ALTAM October 2024 Model Solutions

Question 1

The solution to this question is in the spreadsheet. It should be noted that as stated in the instructions for the exam, only work in the spreadsheet will be graded. Any work on paper is NOT graded for Excel problems.

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Question 2

(a)

(i) Prob. of a claim $= \dfrac{d_{40}^{(1)} + \ldots + d_{44}^{(1)} + d_{40}^{(2)} + \ldots + d_{44}^{(2)}}{l_{40}} = \dfrac{264 + 637}{100,000} = 0.0901$

(ii) Prob. claim is from CI $= \dfrac{637}{901} = 0.707$

(b)

(i) The independent rate of lapse at the year-end is

$$q_{60}^{*(3)} = \frac{d_{60}^{(3)}}{l_{60} - d_{60}^{(1)} - d_{60}^{(2)}} = \frac{400}{10,000 - 50 - 105} = \frac{400}{9845} = 0.04063$$

(ii) The Multiple Decrement Table from age 60 to just before age 61 is

x	l_x	$d_x^{(1)}$	$d_x^{(2)}$
60	10,000	50	105
61^-	9,845		

With constant forces for decrements (1) and (2) we have independent survival probabilities:

$$p_{60}^{*(1)} = \left(p_{60}^{00}\right)^{\frac{p_{60}^{01}}{p_{60}^{0\bullet}}} = \left(p_{60}^{00}\right)^{\frac{d_{60}^{(1)}}{d_{60}^{(1)} + d_{60}^{(2)}}} = (0.9845)^{\frac{50}{155}} = 0.994974$$

$$\Rightarrow q_{60}^{*(1)} = 0.005026$$

(iii) Similarly $p_{60}^{*(2)} = (0.9845)^{\frac{105}{155}} = 0.989474 \Rightarrow q_{60}^{*(2)} = 0.010526$

(c)

(i) Again, first consider exits up to just before the year-end lapses.

$$p_{60}^{00} = p_{60}^{*(1)} p_{60}^{*(2)} = (0.994974)(1 - 0.5(0.010526)) = 0.989737$$

$$\Rightarrow l_{61^-} = 10,000(0.989737) = 9,897.37$$

$$\Rightarrow d_{60}^{(3)} = 0.6(0.04063)(9897.37) = 241.3$$

(ii) Fewer lives will exit through CI diagnosis, leaving more lives exposed to the risk of death. Hence $d_{60}^{(1)}$ will increase.

Examiners' Comments

Part A

Most candidates did well on part A. Those who didn't either mistakenly used just a single age or used the surviving lives at each age as the denominator rather than the beginning lives.

Part B

There were a few common mistakes on part B. Some candidates mistakenly included lapses as a constant force of decrement instead of occurring at the year end. Others incorrectly assumed a uniform distribution.

Part C

On part (i), the most common mistake was changing the force of mortality rather than the q's directly themselves. Candidates lost points in part (ii) for asserting that the reduction in the lapse rate impacted $d_{60}^{(1)}$*; it doesn't since lapses occur at the end of the year.*

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Question 3

(a)

$$\ddot{a}^{(12)}_{65|65} = \ddot{a}^{(12)}_{65} - \ddot{a}^{(12)}_{65:65} \approx \left(\ddot{a}_{65} - \frac{11}{24}\right) - \left(\ddot{a}_{65:65} - \frac{11}{24}\right) = \ddot{a}_{65} - \ddot{a}_{65:65}$$

$$=13.5498\text{-}11.6831=1.8667$$

(b)

$$AL = \left(1100v^{40} + (7400 + 10,500)\,v^{20} + (20,000 + 25,000)\,v^{10}\right)\left(\ddot{a}^{(12)}_{65} + 0.75\,\ddot{a}^{(12)}_{65|65}\right)$$

$$+ 25,000\left(\ddot{a}^{(12)}_{70} + 0.75\,\ddot{a}^{(12)}_{70|70}\right)$$

$$= 34,528.67\left(13.5498 - \frac{11}{24} + 0.75(1.8667)\right)$$

$$+ 25,000\left(12.0083 - \frac{11}{24} + 0.75(12.0083 - 9.9774)\right)$$

$$= 34,528.67(13.0915 + 0.75(1.8667)) + 25,000(11.5500 + 0.75(2.0309))$$

$$= 500,373 + 326,829 = 827,202$$

(c)

The increase in the accrued benefit in 2024 is α=2% of the 2024 salary, for each active employee.

The expected value at 1/1/24 of the increase in the AL in 2024 is

$$(0.02)(1.02)\left(30,000v^{40} + 80,000v^{20} + 100,000v^{10}\right)\left(\ddot{a}^{(12)}_{65} + 0.75\,\ddot{a}^{(12)}_{65|65}\right)$$

$$= 28,322.22$$

Expressed as a % of 2024 payroll, the contribution rate is $\dfrac{28,322.22}{(1.02)(210,000)} = 13.22\%$

(d)

(i) The new accrued benefits are $\alpha \times n \times S_x$ where $\alpha = 0.02$ is the accrual rate, n is past service, and S_x is the salary in 2023, so the new accrued benefits are, by employee

A: (0.02)(2)(30,000) = 1,200
B: (0.02)(10)(40,000) = 8,000
C: (0.02)(15)(40,000) = 12,000
D: (0.02)(25)(50,000) = 25,000
E: (0.02)(30)(50,000) = 30,000

So the new aggregate AL for active members is

$$AL_{actives} = \left(1200v^{40} + 20,000v^{20} + 55,000v^{10} \right) \left(\ddot{a}_{65}^{(12)} + 0.75\, \ddot{a}_{65|65}^{(12)} \right)$$
$$= 41,473.5(13.0915 + 0.75(1.8667))$$
$$= 601,014$$

So the increase is $601,014 - 500,373 = 100,641$

(ii) The NC pays for EPV of the increase in the accrued benefit during the valuation year. The 2024 accruals are unchanged, so the normal contribution is unchanged.

(e)

Under this model, the member's future lifetime distribution is the same as in the independent model, so there is no change to the EPV of their own pension.

However, the possibility of a common shock reduces the expected period of payment of spousal benefits.

Hence the AL would decrease.

Examiners' Comments

Overall, the candidates did poorly on this question. It seems like this is true of every exam. There will always be one pension question on the exam. It is short sighted to not know this material.

Candidates should pay attention to pension questions and understand different defined benefit plans instead of memorizing formulas. Candidates should be able to appreciate the differences of different plans and apply right formulas to tackle the questions and be able to interpret the change in the computed result from changed assumptions.

Pension question usually has a lot of calculations. Since Excel is available, it might be a good idea to use Excel to do the calculations and write down your calculations on the paper answer sheet. Do not just write down the final answer on your answer sheet as this will receive little credit.

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Question 4

(a)

- Set premium to meet a profit objective
- Set or analyze sufficiency of reserves
- Assess profit sharing mechanisms (e.g. for participating or variable insurance)
- Explore implications of changing lapse experience
- Stress testing portfolio or company-wide cashflow projections

(b)

(i)

t	${}_{t-1}V$	P	E	I	EDB	E ${}_tV$	Pr
0			1000			100	-1100
1	100	4300	315	245.1	3398.2	91.7	840.2
2	100	4300	315	245.1	3791.6	0	538.5

(ii)

$\Pi_0 = Pr_0 = \text{-}1100$

$\Pi_1 = Pr_1 = 840.2$

$\Pi_2 = p_{60}^{(\tau)}\, Pr_2 = (0.99660)(0.92)(538.5) = 493.7$

(c)

(i) The NPV is $-1100 + 840.2v_{10\%} + 493.7v_{10\%}^2 \quad = 71.9$

(ii) The IRR is greater than the hurdle rate; the IRR is rate at which NPV =0; NPV is a decreasing function of the hurdle rate.

(iii) $$PM = \frac{NPV}{EPV\text{ Premiums}} = \frac{71.86}{4300\ddot{a}_{x:\overline{2|}}} = \frac{71.86}{4300(1.83352)} = 0.91\%$$

(d) Now assume unknown premium *P*:

$$NPV = -1100 + \{(P(0.95)(1.06) - (3398.2 + 91.69)\}v_{10\%} + \{(P(0.95)(1.06) - 3791.6\}v_{10\%}^2 p_{60}^{(\tau)}$$

$$=P(0.95(1.06))\left(v_{10\%} + p_{60}^{(\tau)} v_{10\%}^2\right) - 7145.71$$

$$= 1.67851P - 7145.71$$

also, the profit margin is $pm = 0.05$

$$\Rightarrow 0.05 = \frac{NPV}{P(1.8335)} = \frac{1.67851P - 7145.71}{P(1.8335)}$$

$$\Rightarrow P(-0.05(1.8335) + 1.67851) = 7145.71$$

$$\Rightarrow P = \frac{7145.71}{1.58679} = 4503.2$$

Examiners' Comments

Part A

Candidates performed well on this part and were able to successfully identify two uses of profit testing in the insurance practice. The most successful candidates listed uses other than to test profit.

Part B

In general, candidates did well on this part. Not all candidates considered the reserve of 100 in the profit calculation at time zero and adjusted the profit signature at time two for both voluntary lapse and mortality.

Part C

Candidates performed well on this part. The most successful candidates knew to discount the profit signature by the hurdle rate, identified that the IRR is greater than the hurdle rate because the IRR is the rate at which the NPV equals zero, and recalled that the profit margin represents the net present value of the contract divided by the expected present value of premiums.

A common mistake was to not consider voluntary lapse in the expected present value of premiums calculation.

Candidates that carried over an error in part b were not penalized twice for that error if the approach was consistent with that prior error. Candidates that tried to show that the net present value was equal to 70 when a prior error would not lead to a net present value of 70 were penalized for this approach.

Part D

The most successful candidates utilized the profit margin formula to solve for the gross premium to achieve a profit margin of 5%. Successful candidates identified that the net present value of the contract needed to be adjusted. Candidates that utilized goal seek or trial and error in Excel and gave a detailed description of the work they completed in Excel were given full credit or the credit demonstrated on the paper answer. The excel file is not reviewed or graded for questions 2-6.

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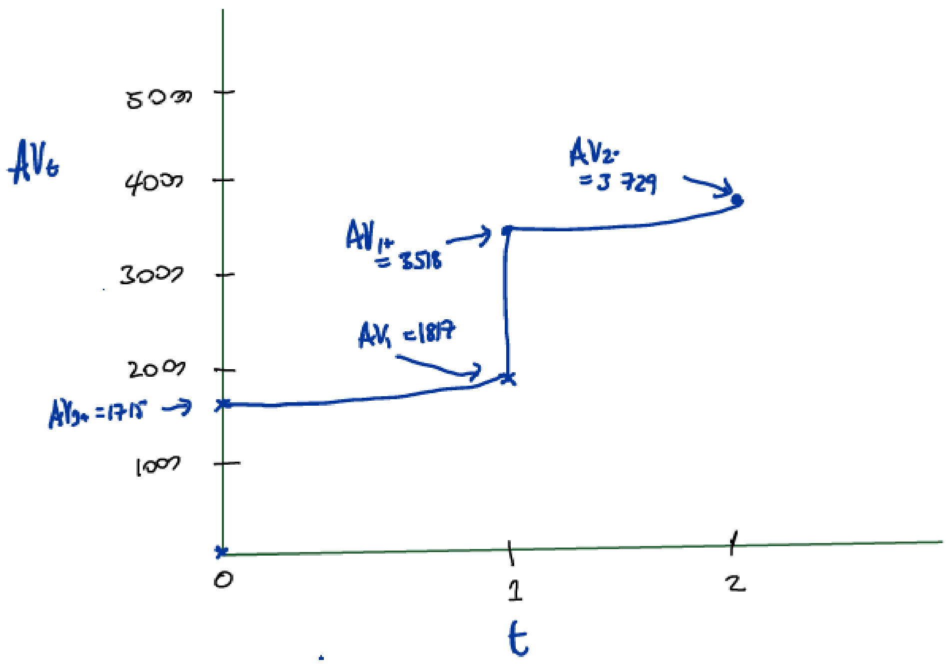
Question 5

(a)

(i) $$AV_1 = \left(0.98P - 100 - 1.2q_{50}(100{,}000)\right)(1.06) = 1817.88$$

$$AV_2 = \left(1817.88 + 0.98P - 100 - 1.2q_{51}(100{,}000)\right)(1.06) = 3729.24$$

(ii)



(b)

$$0.02P + 100 + 1.2q_x(100{,}000) > 2000$$

$$\Rightarrow q_x > \frac{2000 - 40 - 100}{1.2(100{,}000)} = 0.0155$$

$$\Rightarrow x > 73$$

So the youngest age is 74.

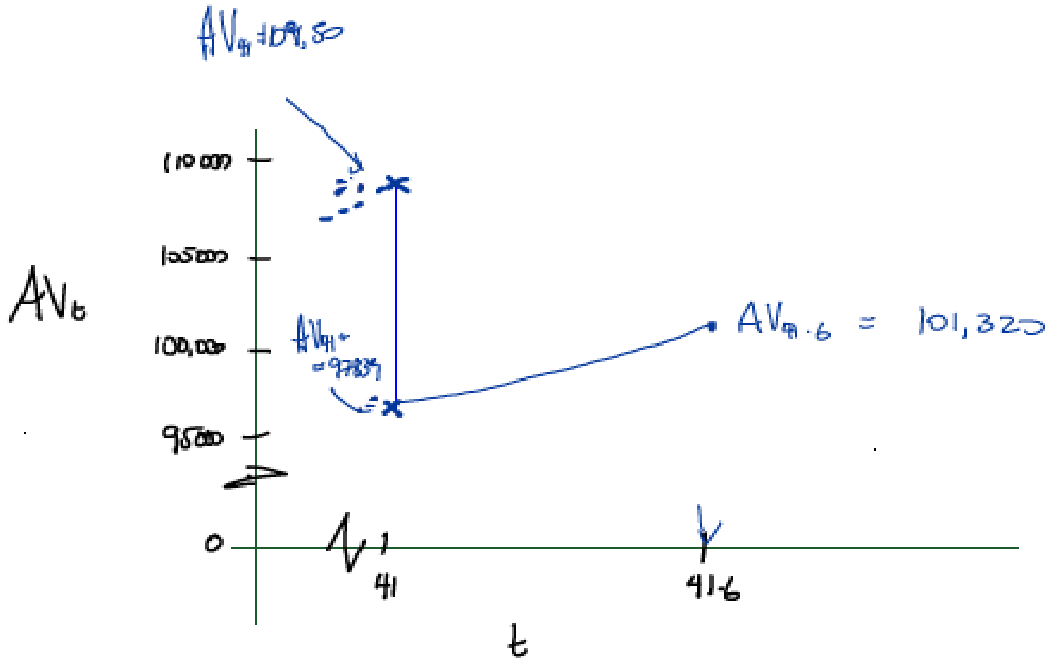
(c)

(i) $AV_{41.6} = \left(109,500 + 0.98(2000) - 100 - 1.2q_{91}(100,000)\right)\left(1.06\right)^{0.6}$

$=101,320$

$DB = AV_{41.6} + ADB = 201,320$

(ii)



(d) Let $\hat{q}_y = 1.2q_y$ and $\hat{p}_y = 1 - \hat{q}_y$

(i) Single Life: $COI = \hat{q}_{x+t}\left(100,000\right)$

Joint Life: $COI = 2\hat{q}_{x+t}\hat{p}_{x+t}\left(50,000\right) + \left(\hat{q}_{x+t}\right)^2\left(100,000\right)$

$$= \hat{q}_{x+t}\left(1 - \hat{q}_{x+t}\right)\left(100,000\right) + \left(\hat{q}_{x+t}\right)^2\left(100,000\right)$$

$$= \hat{q}_{x+t}\left(100,000\right)$$

(ii) They will be the same, as the COI will be the same each year (as in the first year calc above).

(iii) The AVs for the joint life will be greater than for the single life as the ADB and COI will be smaller (ADB is 50,000 instead of 100,000)

Examiners' Comments:

Overall, candidates were well-prepared for this question.

Part A

Candidates did very well on this part, understanding how account value increases and decreases. The best candidates made good graphs that showed vertical jumps in AV at time 0 and time 1, but unfortunately many candidates lost points here. A sizable portion of candidates did not reflect positive interest accumulation within the policy years. Some candidates also misinterpreted "continuous" interest to mean force of interest. Key features of the graph are $AV_0=0$, *jump to* $AV_{0+}=1715$ *, slow increase to* $V_1=1818$, *jump to* $AV_{1+}=3518$*, slow rise to* $AV_2=3729$*. Axes should be appropriately labelled.*

Part B

Candidates did very well on this part as well, understanding how policy deductions are calculated. Occasionally candidates showed how age 74's charges were sufficiently high but, regrettably, did not show or explain how age 73's charges were insufficiently high. Some candidates also misinterpreted "policy deductions" to mean only COIs when in fact other charges should have been included.

Part C

Candidates did reasonably well on this part. The great majority remembered to add the 100,000 face amount to the AV. Mistakes with the mortality charge -- such as applying only a fractional portion of COI charges -- and/or the 0.6 years of interest accumulation were fairly common. As with Part A, the vertical jumps in the graphing portion of this question were commonly misunderstood. Key features of the graph were $AV_{41}=109{,}467$*, jump down to* $AV_{41+}=97{,}839$*, gradual rise to* $AV_{41.6}=101{,}320$*.*

Part D

This part proved difficult for many candidates, including many of those who responded well to parts A through C. In subpart (i), candidates were expected to show more steps in their proofs. Candidates often mistakenly applied the 1.2 factor to the joint life and/or last survivor policy rather than the single life. Several candidates also tried to use the AV in their COI calculations. The best candidates included clear explanations for why their responses were true, specifically calling out the link between the policy's death benefit, its cost of insurance charges, and consequently its account value.

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Question 6

(a)

(i) The hedge cost is $\pi(0) = \left({}_{10}\, p_{60}\right) p(0)$

$$p(0) = Ke^{-rT}\Phi\left(-d_2(0)\right) - P\xi\, \Phi\left(-d_1(0)\right)$$

$K = 1000$; $T = 10$; $r = 0.03$; $\sigma = 0.20$; $\xi = e^{-0.02(10)} = 0.81873$

$$d_1 = \frac{\ln(P\xi/K) + \left(r + \sigma^2/2\right)T}{\sigma\sqrt{T}} = \frac{\ln(0.81873) + \left(0.03 + 0.2^2/2\right)10}{0.2\sqrt{10}} = 0.47434$$

$$d_2 = d_1 - \sigma\sqrt{T} = d_1 - 0.2\sqrt{10} = -0.158115$$

$$\Rightarrow p(0) = 1000e^{-0.03(10)}\ \Phi(0.15811) - 818.73\ \Phi(-0.47432)$$

$$= 1000e^{-0.03(10)}\ (0.562815) - 818.73\ (0.317636) = 156.89$$

$$\Rightarrow \pi(0) = {}_{10}p_{60}\, p(0) = \left(\frac{91,082.4}{96,634.1}\right)(156.89) = 0.94255\ (156.89) = 147.88$$

(ii) The continuous management charge to fund the hedge cost is

$$c = \frac{147.88}{1000\ \bar{a}_{60:\overline{10|}\ \delta=0.02}} = \frac{147.88}{1000\ (8.8569)} = 0.0167$$

(b)

(i) $F_3 = 1000e^{-0.02\times 3}(1.5) = 1412.65$

(ii) The hedge cost is $\pi(3) = {}_{7}p_{63}\, p(3)$ where

$$p(3) = Ke^{-7r}\Phi\left(-d_2(3)\right) - P\xi\ 1.5\Phi\left(-d_1(3)\right)$$

OR

$$p(3) = Ke^{-7r}\Phi\left(-d_2(3)\right) - F_3\, e^{-0.02(7)}\, \Phi\left(-d_1(3)\right)$$

$$d_1(3) = \frac{\ln(F_3 e^{-7(0.02)}/K) + \left(r + \sigma^2/2\right)7}{\sigma\sqrt{7}}$$

$$= \frac{\ln((1412.65)e^{-7(0.02)}/1000) + \left(0.03 + 0.2^2/2\right)7}{0.2\sqrt{7}} = 1.04973$$

$$d_2 = d_1 - \sigma\sqrt{7} = 1.04973 - (0.2)\sqrt{7} = 0.52058$$

$$\Rightarrow p(3) = 1000e^{-0.03(7)}\,\Phi(-0.52058) - P\xi\, 1.5\Phi(-1.04973)$$

$$= 1000e^{-0.03(7)}\,(0.30133) - (1228.10)(0.14692) = 63.81$$

$$\Rightarrow \pi(3) = {}_7p_{63}\, p(3) = \left(\frac{91{,}082.4}{95{,}534.4}\right)p(3) = 0.95340\,(63.81) = 60.84$$

(iii) The value of the future risk charge of 0.0167 is

$$F_3\, c\, \bar{a}_{63:\overline{7|}\delta=0.02} = (1412.65)(0.0167)(6.4021) = 151.03 > \pi(3)$$

The value of risk charge income > value of the guarantee.

Therefore the charge is sufficient.

(c)

(i) If the p/h resets at time 3:

- Their maturity date would be reset to time 13 (minimum 10 year contract)
- Their GMMB would increase to $F_3 = 1412.65$.
- All other policy conditions would (typically) remain the same.

(ii) Advantages:

- Encourage p/h not to surrender the policy when the guarantee is far out of the money.
- Discourage costly lapse and re-entry strategies.
- Useful for marketing, compared with policies that do not offer resets

Disadvantages:

- Keeps outdated contracts on the books
- Many p/h may reset at the same time, eg after a period of very strong returns, which means reset maturity dates will be set to around the same time, creating a time-concentration risk.
- If many p/h reset at the same time, it can create a spike in the guarantee cost of the portfolio, which could be a liquidity or solvency risk.

Examiners' Comments

Part A

Overall, candidates who attempted this part did acceptable on it. Many candidates omitted the entire question. Overall, candidates did a good job of documenting their work which helps in awarding partial credit.

The Black-Sholes formulas are given in the Formula Sheet. The hedge cost is a slight modification of this formula. Many candidates forgot to include the ${}_np_x$ *in the calculation. Also, since the interest rate was a continuous interest rate, the proper adjustment was* $e^{-0.02}$ *instead of 0.98. Use of 0.98 resulted in a small deduction on the question. Part (ii) was an easy calculation even if you could not get the correct answer to part (i). You were given the value of 150 and could use it to get full credit for part (ii). Candidates are encouraged to read the entire question and answer easier parts even if they cannot get the harder parts.*

Part B

Candidates did not do as well on this part. Quite a few candidates used 10 years in the calculations instead of the remaining 7 years. Also, candidates used the 1.5 increase in value without adjusting for the 2% annual charge. Unlike Part A, many candidates did not show sufficient work to earn much partial credit.

Some candidates attempted to use the replicating portfolio instead of the B-S formula to answer this question. A few candidates were successful, but most were not.

"State with reasons" means you must state the reasons to receive the credit. A lot of candidates just stated that the charge was sufficient with stating why.

Part C

Candidates actually did the best on part of the question with those attempting the question getting most or all of the points. This is another example where candidates are encouraged to read the entire question and answer easier parts even if they cannot get the harder parts.

Many candidates did not state that the maturity date would be reset to 13 years. This was necessary to receive full credit.

Part (ii) could be answered from the insurer's standpoint or the annuitant's standpoint. Please note that the question asks for the answer from the insurer's standpoint.