GIRR Model Solutions
Fall 2024

1. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(3c) Identify the types of development triangles that can be used for investigative testing.
(3d) Analyze development triangles for investigative testing.
(3g) Estimate ultimate values using the methods cited in (3e).
(4f) Calculate claim liabilities.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 14, 20, and 24.

Commentary on Question:

This question tests the candidate's understanding of Berquist-Sherman adjustments when there has been a change in case adequacy.

Solution:

(a) Verify that the adequacy of case estimates has increased using two diagnostic tests.

Change in average case:

	Average Case			
Accident Year	12	24	36	48
2020	7,692	14,634	10,588	12,231
2021	7,699	16,098	12,706	
2022	8,688	19,357		
2023	10,425			

1. Continued

	Change in Average Case			
Accident Year	12	24	36	48
2020-2021	0.1%	10.0%	20.0%	
2021-2022	12.8%	20.2%		
2022-2023	20.0%			
Average:	11.0%	15.1%	20.0%	

Evidence of change in case adequacy would show up as a change in the latest diagonals significantly different than 5%, which it is.

Change in average reported claims:

	Average Reported Claims			
Accident Year	12	24	36	48
2020	6,667	9,091	9,706	10,021
2021	6,836	9,632	10,270	
2022	7,438	10,420		
2023	8,374			

	Change in Average Reported Claims			
Accident Year	12	24	36	48
2020-2021	2.5%	6.0%	5.8%	
2021-2022	8.8%	8.2%		
2022-2023	12.6%			
Average:	8.0%	7.1%	5.8%	

Evidence of change in case adequacy would show up as a change in the latest diagonals significantly different than 5%, which it is but not as significantly as change in average case estimates.

(b) Verify that a change in claim settlement pattern has not occurred using one diagnostic test.

Ratio of Closed Counts to Reported Counts:

AY	12	24	36	48
2020	0.567	0.876	0.950	***0.988***
2021	0.567	0.876	***0.950***	
2022	0.567	***0.876***		
2023	***0.567***			

1. Continued

If there has been a change in claim settlement in 2023, the latest diagonal should show noticeable increase or decrease in the ratios. There is no change in these ratios and therefore no evidence of a change in claim settlement pattern.

(c) Calculate the IBNR for accident year 2023 using the reported development method with a Berquist-Sherman adjustment.

Adjusted Average Case Estimates

Accident Year	12	24	36	48
2020	9,006	17,558	12,101	*12,231*
2021	9,456	18,436	*12,706*	
2022	9,929	*19,357*		
2023	*10,425*			

Open Counts

Accident Year	12	24	36	48
2020	650	205	85	20
2021	715	225	93	
2022	785	248		
2023	864			

Adjusted Reported Claims

Accident Year	12	24	36	48
2020	10,853,848	15,599,347	16,628,571	*17,236,120*
2021	12,536,194	18,008,028	*19,204,718*	
2022	14,474,286	*20,808,951*		
2023	*16,723,013*			

Age-to-Age Development Factors

Accident Year	12-24	24-36	36-48	48-Ultimate
2020	1.437	1.066	1.037	
2021	1.436	1.066		
2022	1.438			
Average	1.437	1.066	1.037	
Weighted Avg.	1.437	1.066	1.037	
Selected:	1.437	1.066	1.037	1.000
Age-to-ultimate	1.588	1.105	1.037	1.000

AY 2023 ultimate claims = 16,723,013×1.588 = 26,560,506
AY 2023 IBNR = 26,560,506 – 16,723,013 = 9,837,493

2. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3f) Demonstrate knowledge of good practice related to projecting ultimate values.

(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 15 and 17.

Commentary on Question:

This question tests the estimation of ultimate claims using the expected method.

Solution:

(a) Describe one advantage of using the pure premium approach rather than the claim ratio approach when using the expected method.

Any one of the following is acceptable:

- No adjustment is required for premium rate changes.
- It may be possible to select a pure premium exposure base that is a leading indicator of claims experience.
- It may be possible to choose an exposure base that requires no adjustments.

(b) Provide one reason why the expected method might be preferred over the development method in this scenario for analyzing accident year 2023 claims.

The age-to-ultimate factor at 12 months is very large (or highly leveraged) implying a very immature accident year. The expected method is better for immature years.

2. Continued

(c) Calculate the expected claims for accident year 2023 using the expected method with the following approaches:

(i) Claim ratio

(ii) Pure premium

Accident Year	Project Ultimate Claims from Paid Development Method	Claim Ratio Trend Factor	Trended On-Level Claim Ratios	Trended Pure Premiums
2017	39,794,820	1.194	85.39%	601.90
2018	38,874,654	1.159	81.68%	575.94
2019	40,100,870	1.126	82.42%	580.87
2020	42,901,092	1.093	88.20%	621.93
2021	42,491,743	1.061	80.60%	579.88
2022	43,661,907	1.030	77.78%	588.86
2023	46,092,453	1.000		
		Average:	82.68%	591.56

	(i)	(ii)
Expected claims for accident year 2023:	46,988,824	44,408,679

(d) Estimate accident year 2023 claims expected to be paid between December 31, 2023 and December 31, 2024 using your results from part (c)(ii).

% Paid @ 12 months:	15.7%
% Paid @ 24 months:	26.1%
% Paid between 12 to 24 months:	10.4%
AY 2023 expected paid between 12 to 24 months:	4,619,299

3. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(1l) Understand credibility as used for actuarial work.
(5d) Choose trend rates for claims (frequency, severity, and pure premium) and exposures.
(5e) Calculate trend factors for claims and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 6 and 31.

Commentary on Question:

This question tests the candidate's understanding of loadings for large claims by analyzing claims at various limits.

Solution:

(a) Identify two other considerations in assigning credibility to an experience set of data.

Commentary on Question:

This question is about assigning the credibility and not about what is considered for the complement of credibility.

Any two of the following are acceptable:

- The number of years of claim data underlying the experience
- The stability or variability observed in claims from year to year
- The presence or absence of large or unusual claims
- Changes in the internal or external environment
- The age, relevance, and reliability of the experience
- The age, relevance, and reliability of other data to which the complement of credibility would be applied

(b) Calculate the loadings for 500,000 to total limits for each accident year.

Severity trend for 1,000,000 limit = 7.0%×0.70 + 6.0%×0.30 = 6.7%
Severity trend for total limit = 8.6%×0.50 + 7.0%×0.50 = 7.8%

3. Continued

Accident Year	Trend Period	Severity Trend at: 6.7%	7.8%	Trended Claims at 1,000,000 Limit	Total Limit
2021	4.667	1.353	1.420	5,817,559	6,365,155
2022	3.667	1.268	1.317	5,541,683	6,068,833
2023	2.667	1.189	1.222	5,813,421	6,228,374

Accident Year	Loading for 1,000,000 to Total Limit	Loadings for 500,000 to 1,000,000 Limit	Loadings for 500,000 to Total Limit
2021	1.094	1.196	1.309
2022	1.095	1.165	1.276
2023	1.071	1.185	1.270

(c) Recommend a loading for 500,000 to total limits for ratemaking purposes. Justify your recommendation.

Average of 2022 and 2023 = 1.273
Justification:

- Accident year 2021 loading is much higher than 2022 & 2023
- Therefore, use most recent 2 years as it is more stable, and it uses the most recent data.

4. Learning Objectives:

1. The candidate will understand the key considerations for and key concepts underlying general insurance actuarial work.

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

Learning Outcomes:

(1d) Understand the components of ultimate values.
(2a) Create development triangles of claims and counts from detailed claim transaction data.
(2c) Calculate written, earned, in-force and unearned premiums for portfolios of policies with various policy terms and earnings patterns.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 3, 11, and 12.

Commentary on Question:

This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to calculate reported claim ratios and IBNR.

Solution:

(a) Verify the earned premiums for calendar years 2021, 2022, and 2023.

Written Date	Written Premium	Months Earned in Calendar Year			Earned Premium in Calendar Year		
		2021	2022	2023	2021	2022	2023
1-Apr-21	120	9	12	3	45	60	15
1-May-21	120	8	12	4	40	60	20
1-Jun-21	120	7	12	5	35	60	25
1-Jul-21	120	6	12	6	30	60	30
1-Aug-21	120	5	12	7	25	60	35
1-Sep-21	120	4	12	8	20	60	40
1-Oct-21	120	3	12	9	15	60	45
1-Nov-21	120	2	12	10	10	60	50
1-Dec-21	120	1	12	11	5	60	55
1-Jan-22	120	0	12	12	0	60	60
1-Feb-22	120	0	11	12	0	55	60
1-Mar-22	120	0	10	12	0	50	60
1-Apr-22	120	0	9	12	0	45	60
1-May-22	120	0	8	12	0	40	60
1-Jun-22	120	0	7	12	0	35	60

Written Date	Written Premium	Months Earned in Calendar Year 2021	2022	2023	Earned Premium in Calendar Year 2021	2022	2023
1-Jul-22	120	0	6	12	0	30	60
1-Aug-22	120	0	5	12	0	25	60
1-Sep-22	120	0	4	12	0	20	60
1-Oct-22	120	0	3	12	0	15	60
1-Nov-22	120	0	2	12	0	10	60
1-Dec-22	120	0	1	12	0	5	60
1-Jan-23	120	0	0	12	0	0	60
1-Feb-23	120	0	0	11	0	0	55
1-Mar-23	120	0	0	10	0	0	50
1-Apr-23	120	0	0	9	0	0	45
1-May-23	120	0	0	8	0	0	40
1-Jun-23	120	0	0	7	0	0	35
1-Jul-23	120	0	0	6	0	0	30
1-Aug-23	120	0	0	5	0	0	25
1-Sep-23	120	0	0	4	0	0	20
1-Oct-23	120	0	0	3	0	0	15
1-Nov-23	120	0	0	2	0	0	10
1-Dec-23	120	0	0	1	0	0	5
1-Jan-24	120	0	0	0	0	0	0
1-Feb-24	120	0	0	0	0	0	0
1-Mar-24	120	0	0	0	0	0	0
Total					225	930	1,425

(b) Calculate the unearned premiums as of each year-end for 2021, 2022, and 2023.

	31-Dec-2021	31-Dec-2022	31-Dec-2023
Earned Premiums:	225	930	1,425
Written premiums:	1,080	1,440	1,440
Unearned premiums	855	1,365	1,380

e.g., 1,365 = 1,440 – 930 + 855

(c) Calculate in-force premiums as of December 31, 2023.

There are 24 policies in-force as of December 31, 2023. (#10 through 33)

In-force premiums = 24×120 = 2,880

4. Continued

(d) Describe a scenario where the market analyst's conclusion would be incorrect.

Either of the following is acceptable:

- DEF writes the same volume of written premiums as ABC but annual term policies.
- DEF writes the same volume of written premiums as ABC with 2-year term policies but books annually.

(e) Calculate the reported claim ratios for each of calendar years 2022 and 2023.

Reported claims for CY2022:	319
Reported claims for CY2023:	622

Claim ratio for CY2022:	34.3%
Claim ratio for CY2023:	43.6%

(f) Calculate IBNR for accident years 2022 and 2023.

Ultimate claim ratio for AY2021 = 135/225 = 60%

Accident Year	Earned Premiums	Ultimate Claims	IBNR
2022	930	558	112
2022	1,425	855	427

e.g., $558 = 0.6 \times 930$; $112 = 558 - 446$

5. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6i) Describe the claim ratio and pure premium methods of ratemaking.

(6j) Calculate indicated rates and indicated rate changes using the claim ratio and pure premium methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 32.

Commentary on Question:

This question tests the candidate's ability to calculate the indicated average rate and the differences between the claim ratio and pure premium approaches to ratemaking.

Solution:

(a) Demonstrate that the indicated rate change using the pure premium approach is similar to that using the claim ratio approach (i.e., ±0.5% of 5.91%).

Accident Year	Earned Exposures	Trended Ultimate Claims	Pure Premium
2019	18,640	10,866,820	582.98
2020	18,240	9,735,481	533.74
2021	17,061	9,235,310	541.31
2022	17,992	9,763,870	542.68
2023	17,931	10,191,450	568.37

Average pure premium	553.82
Ratio of ULAE to claims	8.00%
Fixed expenses per exposure = 0.075×13,878,594/17,931 =	58.05
Indicated rate = (553.82×1.08 + 58.05) / (1 – 0.15 – 0.05) =	820.22

2023 trended earned premiums at current rate level	13,878,594
2023 earned exposures	17,931
Current average rate = 13,878,594 / 17,931 =	774.00
Indicated Rate Change = 820.22 / 774.00 – 1 =	5.97%

This is withing 0.5% of 5.91%.

5. Continued

(b) Describe one such reason.

The premium adjustment factors for trend and on-level factors are both approximations used to restate historical earned premiums as if they were at the current rate level and mix of exposures for the forecast period.

(c) Calculate the profit and contingencies ratio implied by increasing the rates by 2%.

First, solve for CR: $(CR + F/R_c) / (1 – V – Q) – 1 = 5.91\%$
$CR = 77.23\%$
Solve for Q, where $(CR + F/R_c) / (1 – V – Q) – 1 = 2\%$
$Q = 1 – V – (CR + F/R_c) / (1.02) = 1.93\%$

(d) Explain how implementing a lower rate change than indicated will result in higher rate indications for the next rate review using the claim ratio approach.

Implementing a lower rate increase than indicated would mean charging lower premiums than needed to achieve the required profit. This will lead to higher claim ratios which will lead to higher rate indications for the next review than would have been had the full rate change been implemented.

6. Learning Objectives:

6. The candidate will understand how to apply the fundamental ratemaking techniques of general insurance.

Learning Outcomes:

(6d) Quantify different types of expenses required for ratemaking including expense trending procedures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 30.

Commentary on Question:

This question tests the candidate's understanding of expenses used for ratemaking.

Solution:

(a) Calculate the total variable expense ratio for each of calendar years 2019 to 2023.

	(1)	(2)	(3)	(4) = (2) + (3)
	General Expenses			
Calendar Year	Variable	As a % of Premiums	Commission and Premium Tax Expense Ratio	Total Variable Expense Ratio
2019	870,000	4.58%	13.0%	17.58%
2020	852,000	4.55%	13.0%	17.55%
2021	864,000	4.74%	12.5%	17.24%
2022	852,000	4.80%	12.0%	16.80%
2023	834,000	4.78%	12.0%	16.78%

Notes: (1) = 60%×(General Expenses)
(2) = (1) / (Direct Earned Premium)
(3) = (Total Commission Expenses and Premium Taxes) / (Direct Written Premium)

6. Continued

(b) Recommend the total variable expense ratio to use in ratemaking. Justify your recommendation.

Calendar Year	Variable General Expense Ratio	Commission and Premium Tax Expense Ratio	Total Variable Expense Ratio
2019	4.58%	13.00%	17.58%
2020	4.55%	13.00%	17.55%
2021	4.74%	12.50%	17.24%
2022	4.80%	12.00%	16.80%
2023	4.78%	12.00%	16.78%
Average	4.69%	12.50%	17.19%
Selection:	4.77%	12.00%	16.77%

Justification:

- Latest 3 years average for variable general expense ratio due to the increase over the last 3 years.
- Select 12% for commission and premium tax expense ratio, as there has been a change to these ratios.

6. Continued

(c) Recommend the fixed expense per exposure to use in ratemaking. Justify your recommendation.

Fixed expenses are incurred at the time each policy is written.
Therefore, need to trend from the average written date in each calendar year to the average written date in the future rating period.
Average written date in calendar year 2023: July 1, 2023
Average written date in future rating period: January 1, 2026
Therefore, trend period for 2023: 2.5 years

	(4)	(5)	(6)	(7)	(8)
Calendar Year	Fixed General Expense	Fixed General Expense Per Exposure	Trending Period	Fixed Expense Trend Factor	Trended Fixed Expenses
2019	580,000	22.48	6.5	1.13737	25.57
2020	568,000	23.18	5.5	1.11507	25.85
2021	576,000	24.94	4.5	1.09320	27.26
2022	568,000	25.94	3.5	1.07177	27.80
2023	556,000	26.80	2.5	1.05075	28.16

Selection: 27.74
Justification: Select the latest 3 years average due to the increase over the last 3 years.

Notes: (4) = 40%×(General Expenses)
(5) = (4) / (Earned Exposures)
(7) = $1.02^{(6)}$
(8) = (5)(7)

7. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3h) Explain the effect of changing conditions on the projection methods cited in (3e).
(3i) Assess the appropriateness of the projection methods cited in (3e) in varying circumstances.
(3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 21 and 22.

Commentary on Question:

This question tests the candidate's understanding of changing conditions on different projection methods.

Solution:

(a) Provide two reasons why actuaries use multiple methods to estimate ultimate claims.

Any two of the following are acceptable:

- where required by actuarial standards
- each method has different underlying assumptions, none of which are usually perfectly true
- to allow the results of different methods to be compared
- to better reflect the complexities of the business being modelled
- to identify sensitivity to the underlying assumptions

(b) Provide two areas in which an actuary can exercise professional judgement in estimating ultimate claims, other than the selection of methods.

Any two of the following are acceptable:

- reviewing reasonableness of results
- choosing assumptions (e.g., trend)
- assessing reasonableness of information provided
- evaluating estimates from different methods
- final selection of estimates
- determining relevance of information
- what relevant information is sufficient
- whether to supplement available information
- whether to assess the needs of users
- the level and detail of documentation

7. Continued

(c) Explain how effective each of the following projection methods will be in responding to the recent changes at XYZ:

(i) Paid development method

change in claim ratio	responsive
volume change	responsive
change in reporting pattern	will distort factors, so not responsive
overall	should be mostly responsive

(ii) Expected method

change in claim ratio	not responsive
volume change	responsive
change in reporting pattern	not responsive, but results will be fine if expected ultimate claim ratio is adjusted.
overall	likely not responsive

(iii) Reported Bornhuetter Ferguson method

change in claim ratio	not responsive
volume change	responsive
change in reporting pattern	not responsive
overall	likely not responsive

8. Learning Objectives:

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(5e) Calculate trend factors for claims and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 26.

Commentary on Question:

This question tests the calculation of pure premium trend, as well as considerations when selecting data points to include in trending procedures.

Solution:

(a) Describe one reason for relying on a longer period of time when trending a long-tailed line of business.

One reason for using more data points is to account for the greater uncertainty inherent in the projection of ultimate claims for long-tail coverages, particularly for the most recent years in the experience period.

(b) Provide an example where a longer period of time may not be appropriate for trending a long-tailed line of business.

Due to potential changes in coverage as well as in the economic, regulatory, and legal environments over time.

8. Continued

(c) State two considerations when selecting which data points to include in trending procedures.

Any two of the following are acceptable:

- Balance the need for stability with the need for responsiveness to the most recent experience.
- Assign greater weight to the most recent experience for short-tail lines of business.
- Have a sufficient number of data points in the experience period to determine a pattern for the annual change.
- Consider both long-term and short-term trend indications for long-tail lines of business.
- Consider the effect of changes in coverage, economic, regulatory and legal environments over time.
- The experience of the most recent data points may be too immature for long-tail lines of business.
- Consider excluding outliers.

(d) Calculate the pure premium trend factor for accident year 2022.

months trending period:

12-month policies (given):	45
6-month policies:	42
Weighted average # months:	43.2 (0.4×45 + 0.6×42)

Exponent = 43.2 / 6 = 7.2

Pure premium trend factor = $e^{(0.045\times 7.2)} \bullet e^{(-0.007\times 7.2)} = 1.31469$

9. Learning Objectives:

4. The candidate will understand financial reporting of claim liabilities and premium liabilities.

Learning Outcomes:

(4a) Describe the key assumptions underlying ratio and count-based methods for estimating unpaid unallocated loss adjustment expenses.

(4b) Estimate unpaid unallocated loss adjustment expenses using ratio and count-based methods.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapter 23.

Commentary on Question:

This question tests the candidate's understanding of estimating unpaid ULAE using the classical paid-to-paid method, as well as the Wendy Johnson count-based method.

Solution:

(a) Describe why unallocated loss adjustment expenses (ULAE) are usually analyzed on a calendar year basis.

ULAE is not assigned to specific claims, therefore, no accident year can be assigned.

(b) Describe a weakness of the classical paid-to-paid method that the Kittel refinement is intended to address.

ULAE are not simply associated with the payment of claims, but other activity as well.

(c) Describe these two major drawbacks.

Ratio-based methods do not recognize that the amount of ULAE does not depend solely on the magnitude of the total claims in the portfolio.

ULAE from ratio-based methods will fluctuate in response to changes in the estimates of unpaid claims.

9. Continued

(d) Estimate unpaid ULAE as of December 31, 2023 using the classical paid-to-paid method.

Calendar Year	Paid Claims	Paid ULAE	Paid-to-Paid ULAE Ratio
2021	30,400,000	1,489,600	4.9%
2022	31,698,113	1,680,000	5.3%
2023	28,000,000	1,596,000	5.7%
	90,098,113	4,765,600	5.3%
		Selected:	5.5%

	As of Dec. 31, 2023	Expense Multiplier	Unpaid ULAE
Case Reserves	19,507,585	75%	804,688
IBNER	7,861,668	75%	324,294
IBNYR	4,812,040	100%	264,662
	41,919,318		1,393,644

(e) Demonstrate that the projected open counts for calendar years 2024, 2025, and 2026 are calculated correctly based on newly reported claims and closed claims.

Calendar Year	Projected Open Counts
2024	1,044
2025	323
2026	0

e.g., 1,044 = 1,402 + 1,067 – 1,425

9. Continued

(f) Estimate unpaid ULAE as of December 31, 2023 using the Wendy Johnson method.

Calendar Year	Paid ULAE	Counts: Newly Reported	Counts: Open	Counts: Closed	Counts: Weighted Total	Avg ULAE Per Weighted Count
Weights:		30%	50%	20%		
2021	1,489,600	2,325	1,336	2,370	1,840	810
2022	1,680,000	2,550	1,391	2,495	1,960	857
2023	1,596,000	2,528	1,402	2,517	1,963	813
				Selected Average ULAE per Weighted Count:		827

e.g., 810 = 1,489,600 / 1,840

Calendar Year	Counts: Newly Reported	Counts: Open	Counts: Closed	Counts: Weighted Total	Trending Period in Years	Prospective Trend Factor	Trended Avg. ULAE	Estimated Unpaid ULAE
2024	1,067	1,044	1,425	1,127	1	1.0200	843	950,475
2025	122	323	843	367	2	1.0404	860	315,420
2026	-	-	323	65	3	1.0612	877	56,678
Total								1,322,572

e.g., $843 = 827 \times 1.02$
$950{,}475 = 843 \times 1{,}127$

10. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(2a) Create development triangles of claims and counts from detailed claim transaction data.

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 11 and 15.

Commentary on Question:

This question tests the candidate's understanding of certain details of individual insurance policies and ability to make correct calculations of earned exposures, earned premium, unearned premium and written premium for various policies. The candidate also needs to understand earned premiums adjusted to current rate level.

Solution:

(a) Provide an example of another line of business that often has a long lag between the occurrence date and the report date.

Any of the following are acceptable:

- Errors & Omissions
- Medical malpractice
- Any type of bodily injury liability only coverage

(b) Provide an example of a line of business where claim files are commonly reopened.

Any of the following are acceptable:

- Workers compensation
- Any type of bodily injury liability only coverage

10. Continued

(c) Construct a revised cumulative paid claims triangle adjusted for the legislative change.

Commentary on Question:
It is necessary to start with incremental paid claims, as the reform affects claims paid after a certain date and not the cumulative of all claims paid to that date.

Accident Year	Incremental Paid Claims					
	12	24	36	48	60	72
2018	1,518,006	1,766,528	1,553,804	1,308,213	798,483	204,638
2019	1,582,770	1,969,314	1,523,378	1,064,621	903,118	
2020	1,573,601	2,034,384	1,315,593	1,284,989		
2021	1,608,502	1,795,820	1,492,737			
2022	1,448,977	1,890,519				
2023	1,791,306					

Accident Year	Adjustment Factors for Tort Reform					
	12	24	36	48	60	72
2018	0.80	0.80	0.80	0.90	1.00	1.00
2019	0.80	0.80	0.90	1.00	1.00	
2020	0.80	0.90	1.00	1.00		
2021	0.90	1.00	1.00			
2022	1.00	1.00				
2023	1.00					

Accident Year	Adjusted Incremental Paid Claims = Incremental Paid Claims × Adjustment Factors					
	12	24	36	48	60	72
2018	1,214,405	1,413,222	1,243,043	1,177,392	798,483	204,638
2019	1,266,216	1,575,451	1,371,040	1,064,621	903,118	
2020	1,258,881	1,830,946	1,315,593	1,284,989		
2021	1,447,652	1,795,820	1,492,737			
2022	1,448,977	1,890,519				
2023	1,791,306					

Accident Year	Adjusted Cumulative Paid Claims					
	12	24	36	48	60	72
2018	1,214,405	2,627,627	3,870,670	5,048,062	5,846,545	6,051,183
2019	1,266,216	2,841,667	4,212,707	5,277,328	6,180,446	
2020	1,258,881	3,089,826	4,405,419	5,690,408		
2021	1,447,652	3,243,472	4,736,209			
2022	1,448,977	3,339,496				
2023	1,791,306					

10. Continued

(d) Verify the projected ultimate claims for accident years 2024 and 2025.

Accident Year	Projected Frequency	Projected Counts	Projected Severity	Projected Ultimate Claims
2024	10.57%	1,120.23	6,342.50	7,105,054
2025	10.54%	1,128.46	6,818.19	7,694,043

e.g., $10.57\% = 10.6\% \times (1 - 0.3\%)$
$1{,}120.23 = 10.57\% \times 10{,}600$
$6{,}342.50 = 5{,}900 \times (1 + 7.5\%)$
$7{,}105{,}054 = 1{,}120.23 \times 6{,}342.50$

(e) Calculate the claims expected to be paid in calendar years 2024 and 2025, using the results from part (c).

Commentary on Question:
Age-to-ultimate factors are calculated by dividing the given ultimate claims by cumulative paid claims to date (i.e., the latest diagonal).

Accident Year	Cumulative Paid Claims						Projected Ultimate Claims
	12	24	36	48	60	72	
2018	1,518,006	3,284,534	4,838,338	6,146,551	6,945,034	*7,149,672*	7,149,672
2019	1,582,770	3,552,084	5,075,462	6,140,083	*7,043,201*	7,289,724	7,289,724
2020	1,573,601	3,607,985	4,923,578	*6,208,567*	7,231,724	7,484,846	7,484,846
2021	1,608,502	3,404,322	*4,897,059*	6,280,054	7,314,992	7,571,028	7,571,028
2022	1,448,977	*3,339,496*	4,873,746	6,250,157	7,280,168	7,534,985	7,534,985
2023	*1,791,306*	4,087,339	5,965,167	7,649,810	8,910,480	9,222,361	9,222,361
2024	1,380,051	3,148,951	4,595,660	5,893,535	6,864,776	7,105,054	7,105,054
2025	1,494,453	3,409,990	4,976,627	6,382,093	7,433,847	7,694,043	7,694,043

	12-24	24-36	36-48	48-60	60-72	72-84
Age-to-age:	2.282	1.459	1.282	1.165	1.035	1.000
Age-to-ult:	*5.148*	*2.256*	*1.546*	*1.206*	*1.035*	*1.000*

e.g., $5.148 = 9{,}222{,}361 / 1{,}791{,}306$
$2.282 = 5.148 / 2.256$

10. Continued

Accident Year	Incremental Paid Claims					
	12	24	36	48	60	72
2018	1,518,006	1,766,528	1,553,804	1,308,213	798,483	204,638
2019	1,582,770	1,969,314	1,523,378	1,064,621	903,118	246,523
2020	1,573,601	2,034,384	1,315,593	1,284,989	1,023,157	253,122
2021	1,608,502	1,795,820	1,492,737	1,382,995	1,034,938	256,036
2022	1,448,977	1,890,519	1,534,250	1,376,411	1,030,011	254,817
2023	1,791,306	2,296,033	1,877,828	1,684,643	1,260,671	311,881
2024	1,380,051	1,768,900	1,446,709	1,297,876	971,241	240,278
2025	1,494,453	1,915,537	1,566,637	1,405,466	1,051,754	260,196

CY2024 paid claims: 7,863,009
CY2025 paid claims: 7,805,652

11. Learning Objectives:

2. The candidate will demonstrate the ability to prepare claims and exposure data for general insurance actuarial work.

5. The candidate will understand trending procedures as applied to ultimate claims, exposures and premiums.

Learning Outcomes:

(2d) Adjust historical earned premiums to current rate levels.
(5b) Identify the time periods associated with trending procedures.
(5e) Calculate trend factors for claims and exposures.

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 13 and 27.

Commentary on Question:

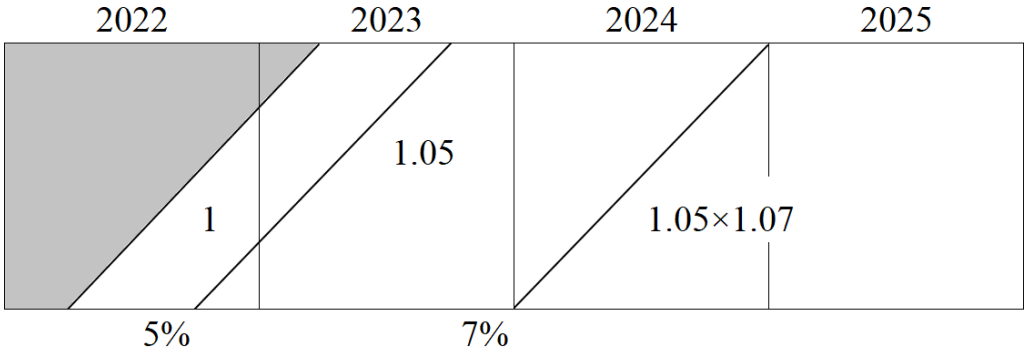
This question tests the candidate's ability to adjust premium to current rate levels and adjust premiums for trend for ratemaking purposes.

Solution:

(a) Calculate the on-level premium factors for calendar year 2022 and 2023.

Commentary on Question:

Since the company started writing a new line of business on March 1, 2022, the shaded area in the diagram below has no earned premiums and should not be included in estimating the percent of premiums earned in each calendar year.



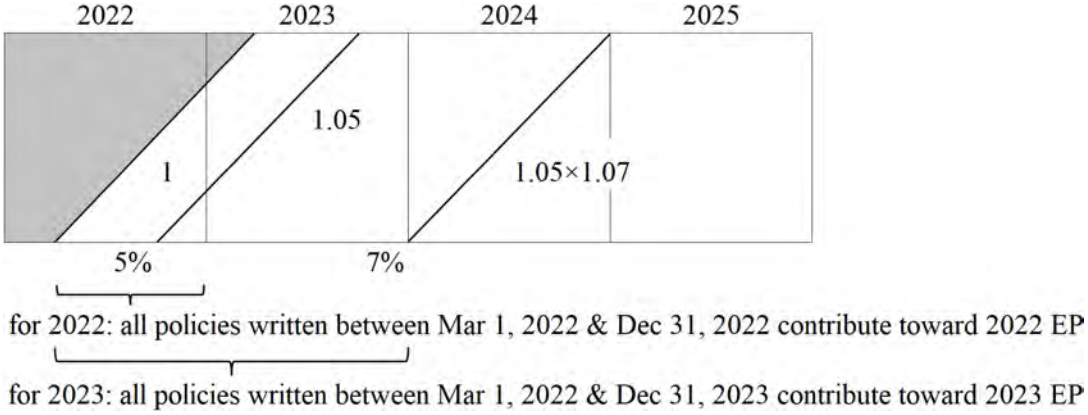
11. Continued

Rate Change History			Percent Premium Earned in Each Calendar Year (CY) at Rate Level	
Effective Date of Rate Change	Rate Change %	Rate Level Index	2022	2023
Prior to Mar 1/22			65.28%	1.39%
		1.00000	29.17%	20.83%
1-Sep-22	5%	1.05000	5.56%	77.78%
1-Jan-24	7%	1.12350	-	-
Total			34.72%	98.61%
Average Rate Level in each CY:			1.00800	1.03944
On-Level Factors:			1.1146	1.0809

Notes:

- $5.56\% = 0.5(4/12)^2$
- $29.17\% = 0.5(10/12)^2 - 5.56\%$
- Avg rate level in CY 2022 = (1.0000×29.17% + 1.0500×5.56%) / 34.72% = 1.0080
- $20.83\% = 0.5(8/12)^2 - 0.5(2/12)^2$
- $77.78\% = 1 - 0.5(8/12)^2$
- Avg rate level in CY 2023 = (1.0000×20.83% + 1.0500×77.78%) / 98.61% = 1.0394

(b) Calculate premium trend factors for calendar year 2022 and 2023.



Trend from average written date in experience period to average written date in future rating period.

11. Continued

Calendar Year	Average Written Date in Experience Period	Average Written Date Rating Period	Trending Period in Months	Trending Period in Years	Trend Factor
2022	1-Aug-22	1-Oct-25	38	3.167	0.98425
2023	1-Feb-23	1-Oct-25	32	2.667	0.98672

e.g., $0.98425 = (1 - 0.005)^{3.167}$

12. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3f) Demonstrate knowledge of good practice related to projecting ultimate values.
(3g) Estimate ultimate values using the methods cited in (3e).
(3j) Evaluate and justify selections of ultimate values based on the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 18, 18, 19, and 22.

Actuarial Standards of Practice, Actuarial Standards Board of the American Academy of Actuaries, No. 25, Credibility Procedures, 2013.

Commentary on Question:

This question tests the candidate's understanding of estimating ultimate claims using the development method, the Bornhuetter Ferguson method, and the Cape Cod method, including adjustments because of a large claim.

Solution:

(a) Calculate projected ultimate claims for all accident years using the development method.

Accident	Reported Claims, Adjusted for Large Claim				
Year (AY)	12	24	36	48	60
2019	540,061	575,731	648,087	683,622	702,734
2020	554,275	591,019	665,056	701,405	
2021	567,907	***606,134***	***681,837***		
2022	581,936	621,002			
2023	596,836				

	Reported Claims Age-to-Age factors				
AY	12-24	24-36	36-48	48-60	60-ultimate
2019	1.066	1.126	1.055	1.028	
2020	1.066	1.125	1.055		
2021	1.067	1.125			
2022	1.067				
Simple All Years Avg.	1.067	1.125	1.055	1.028	
Vol. Wtd. Avg.	1.067	1.125	1.055	1.028	
Selected	1.067	1.125	1.055	1.028	1.028
Age-to-ult. (CDF)	1.338	1.254	1.115	1.057	1.028

12. Continued

AY	Reported Claims Excluding Large Claim	Age-to-Ultimate Development Factors	Ultimate Claims Excluding Large Claim	Ultimate Claims Including Large Claim
2019	702,734	1.028	722,380	722,380
2020	701,405	1.057	741,172	741,172
2021	681,837	1.115	759,935	***809,935***
2022	621,002	1.254	778,838	778,838
2023	596,836	1.338	798,464	798,464
Total	3,303,814		3,800,789	3,850,789

(b) Critique the appropriateness of selecting the development method for this line of business.

The method seems appropriate after adjusting for large loss because development factors are relatively stable.

(c) Calculate projected ultimate claims for all accident years using the Bornhuetter Ferguson method.

Commentary on Question:
The claim ratio to use is the given expected claim ratio based on industry data of 65% for all accident years.

AY	% Claims Unreported	Reported Claims	Earned Premiums	Claim Ratio	BF Method Ultimate Claims
2019	0.027	702,734	1,000,000	65%	720,412
2020	0.054	701,405	1,040,000	65%	737,675
2021	0.103	731,837	1,082,000	65%	804,115
2022	0.203	621,002	1,125,000	65%	769,194
2023	0.253	596,836	1,170,000	65%	788,877
Total		3,353,814	5,417,000		3,820,273

e.g., $804{,}115 = 731{,}837 + 0.103 \times 1{,}082{,}000$

(d) Critique the appropriateness of selecting the Bornhuetter Ferguson method for this line of business.

The BF method may not be appropriate as management is uncertain that expected claim ratio from industry data is representative of this book of business. Alternatively, the BF method correctly reflects large claim.

12. Continued

(e) Calculate projected ultimate claims for all accident years using the Cape Cod method.

	Rate Level	Percent Premium Earned in Each CY at Rate Level				
	Index	2019	2020	2021	2022	2023
	1.00000	100.00%	100.00%	100.00%	50.00%	-
	0.95000	-	-	-	50.00%	100.00%
	Total	100.00%	100.00%	100.00%	100.00%	100.00%
Average rate level in each CY:		1.0000	1.0000	1.0000	0.9750	0.9500
On-level factors:		0.9500	0.9500	0.9500	0.9744	1.0000

	(1)	(2)	(3) = (1)(2)	(4)	(5) = 1/(4)	(6) = (3)(5)
AY	Earned Premiums	On-level Factors	On-Level Earned Premiums	Reported CDF	Expected % Claims Reported	Used Up On-Level Earned Premiums
2019	1,000,000	0.9500	950,000	1.028	97.3%	924,163
2020	1,040,000	0.9500	988,000	1.057	94.6%	934,990
2021	1,082,000	0.9500	1,027,900	1.115	89.7%	922,263
2022	1,125,000	0.9744	1,096,154	1.254	79.7%	874,012
2023	1,170,000	1.0000	1,170,000	1.338	74.7%	874,552
Total	5,417,000		5,232,054			4,529,980

	(7)	(8)	(9)=(7)(8)	(10) = 0.6995×(3)/(8)
AY	Reported Claims Excluding Large Claim	Claim Trend Factors	Adjusted Claims	Expected Claims
2019	702,734	0.9224	648,179	720,426
2020	701,405	0.9412	660,157	734,258
2021	681,837	0.9604	654,836	748,632
2022	621,002	0.9800	608,582	782,376
2023	596,836	1.0000	596,836	818,381
Total	3,303,814		3,168,590	3,804,073

Adjusted expected claim ratio = 3,168,590 / 4,529,980 = 0.6995

12. Continued

AY	(11) = 1 – (5) Expected % Unreported	(12) = (10)(11) Expected Unreported Claims	(13) = (7) + (12) Ultimate Claims Including Large Claim
2019	2.7%	19,593	722,327
2020	5.4%	39,396	740,801
2021	10.3%	76,937	***808,774***
2022	20.3%	158,553	779,555
2023	25.3%	206,657	803,493
Total		501,136	3,854,950

Note: for AY2021 = 681,837 + 76,937 + 50,000 = 808,774

(f) Critique the appropriateness of selecting the Cape Cod method for this line of business.

Any of the following is acceptable:

- Method is appropriate for newer lines of business (immature experience periods)
- Method is better than BF because expected claim ratio is experience-based
- Method is appropriate because it explicitly adjusts for trend
- Method is appropriate but usually used for longer-tailed lines

13. Learning Objectives:

3. The candidate will know how to calculate and evaluate projected ultimate values.

Learning Outcomes:

(3e) Describe the key assumptions underlying the following projection methods: development method, frequency-severity methods, expected method, Bornhuetter Ferguson method, Benktander method, Cape Cod method, Generalized Cape Cod, and Berquist-Sherman adjustments to the development method.

(3g) Estimate ultimate values using the methods cited in (3e).

Sources:

Fundamentals of General Insurance Actuarial Analysis, Second Edition (2022), J. Friedland, Chapters 16 and 20.

Commentary on Question:

This question tests the candidate's understanding of frequency-severity method for estimating ultimate claims.

Solution:

(a) Recommend an annual claim frequency trend.

Accident Year (AY)	Frequency	Change from Court Ruling	Court Ruling Adjusted Frequency	Annual Trend
2017	0.04263	1.05	0.0448	
2018	0.04284	1.05	0.0450	0.51%
2019	0.04305	1.05	0.0452	0.48%
2020	0.04327	1.05	0.0454	0.51%
2021	0.04349	1.05	0.0457	0.50%
2022	0.04370	1.05	0.0459	0.49%
2023	0.04611	1.00	0.0461	0.49%
			All years average	0.50%
			Average excluding 2023	0.50%
			Recommended Trend	0.50%

13. Continued

(b) Calculate the projected ultimate claims for all accident years using the development-based frequency-severity method.

AY	Frequency	Severity	Frequency Trend @0.5%	Severity Trend @4.0%	Change from Court Ruling	Trended Frequency	Trended Severity	Ultimate Claims
2017	0.04263	28,747	1.0304	1.2653	1.05	0.0461	36,374.31	5,011,553
2018	0.04284	29,953	1.0253	1.2167	1.05	0.0461	36,442.61	5,451,088
2019	0.04305	31,137	1.0202	1.1699	1.05	0.0461	36,425.31	5,732,349
2020	0.04327	32,388	1.0151	1.1249	1.05	0.0461	36,432.17	6,219,950
2021	0.04349	33,732	1.0100	1.0816	1.05	0.0461	36,484.55	6,502,557
2022	0.04370	35,093	1.0050	1.0400	1.05	0.0461	36,496.90	7,148,701
2023	0.04611	36,175	1.0000	1.0000	1.00	0.0461	36,175.18	7,909,279
					All years average:	0.0461	36,404.44	
					Average excluding 2023:	0.0461	36,442.64	
					Selected:	0.0461	36,442.64	

e.g., Ultimate claims for AY2022:

$$7{,}148{,}701 = (0.0461)(4{,}668)(36{,}442.64)/(1.0050\times1.0400\times1.05)$$

(c) Describe how to calculate the *proportion of closed counts* triangle when using the frequency-severity closure method.

Instead of using development triangles with cumulative data, the closure method relies on triangles of incremental counts and claims. The proportion is the percent of counts closed of the remaining open counts.

(d) Describe how to calculate the triangle of *disposal ratios* when using the Berquist-Sherman adjustment for changing settlement rates.

The proportion closed counts is equal to the ratio of the counts closed at each maturity age to the counts remaining as of the prior maturity age. The disposal ratio is the percent closed of ultimate counts.