)$

EIU for Option A = ln(4500) * 0.5 + ln(5500) * 0.95 – ln(5000) = 0.0853
EIU for Option B = ln(500) * 0.01 + ln(1000) * 0.05 + ln(5000) * 0.1 + ln(6000) * 0.84 – ln(5000) = 0.0497

(c) Explain whether the suggested probabilities are consistent with the findings based on the cumulative prospect theory.

Commentary on Question:
Candidates generally performed below average on this part of the question. Most candidates could not identify features of cumulative prospect theory or apply them to the situation. In addition, many candidates made the mistake of identifying the payoff of $6000 as an extreme scenario when it was assigned the highest probability.

Yes, the findings are consistent. Very small probabilities have been set to 0 consistent with the observation that unlikely probabilities are often categorized as will not happen. Within the may happen zone, small probabilities are overweighted. Probabilities are also set equal to each other when they are similar.

2. Learning Objectives:

1. The candidate will understand and be able to identify and describe types of risk present in investment management.

Learning Outcomes:

(1b) Describe and apply various risk identification tools.

(1d) Identify environmental, social and governance (ESG) risks and explain how they factor into investment management.

Sources:

Financial Enterprise Risk Management, Sweeting, Paul – Chapter 8, Risk Identification

QFII-131-24: Introduction to ESG (CFA Society UK), Ch. 1

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a) Assess whether each of the company's actions above achieves its goal of following the PRI.

Commentary on Question:

Candidates are able to identify the first two actions to be in compliance with the PRI principles. However, most candidates are not able to explain why the third action may cause inconsistencies with the PRI principles.

The first action helps achieve the goal by addressing principle 1. The new rules help ensure that the company incorporates ESG into its investment analysis and decision-making processes.

The second action helps achieve the goal by addressing principles 2, 4, and 5. By joining the industry working group, the company can engage with peer companies and help to advise on future policies.

The third action does not help achieve the goal. By developing its own metrics for internal use and making them proprietary, the company is potentially creating different standards for itself.

Overall, the company has made progress towards the goal, but these three steps are not sufficient by themselves.

2. Continued

(b) Recommend additional actions that would help MNO become more compliant with the PRI.

Commentary on Question:
Most candidates are not able to pinpoint the discrepancies of the actions from PRI principles 3 and 6.

We are missing representation of principles 3 and 6 in the company's actions.

For principle 3 – The company needs to actively seek ESG related disclosures from the company in which it invests. A possible action is to request disclosure from companies on their compliance with one of the many ESG compliance or reporting initiatives.

For principle 6 – The company needs to report on its own activities and progress toward implementing the PRI. The company can take the third action a step further and report this to peers and stakeholders. One possible action is to publish an annual report on ESG related efforts and progress, which is distributed to both shareholders and policy owners.

(c)

(i) Explain why the CIO's approach to create a risk checklist is inadequate.

(ii) Recommend two changes to the CIO's approach that could help ensure the risk checklist is adequate.

Commentary on Question:
Most candidates can explain why the checklist is inadequate and recommend changes to rectify.

Solutions:

(i) The proposed approach may lead to a risk check list that inadequately captures the risks that need to be considered. Under the proposed approach, we are primarily relying on the experiential knowledge of the panelist who lacks relevant experience investing in the transportation sector.

2. Continued

(ii)

- The CIO could hire external resources with more knowledge and approach relies on experiential knowledge, which may be limiting.
- The CIO could have the panelists prepare additional research on other documented information and resources to supplement the panel discussion with documented knowledge.

(d)

(i) Explain how environmental risks could have a negative impact on the company's long-term investment returns.

(ii) Explain the potential conflict between the fiduciary duty of MNO's investment team and ESG factors.

(iii) Recommend which of the two companies is a better investment option for MNO.

Commentary on Question:
Most candidates can articulate the impact of the environmental risks, the conflicts between company's fiduciary duty of the investment team and ESG factors, and analyze which airline company is a better investment option.

(i) Environmental risks may reduce long-term investment returns by raising operational costs and limiting future economic growth.

One way economic risks can translate to immediate investment returns is through government intervention to combat environmental impacts. A possible example is that regulators may impose taxes or fines on companies that exceed certain emission thresholds, which could impact profits for companies with high CO2 emissions, such as Airline Company A.

It's also necessary to consider the how a company's operations impact the broader environment. Carbon emissions and noise pollution contribute to environmental damage, which can reduce the value of capital markets and future economic growth around the world. This will reduce the expected returns on all firms.

(ii) While fiduciary duty generally implies that fiduciaries (financial institutions, related professions) benefit their stakeholders' interests by seeking to maximize financial returns, ESG factors may have long-term financial impacts that are not immediately obvious or realized in the short-term. As many ESG factors can create value in the long-term, considering ESG factors is an action that helps to maximize long-term financial returns.

2. Continued

(iii) Based on the information provided, Company B is the better fit in the company's investment portfolio. From a long-term financial perspective, the smaller carbon footprint of company B work. This in turn may help reduce the potential threats to natural resources and capital, which could have secondary impacts on other investments the company has made. As the company is actively working to integrate ESG practices into its decision making and adhering to the PRI principles, Company B is a better strategic fit.

Alternatively : Company A is a better option
with a higher expected equity return in the short term, however, longer term ESG impact outlined in (i) may reduce the expected long-term equity

3. Learning Objectives:

2. The candidate will understand and be able to apply different approaches to measuring and assessing risk exposures.

Learning Outcomes:

(2f) Evaluate a company's or a portfolio's exposures to various risks, including liquidity risk.

Sources:

Investment Risk Management, Baker, Kent and Filbeck, Greg, 2015
Ch. 8: Liquidity Risk p. 144

Quantitative Enterprise Risk Management, Mary Hardy and David Saunders, 2022, Ch. 3: Risk Measures

Commentary on Question:

This question tests candidates' knowledge on measures for evaluating a company's exposure to various risks, including liquidity risk and investment risk. To receive full credit, candidates needed to provide support for their analysis.

Solution:

(a) Compare and contrast liquidity ratio (LR) and Martin liquidity (ML).

Commentary on Question:

Candidates performed well on identifying the similarities and differences between the two measures.

Liquidity ratio and Martin liquidity are both volume-based measures, they are simple and easy to obtain because of widespread availability of volume data. However, they as a measure suffer from correlation to volatility and double counting problem.

Liquidity ratio is asset-specific measure while Martin liquidity is market-specific measure.
Liquidity ratio is often measured at a monthly frequency while Martin liquidity is better measured on a daily basis.

A relatively high liquidity ratio means that a high volume of trade has a relatively low impact on asset price change, which is indicative of high liquidity. ML is inversely related to LR, therefore higher ML means lower LR and lower liquidity.

3. Continued

(b) Recommend an appropriate category of liquidity measures in light of the new regulation.

Commentary on Question:
Candidates struggled with this question, as many failed to recognize the necessity of using the Market Impact Measure and to link it to the rationale behind unanticipated changes stemming from new regulations.

ABC should use Market-impact measures. It is designed to address the shortcomings of volume-based liquidity measures, which do not differentiate between price changes due to anticipated versus unanticipated trade volume. Market-impact measures will better capture the unanticipated trade volume resulting from the change in capital requirement, which is a market specific new information that causes unanticipated trade volume.

(c)

(i) Calculate the HH liquidity index for the trade.

Commentary on Question:
Candidates had mixed performance on this question. Candidates made various minor mistakes in applying the formula.

$$(P_{max} - P_{min})/P_{min} = (10 - 7)/7 = 0.43$$
$$\frac{V}{N} * P_{avg} = (1/5)*8.6=43$$
HH=0.009966777

(ii) Assess the level of liquidity for this trade based on the HH liquidity index calculated above.

Commentary on Question:
Candidates performed very well in this question, with most candidates being able to identify the negative relationship between the HH index and liquidity.

The higher the HH index is, the lower the liquidity will be. Since an HH value of 0.009967 is much lower than 1 (average level of liquidity), this indicates very high liquidity.

3. Continued

(iii) Critique the use of HH liquidity index as a liquidity measure for this trade.

Commentary on Question:
Candidates performed poorly on this question. Very few candidates were able to identify the appropriate reasoning to critique the use of HH in this question.

The HH measure encompasses other volume-based liquidity measures that link trade volume to asset price but suffer from the shortcoming that a five-day period is long enough to allow for stock prices to adjust to illiquidity problem, as is in this case where the price return to the original level after 5 days. Therefore, this is not an appropriate liquidity measure for this trade.

(d)

(i) Calculate the probability that the company's portfolio will experience a loss at the end of the year.

Commentary on Question:
Candidates performs quite well in the question, with most of the candidates were able to obtain the correct answer to this question.

$\Pr[L > 0] = \Pr[S < S_0] = \Phi\left[\frac{\log(1)-\mu}{\sigma}\right] = 0.3085.$

(ii) Calculate the economic capital according to the company's risk appetite.

Commentary on Question:
Candidates did just ok on this question. Some candidates knew that the normal quantile needs to be used, but failed to apply the right formula.

$$\Pr[L < Q_{0.95}] = 0.95$$
$$\Pr[10{,}000{,}000(1-S) < Q_{0.95}] = 0.95$$
$$\Pr\left[S > 1 - \frac{Q_{0.95}}{10{,}000{,}000}\right] = 0.95$$
$$\Phi[\frac{log\left(1 - \frac{Q_{0.95}}{10{,}000{,}000}\right) - \mu}{\sigma}] = 0.05$$
$$\frac{log\left(1 - \frac{Q_{0.95}}{10{,}000{,}000}\right) - \mu}{\sigma} = -1.645$$

$$Q_{0.95} = 2{,}046{,}481.33$$

3. Continued

(e)

(i) Calculate the 95% expected shortfall of the loss in portfolio value.

Commentary on Question:
Very few students were able to identify the right formula to apply for solving this question.

$$ES_\alpha = 10{,}000{,}000\left\{1 - \frac{e^{\mu+\frac{\sigma^2}{2}}}{1-\alpha}\Phi(z_{1-\alpha} - \sigma)\right\}$$

$$ES_\alpha = 2{,}664{,}636.95.$$

(ii) Critique the CFO's suggestion.

Commentary on Question:
Candidates did just ok on this question. Many candidates were not able to clearly explain the difference between VaR and ES.

The CRO's concern is correct, the VaR risk measure is the 'worst case' loss, where worst case is defined as the event with probability 5%. If this is sufficient, then VaR can be an adequate risk measure. But since VaR does not take into consideration the loss distribution above the quantile. The Expected Shortfall can be a better risk measure to capture the tail risk of the loss as it takes the average loss, given that the loss lies in the right tail of the distribution.

4. Learning Objectives:

2. The candidate will understand and be able to apply different approaches to measuring and assessing risk exposures.

Learning Outcomes:

(2c) Analyze and evaluate the use and misuse of correlation, integrated risk distributions and copulas.

(2f) Evaluate a company's or a portfolio's exposures to various risks, including liquidity risk.

Sources:

QFII-104-14: Correlation: Pitfalls and Alternatives

Quantitative Enterprise Risk Management, Mary Hardy and David Saunders, 2022, Ch. 6: Copulas

Commentary on Question:

Candidates typically did well on parts (a) through (c) but struggled on the remaining two parts.

Solution:

(a) Describe two advantages of using copulas.

Commentary on Question:

Most candidates received full credit on this part by providing two advantages with some explanation.

Possible answers include:

- Copulas are useful for modeling dependent risks
- Copulas can be used to create multivariate distributions when the marginal distributions are not in the same family. In risk management this bottom-up capability can be very useful. It allows us to use copulas to bring together the risks from different parts of the firm, to generate a firm-wide model.
- A copula may be used to bring together the individual loss distributions, to create a model of the joint distribution of aggregate losses from all lines of business across the enterprise.

(b) Describe four drawbacks of using correlation as a measure of the relationship among risks.

Commentary on Question:

Most candidates provided at least four drawbacks to using correlation.

4. Continued

Drawbacks include:

Correlation is a scalar measure of dependency
Possible values depend on the marginal distributions
Perfectly positive dependent risks don't necessarily have a correlation of 1
A correlation of zero does not indicate independence of risks
Correlation is not invariant under transformations of risks
Correlation is only defined when variances of risks are finite

(c) A colleague is modeling joint risks and suggests using a multivariate normal distribution is appropriate when the marginal distributions are normally distributed.

Critique your colleague's suggestion.

Commentary on Question:
To receive full credit on this part, candidates had to correctly identify the weakness in the colleague's suggestion.

Marginal normal distributions do not necessarily imply that the joint distribution has a multivariate normal distribution.

(d) Describe two tests to determine if the multivariate normal distribution is appropriate.

Commentary on Question:
Few candidates were able to identify two tests and describe them in enough detail to receive full credit for this part.

Use the Jarque-Bera Test to determine if each marginal distribution is normally distributed. Each marginal must be tested. Test statistic involves skewness and kurtosis and the test statistic has a chi-square distribution with 2 degrees of freedom.

Follow up using Mardia's test on joint multivariate samples. Test statistics based upon empirical multivariate measures of skewness and kurtosis. There are two test statistics, one follows a chi-square distribution and the other follows a standard normal distribution.

(e) List three properties of the generating function $\phi(u)$ of an Archimedean copula.

Commentary on Question:
Candidates struggled to identify three properties of the Archimedean copula.

4. Continued

Possible properties include:
$\phi(u)$ maps [0,1] to $[0, \infty]$;
$\phi(0) = \infty$ and $\phi(1) = 0$,
ϕ is continuous,
ϕ is strictly decreasing,
ϕ must be convex for bivariate distributions

5. Learning Objectives:

2. The candidate will understand and be able to apply different approaches to measuring and assessing risk exposures.

3. The candidate will understand and be able to apply the components of an effective risk management system to investment management.

Learning Outcomes:

(2a) Explain the advantages and limitations of different risk metrics.

(2b) Explain how different approaches and tests form a set of complementary investment risk metrics.

(2d) Explain the difference between real-world and risk-neutral processes and select appropriate market risk models.

(2g) Apply different techniques of assessing tail risks, including stress testing and scenario analysis.

(3b) Identify and describe various approaches for managing portfolio risks including VaR/ES methods, position limits, etc.

Sources:

Quantitative Enterprise Risk Management, Mary Hardy and David Saunders, 2022, Ch. 8: Market Risk Models

Commentary on Question:

This question aims to test candidates' understanding on the advantages and limitations of various risk metrics. To receive maximum points, candidates must not only demonstrate the ability to select the appropriate market risk models, but also provide explanation as to how different approaches form a set of complementary investment metrics. Overall, candidates did well in question (a), but most struggled to receive full credit in question (b) and (c).

Solution:

(a) Explain which models above are appropriate for short-term risk assessment and which are appropriate for long-term risk assessment.

Commentary on Question:

The candidate performed well on this question overall. Most were able to correctly identify that the ILN model is suitable for short-term applications, the RSLN model is better for longer-term horizons, and the GARCH model works for both. However, many candidates did not provide sufficient explanation to justify these choices, resulting in partial credit.

5. Continued

The ILN model is typically used for very short-term applications with higher frequency time steps.

- It is similar to geometric Brownian motion (GBM) and forms the basis for the Black-Scholes formula. It provides a good fit for stock data over short horizons, assuming no major market events occur.
- ILN is scalable, converging to GBM as time steps increase, offering tractability. However, it doesn't fit well for longer time periods and fails to capture volatility clustering or extreme market disruptions, making it unsuitable for long-term risk assessment.

The RSLN model needs more data for an adequate fit, and would be used for longer horizons.

- RSLN assumes that the stochastic log-return process randomly switches between K different underlying processes, each with different parameters. Each process represents a different regime for the state of the economy. The model can be used for continuous or discrete time processes.
- Regime switching lognormal models with 2 or 3 regimes have proved quite robust for fitting stock prices over longer time periods, and are also relatively tractable. Generally, more frequent data requires more regimes.
- Like GARCH model, the RSLN model allows sudden jumps in volatility. A major difference between the models lies in the subsequent behavior. In the GARCH model, the volatility will trend back to a lower value over some period. Under the RSLN model, the volatility switches suddenly from high to low. Goodness of fit tests for long-run returns have been found to favor the regime switching framework, because the swift change from low to high volatility, and from high to low volatility in the RS model is more consistent with markets, where sudden upward jumps in volatility are common. Therefore, RSLN is a good fit for long term risk assessment.

The GARCH model is flexible, and is used in a wide range of short and longer term settings.

- GARCH is a part of a family of discrete time models with time-varying volatility.
- Unlike ILN, where stock returns/log-returns are independent and identically distributed normal random variables, the GARCH model incorporates stochastic volatility and volatility clustering, through the dependence on stock returns at a given time t.
- Even with fixed parameters, GARCH can take a wide range of paths, and are therefore more flexible in both short- and long-term risk assessment.

5. Continued

(b) Describe practical considerations in deciding which model to use.

Commentary on Question:

Candidates performed poorly on this question, primarily because they did not effectively connect the practical considerations to the overall context, where the manager asked for a recommendation on the best model to evaluate equity risk using the expected shortfall measure. However, most candidates earned partial credit for discussing general factors to consider when choosing a model.

The GARCH and RSLN models are fatter tailed than the ILN model, and generate serial correlation in the log-returns and in the volatility. If this is not critical to the model results, then using the ILN model may be adequate.

When calculating a VaR risk measure, it may not be necessary or worthwhile to ensure that a model provides a good fit in the extremes of the distribution, beyond the relevant α-quantile. On the other hand, the Expected Shortfall risk measure takes the full tail of the loss distribution into consideration, so it is more important to fit a fat-tailed distribution to fat-tailed data.

(c) Recommend one of these models using both of Akaike Information Criterion and Bayes Information Criterion, based on:

(i) daily data

(ii) monthly data

Commentary on Question:

The candidate performed poorly on this question, as most were unable to correctly calculate the highest AIC or BIC. However, many earned partial credit for demonstrating the correct reasoning in selecting the appropriate model, assuming they had calculated the AIC and BIC correctly.

Akaike Information Criteria (AIC) or Bayes Information Criterion (BIC) should be used to choose the model with the highest AIC or BIC. AIC (= ll – k) and BIC = ll – (k log(n))/2, where ll is the maximum log-likelihood, k is the number of parameters, and n is the number of data points in the sample.

5. Continued

Model	Maximum Log-likelihood (Monthly)	Maximum Log-likelihood (Daily)	Number of Parameters	Daily AIC	Daily BIC	Monthly AIC	Monthly BIC
ILN	595	1335	2	1,333	1,332	593	592
GARCH(1,1)	611	1609	4	**1,605**	**1,604**	607	606
RSLN	619	1579	6	1,573	1,571	**613**	**611**

ILN fit is poor for both data sets. Therefore, it is not recommended.

Daily, recommend: GARCH

The GARCH model provides a much better overall fit for the daily data, using both AIC and BIC, compared to the other two models.

Monthly, recommend: RSLN
Using AIC and BIC criteria, RSLN model provides a slightly better fit for monthly data. The monthly data is more relevant to your company's assessment on stock monthly return.

6. Learning Objectives:

3. The candidate will understand and be able to apply the components of an effective risk management system to investment management.

Learning Outcomes:

(3a) Explain the best practices of investment risk management.

Sources:

QFII-120-20: IAA Note on ERM for Capital and Solvency Purposes in the Insurance Industry; The Top Ten Operational Risks: A Survival Guide for Investment Management Firms and Hedge Funds.

Commentary on Question:

Commentary listed underneath question component.

Solution:

(a) Identify the two key features according to the IAIS standard that relate to your company's ERM framework.

Commentary on Question:

Candidates performed poorly on this question. Most candidates were not able to identify and write down the items relevant to the ERM framework.

Key Feature 1: A sound ERM framework should:

- be appropriate to the nature, scale, and complexity of the business and its risks.
- be integrated with business operations, reflecting culture and behavioral expectations and addressing all reasonably foreseeable and relevant material risks.
- led and overseen by board and senior management.
- include provision for the quantification of risk for a sufficiently wide range of outcomes.

Key Feature 4: ERM framework should be responsive to change. It should incorporate the feedback loop and enable insurer to take actions in a timely manner.

(b) Recommend three changes to the ERM framework for your company.

Commentary on Question:

Candidates performed very well on this question. Full marks were awarded if candidates have recommended changes specific to the ERM framework with supporting arguments.

6. Continued

The ERM framework from five years ago is likely outdated given changing market conditions and emerging risks. The framework should be reviewed and updated at least every 1–2 years.

The current ERM framework likely addresses some risks only applicable to large multinational insurers and assumes a greater separation and formalization of different functions. Given our company is mid-sized, the framework should be tailored to account for risks and structures specific to us.

The CRO position should be reinstated as it ensures the independence of risk management functions and alignment with regulatory best practices.

(c) Your team provides three suggestions to improve the ERM framework:

- The framework's risk language should utilize terms already used within the organization to promote a smooth integration.
- The internal audit team should lead the development of the risk management framework because of the skill set match between auditors and those needed to implement ERM.
- The company operates on a conservative basis as compared with peers. Risk tolerances should therefore be set more aggressively than company practice in the ERM framework. This will ease adoption and promote a strong risk culture.

Critique these suggestions.

Commentary on Question:
Candidates performed as expected on this question. Most were able to discuss some strengths and weaknesses of the suggestions. Partial credit was given to responses with reasonable justifications.

(i) While leveraging familiar terminology can ease adoption, inconsistent or ambiguous interpretations of risk terms can undermine the ERM framework's effectiveness. This can lead to confusion, reinforce silos, and weaken management buy-in. Establishing a unified and consistent risk language across the organization is essential to ensure clarity and promote integration.

6. Continued

(ii) The internal audit team possesses valuable skills that may support initial ERM implementation. However, tasking them with leading ERM development could compromise the independence of the risk function and position ERM as merely a compliance or assurance exercise. Clearly defining the distinct roles of internal audit and risk management is critical to creating a sustainable and effective ERM framework in the long term.

(iii) The risk tolerance policy must be developed according to each company's own circumstances and decides by the board. It should not be based on peers nor the opinions of external parties. Adopting a more aggressive risk tolerance could conflict with the company's historically conservative culture, leading to misalignment in risk appetite and operational practices, which may ultimately hinder adoption and the development of a strong risk culture.

7. Learning Objectives:

3. The candidate will understand and be able to apply the components of an effective risk management system to investment management.

Learning Outcomes:

(3c) Explain and manage model risk.

Sources:

Quantitative Enterprise Risk Management, Mary Hardy and David Saunders, 2022, Ch. 14: Model Risk and Governance

QFI-119-19 Chapter 3 of The Known, the Unknown

Commentary on Question:

This question assessed candidates' understanding of model risk as it relates to parameter estimation and choice of model. Candidates generally understood risks inherent in parameter estimation, advantages and disadvantages of MLE vs. Bayesian Methods, and considerations in model choice between ILN and RSLN, but struggled to complete the Excel component requiring simulation of stock values based on provided parameters and simulated standard normal variates.

Solution:

(a) Identify three parameter estimation risks.

Commentary on Question:

Candidates did not perform as well as expected on this question. To get maximum points, candidates needed to provide answers relevant or specific to parameter estimation risks.

- Lack of quality data around key events, e.g., tail events
- Inadequate amount of historical data, rendering estimates highly uncertain
- Structural changes due to such things as government and regulatory policy may render parameters irrelevant or misleading
- Choice of distribution may be biased, for example the prior distribution in Bayesian MCMC methods may be mis-specified.

(b) Describe the advantages and disadvantages of Bayesian MCMC and MLE for parameter estimation.

Commentary on Question:

Candidates generally did well in this question. However, very few candidates were able to recall the finer aspects of Bayesian MCMC and MLE.

7. Continued

MLE
Advantages
- Easier to compute, has an analytical/tractable solution; easier to communicate.
- Uncertainty in MLE parameters can be quantified easily using standard error, as MLE estimates are typically asymptotically normally distributed.

Disadvantages
- Only provides a point estimate.
- For small sample sizes or where parameters are close to the boundary of the parameter space, asymptotic properties may be quite inaccurate.

Bayesian MCMC
Advantages
- Provides a direct reflection and representation of posterior parameter uncertainty in underlying distributions (i.e., get a distribution of parameters instead of a point estimate). Therefore, it's a richer framework.

Disadvantages
- Requires empirical methods and simulation to quantify parameter uncertainty, as parameters are treated as random values and the resutling distributions are usually not analytically tractable. Therefore, computationally intensive and can be hard to interpret.
- Using an inappropriate prior distribution will generate inappropriate results.

(c)

(iii) Calculate the expected value and standard deviation of the stock value at T=5, using the five simulations and MLE parameters.

(iv) Calculate the expected value and standard deviation of the stock value at T=5, using the five simulations and Bayesian MCMC parameters.

(v) Assess the impact of simulating results using Bayesian MCMC versus with MLE-estimated parameters.

Commentary on Question:
Candidates struggled with this question. Common errors included mixing and matching variance and standard deviation, averaging parameters and standard normal variates, formula errors and/or only attempting one or a few scenarios. To get full points, candidates had to show their work across all simulations.

7. Continued

(i)

Detailed answer in accompanying excel workbook.

1. At time t for parameter:
   For each simulation *sim*:
   Compute $\sigma^2(sim,t)$ recursively as:
   $\alpha 0 + \alpha 1 \times (Y(sim,t-1) - \mu)^2 + b\sigma^2(sim,t)$

   Compute $Y(sim,t) = \mu + \sigma(sim,t) \times Z(sim,t)$
   where $Z(sim,t)$ is the corresponding standard normal variate.

2. At time t, compute S(sim, t) as:
   $S(sim,t) = S(sim, t-1) \times EXP(Y(sim,t-1))$

   $S(1,1) = S(0) \times EXP(Y0)$
   $S(1,2) = S(1,1) \times EXP(Y(1,1))$
   where $Y(sim, t) = \mu + \sigma(sim,t) \times Z(sim,t)$

   $S(1,5) = S(1,4) \times EXP(Y(1,4))$
   Alternatively, $S(1,5) = S(0) \times EXP(Y(1,1)+Y(1,2)\ldots+Y(1,4))$

3. Repeat across all 5 simulations.
4. Take average and standard deviation of S(5) across simulations.

   Expected Value = 105.8002
   Standard Deviation = 22.1735

(ii)

1. At time t for parameter:
   For each simulation *sim*:
   Compute $\sigma^2(sim,t)$ recursively as:
   $\alpha 0(sim) + \alpha 1(sim) \times (Y(sim,t-1) - \mu)^2 + b\sigma^2(sim,t)$

   Compute $Y(sim,t) = \mu + \sigma(sim,t) \times Z(sim,t)$
   where $Z(sim,t)$ is the corresponding standard normal variate.

2. At time t, compute S(sim, t) as:
   $S(sim,t) = S(sim, t-1) \times EXP(Y(sim,t-1))$

   $S(1,1) = S(0) \times EXP(Y0)$
   $S(1,2) = S(1,1) \times EXP(Y(1,1))$
   where $Y(sim, t) = \mu + \sigma(sim,t) \times Z(sim,t)$

7. Continued

$S(1,5) = S(1,4) \times EXP(Y(1,4))$
Alternatively, $S(1,5) = S(0) \times EXP(Y(1,1)+Y(1,2)\ldots+Y(1,4))$

3. Repeat across all 5 simulations.
4. Take average and standard deviation of S(5) across simulations.

Expected Value = 106.2285
Standard Deviation = 22.7147

(iii)

- Bayesian MCMC parameter sets results in a similar overall stock price and slightly higher standard deviation.
- Differences in this case do not demonstrate a large impact of parameter uncertainty.

(d) Your colleague states that the ILN has less model and parameter uncertainty and thus we should choose the ILN over the RSLN-2.

Critique your colleague's statement.

Commentary on Question:
Candidates performed fairly well in this question. They generally understood that the RSLN model was more suited for the exercise than the ILN model. However, many did not explicitly call out that the statement was inaccurate, or did not provide correct reasoning for why the statement was inaccurate.

- The statement is not accurate.
- While the standard error might indicate that the ILN is more stable, it only measures idiosyncratic risk and not model accuracy.
- The ILN model is thinner-tailed and is likely to understate risk exposure in the tail as measured by Expected Shortfall/VAR. This would contribute to the lower standard error calculated.
- If there were structural changes in the economy, the Regime Switching model may be better suited to capturing that given the ILN model would use one set of parameters across both regimes.

8. Learning Objectives:

4. The candidate will understand and be able to apply different approaches to mitigate investment risks using derivatives.

Learning Outcomes:

(4a) Explain and implement techniques used to mitigate market risks.

(4b) Understand interest rate derivatives and use them to mitigate interest rate risk.

Sources:

Quantitative Enterprise Risk Management, Mary Hardy and David Saunders, 2022, Ch. 15: Risk mitigation using options and derivatives

Commentary on Question:

This question intends to test the candidate's understanding of different hedging options with both qualitative and quantitate assessments. Detail comments can be found below for each part.

Solution:

(a)

(i) Describe each of the three hedging options above.

Commentary on Question:

Most candidates did not describe the details of Hedging with option combinations

(i)

Delta-neutral: Construct a portfolio with stock and options on the underlying to have zero delta

Delta-gamma-neutral hedging: Construct a portfolio with stock and options on the underlying to limit both delta and gamma value to zero

Hedging with option combinations:

- Construct a portfolio of options on the underlying
- Gives up some return to protect against tail loss due to equity prices drop

(ii) Assess the suitability of each option, in light of the risk committee's considerations.

Commentary on Question:

Candidates need to assess both difficulty of rebalancing and risk-return trade-off to get full points. Most candidates considered only 1 area against risk committee's requirement hence earned partial points.

8. Continued

(ii)

Delta-neutral hedging is not suitable

- Removes the delta and does not prioritize the risk-return trade-off
- Requires constant rebalancing to keep zero delta
- Not sustainable for the small team

Delta-gamma-neutral hedging is not suitable

- Same shortfall as delta-neutral hedge
- Extra difficulty in rebalancing to keep zero gamma
- Further lowers the risk and drives down the return significantly

Hedging with option combinations is suitable

- Does not require frequent rebalancing and can be efficiently managed by the small investment team
- Can be constructed to provide a balance between risk-return trade-off to meet risk committee's requirement

(b)

(i) Verify that the portfolio value at time T=0 was $5,000.

(ii) Verify that the portfolio is delta-neutral at time T=0.

Commentary on Question:
Most candidates earned full points on this part

	Time = 0
Investment	5,000
Unit of stock	202.8
Unit of put	624.3
Stock price (S_0)	20
Strike price (K)	19
Option term (T)	1.00
Risk free rate (r)	0.04
Volatility (σ)	0.3
N(-d1)	0.3248
N(-d2)	0.4387
Trading days per year	250
$z_{99\%}$	2.3263

8. Continued

(i)

put option price: $p_0 = K * \exp(-r*T) * N(-d2) - S_0 * N(-d1) = 1.5121$

portfolio value: $S_0 * 202.8 + p_0 * 624.3 = 5{,}000$

(ii)

delta of put: $-N(-d1) = -0.3248$

portfolio delta: $202.8 - 624.3 * N(-d1) = 0$

(c) Calculate the return of the portfolio with and without delta-neutral hedging.

Commentary on Question:
For the candidates attempted this part, most of them were able to calculate the return for the portfolio without hedging. About half of the candidates were able to calculate the put option price at t=0.05 and the return for the hedging portoflio. For the candidates calculated the wrong d1 at t=0.05, they were not penalized for this again if the following calculations were correct based on the d1 they had.

Without hedging at t = 0.05, S = 22

- Units of stock = investment_0 / S_0 = 5000 / 20 = 250
- Return = 250 * 22 / 5,000 = 500 or 10%

With hedging at t = 0.05, S = 22

- $d1 = (\ln(S / K) + (r + (\sigma^2) / 2) * (T - t) / (\sigma * \sqrt{T - t})) = 0.7775$
- $d2 = d1 - \sigma * \sqrt{T - t} = 0.485$
- $N(-d1) = 0.2184$ and $N(-d2) = 0.3138$
- $p = K * \exp(-r * (T - t)) * N(-d2) - S * N(-d1) = 0.9345$
- Portfolio Value = $S * 202.8 + p * 624.3 = 5{,}045$
- Return = 5,045 / 5,000 = 45 or 0.90%

(d) Calculate the 1-day 99% VaR for the portfolio with and without delta-neutral hedging, as a percentage of the portfolio value, using the delta-normal method.

Commentary on Question:
Most candidates did not do well or did not attempt this part. For the candidates attempted this part, some of them were able to calculate the portfolio deltas but did not apply the correct formula for 1-day VaR. For the Candidates correctly calculated the 1-day VaR, most of them did not express the VaR in percentage of the portfolio value as requested in the question.

8. Continued

Without hedging at t = 0.05, S = 22

- Delta = 250 * 22 = 5,500
- 1-day VaR_99% = delta * z_99% * sqrt(1/250) * σ = 242.7644
- As % of portfolio value = 242.7644 / 5,500= 4.41%

With hedging at t = 0.05, S = 22

- Delta = *202.8 – 624.3 * N(-d1) = 66.4389*
- *1-day VaR_99%* = delta * z_99% * sqrt(1/250) * σ = 64.5166
- As % of portfolio value = 64.5166 / 5,045 = 1.28%

9. Learning Objectives:

5. The candidate will understand the importance of risk culture and governance.

Learning Outcomes:

(5b) Identify sources of unethical conduct and explain the role of a fiduciary.

(5c) Compare the interests of key stakeholders and describe governance mechanisms that attempt to address conflicts.

(5d) Explain how governance may be structured to gain competitive advantages and efficiencies.

Sources:

Investment Ethics Chapter 2

Financial Enterprise Risk Management, Sweeting, Chapter 20 Case Studies

Commentary on Question:

Candidates performed reasonably well on this question. The question tests candidates' knowledge of various topics:

- *The role of a fiduciary and sources of unethical conducts*
- *Governance structure that may improve team performance as well as*
- *Sources of unethical or misleading conducts when demonstrating performance*

However, there were instances when the valid responses were written in other sub-questions.

Solution:

(a)

(i) Identify the principal-agent relationship.

(ii) Explain each party's incentive in this situation.

Commentary on Question:

Candidates performed very well on this part. Most candidates received full credits. Common credit deductions were because some answered that the pensioner as the principal; some missed the keyword "safe," i.e., they used generic terms such as investment strategy or maximized investment return. A few candidates described the roles and responsibilities but didn't identify the parties.

- The pension fund trustee is the principal, and the asset management firm is the agent.
- The pension fund trustee asked the asset management firm to manage the pension fund on their behalf. In return, the trustee pays the asset manager a certain fee.

9. Continued

- The pension fund trustee would want to grow the pension fund with **safe** investment strategy to benefit the pensioner, but the fund manager would want to grow the fund to benefit from higher fees by spending time doing investment research.

(b)

(i) Explain the potential unethical behavior in Bob's suggestion.

(ii) Critique Ron's proposal.

(iii) Recommend an action that Ron should take to avoid any unethical behavior.

(b) (i)

Most candidates received full credits. Candidates who performed well on this part when they know and understand or recall the material. Points around churning and soft dollars were made by well prepared candidates. Very few candidates explored the idea that generating additional commissions per se is not illegal.

- Bob is offering Ron the soft dollar to get the research materials for free because research produced by brokers can act as a bond that they will effectively execute trades. Broker can earn a return on the research only by attracting clients willing to place trades with them.
- Bob's proposal will generate additional commissions for the broker. It is fine to earn a commission but not at the client's expense. We are not sure if an active trading strategy is suitable for the pension.
- What Bob proposes may be unethical and is called churning -- when a broker encourages excessive buying and selling to earn commissions from the trades.

(b) (ii)

Candidates performed well on this part as it is straightforward application of investment principles. Only a very few candidates specifically called out both the points.

- Ron needs to carry out the fiduciary duties. The fact that Ron carves out a portion of the fund to test out the strategy could potential hurt the pensioner of the fund. He is testing the strategy using real money at the expense of the pension fund.
- The risk that Ron is taking could be substantial even though it's only 10% of the fund. The risk that Ron is taking may not be what the trustees are willing to accept.

9. Continued

(b) (iii)

Candidate performance was fair for this part, differentiating candidates' understanding, only a few candidates covered all the bullets and received full credits.

- Ron should disclose his testing strategy before engaging with Bob.
- He should disclose the estimated cost and risk associated with this new active trading strategy.
- He should disclose the value of the free research he can collect from Bob.
- Rob should investigate if Bob's pricing is competitive.

(c) Explain the pros and cons of increasing the weight towards the team's performance in the bonus structure.

Commentary on Question:
Candidates performed very well on this part. Most candidates received full credits as the answers are straightforward and follow common sense..

- Pros: Promote good team work and cooperative environment as everyone is working towards the same goal.
- Cons: If the bonuses were mainly based on team performance, it would cause the individual to not differentiate themself from the team. The individual will then just copy the rest of the team and will benefit in good times and get protection in bad times.

- It leads to less diversity of opinion and discourages individuals from identifying issues in the team.

(d) Explain the potential unethical behavior that Fund A's manager might be exhibiting.

Commentary on Question:
Most candidates performed very well on this part and received full credit.

- The fund A manager may intentionally hide the returns for some clients where the performance may not be as ideal. He may be subject to "cherry-picking" returns, which is unethical.

9. Continued

(e)

(i) Recommend two improvements to the average return calculation.

(ii) Describe two other factors Ron should consider before choosing the fund to invest in.

(e) (i)

Commentary on Question:
Candidates performed very well on the first bullet, but only a few candidates touched on the annual geometric return (the second bullet). Rather, a lot of candidates suggested asset-weight return, which is close to the correct answer, but not quite the same as annualized geometric return.

- When calculating return, we should consider the management fee.
- Arithmetic average return is deceiving because it doesn't tell us the true ending balance after the return. We should use the accumulated wealth factor or annual geometric return.

(e) (ii)

Commentary on Question:
Candidates performed fair on this question. Only a few candidates made both points.

- Ron should not only look at the absolute return of the two funds but also relative to the benchmark'
- Should evaluate the risk of the two funds, such as by calculating the risk-adjusted return.
- There may be potential leverage, short- selling, and low liquidity in the fund, so Ron should be aware of the potential risk that the fund is bearing.
- Other things to consider: stability of investment team, client inflows and outflows……

(f) Explain additional considerations in selecting a fund, in light of Fund A's benchmark change and Fund B's backfill of historical performance data.

Commentary on Question:
Candidates performed fair on this question. Whilst some flavor of second point was made by well-prepared candidates, the point around benchmark change in first point was often skipped.

9. Continued

- Changing the benchmark may imply a change in investment strategy or that the fund manager is trying to unethically illustrate better performance. The fact that Fund A outperforms the benchmark every year could potentially indicate a style shift in the investment strategy or showing better performance.
- Ron would need to investigate and compare the benchmark for Fund A before the replacement to see if Fund A is taking on more risk.
- Fund B backed fill the historical return data where Ron needs to be careful about data mining to show how close they track the benchmark. Ron would need to ask for details on how the return is being calculated to assess the risk.

(g) Explain what would cause Fund A to have a large interest expense.

Commentary on Question:
Candidates performed well on this question with several of them receiving full credit. The point around leveraging was made by well-prepared candidates.

- The large interest expense implies the fund is borrowing money to invest, which implies larger risk-taking behavior.
- Ron should ask Fund A manager to disclose his transactions and be clear to Fund A manager that leverage is not allowed in the pension fund.